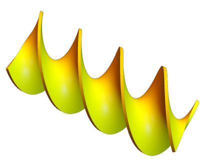
Homework 9: Parametrized Surfaces

This homework is due Monday, 9/30.

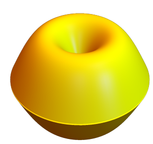
1 a) Identify the surface $\vec{r}(u,v) = [\sqrt{v}\sin(u), v\cos^2(u), \sqrt{v}]$.
b) Identity the surface $\vec{r}(s,t) = [t^4, s^{12} + t^8, s^6]$.

Solution:
a) It is a hyperbolic paraboloid $x^2 + y = z^2$.
b) It is an elliptic paraboloid because $y = x^2 + z^2$.

2 Match the following surfaces:



1



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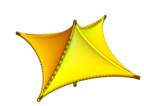

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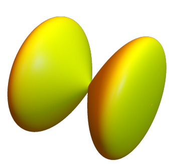

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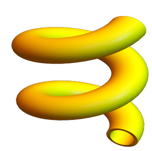

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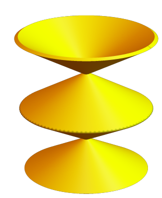


6

$\vec{r}(u,v) =$	1-6
$[(3+\cos(v))\cos(u), (3+\cos(v))\sin(u), \sin(v)+u]$	
$[\sin(v), \cos(u)\sin(2v), \sin(u)\sin(2v)]$	
$[\lvert 1-\lvert u\rvert\rvert\cos(v), \lvert 1-\lvert u\rvert\rvert\sin(v), u]$	
$[u\cos(v), u\sin(v), 2\sin(u)]$	
$[\cos(u)^3\cos(v)^3, \sin(u)^3\cos(v)^3, \sin(v)^3/2]$	
$[v, u\cos(v), u\sin(v)]$	

Solution:
5,4, 6, 2, 3,1

3 Find parametric equations for the surface obtained by rotating the curve $x = f(y) = 4y^2 - y^5, -2 \leq y \leq 2$, about the y-axis and use the graph of f to make a picture of the surface.

Solution:
Letting θ be the angle of rotation about the y-axis, we can see that the xz plane cross-sections are circles. Therefore, we can write the parametrization $x = (4y^2 - y^5)\cos\theta$, $y = y$, $z = (4y^2 - y^5)\sin\theta$, $-2 \leq y \leq 2$, $0 \leq \theta \leq 2\pi$.

4 Let us draw the surface with parametric equations

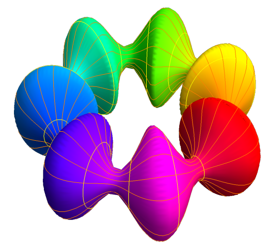
$$\vec{r}(u, v) = [(2+v\cos(u/2))\cos(u), (2+v\cos(u/2))\sin(u), -v\sin(u/2)]$$

with $-1 \leq v \leq 1$ and $0 \leq u \leq 2\pi$. It is called the Moebius strip. You might have seen it in the movie "Avengers, End Game". Please draw it or build it with paper. In the later case, you can also just glue your creation onto your paper and turn it in.

Solution:

5 Find a parametrisation of the **bumpy torus**, given as the set of points which have distance $5 + 2\cos(7\theta)$ from the circle $[10\cos(\theta), 10\sin(\theta), 0]$, where θ is the angle occurring in cylindrical and spherical coordinates.

Hint: Use r, the distance of a point (x, y, z) to the z-axis. This distance is $r = (10 + (5 + 2\cos(7\theta))\cos(\phi))$ if ϕ is a suitable angle. Make a picture to see also $z = (5 + 2\cos(7\theta))\sin(\phi)$. To finish the parametrization problem, translate back to Cartesian coordinates.



Solution:
$\vec{r}(\theta, \phi) = [(10 + (5 + 2\cos(7\theta))\cos(\phi))\cos(\theta), (10 + (5 + 2\cos(7\theta))\cos(\phi))\sin(\theta), (5 + 2\cos(7\theta))\sin(\phi)]$.

Main definitions

A **parametrization** of a surface is given by

$$\vec{r}(u, v) = [x(u, v), y(u, v), z(u, v)] ,$$

where $x(u, v), y(u, v), z(u, v)$ are three functions.

Plane: $\vec{r}(s, t) = \vec{OP} + s\vec{v} + t\vec{w}$
Sphere $\vec{r}(u, v) = [\rho\cos(u)\sin(v), \rho\sin(u)\sin(v), \rho\cos(v)]$.
Graph: $\vec{r}(u, v) = [u, v, f(u, v)]$
Surface of revolution: $\vec{r}(u, v) = [g(v)\cos(u), g(v)\sin(u), v]$