## Homework 31: Divergence Theorem

This last homework is due Monday, December 2, 2019. This is the last lecture.

1 Find the flux of the field $\vec{F}(x, y, z) = [z^3 + x^3, y^3 - 3yz^2, z^3 - \sin(x^3)]$ through the boundary of the solid bounded by paraboloid $z = 16 - x^2 - y^2$ and the xy-plane.

2 Find the flux of the vector field $\vec{F}(x, y, z) = [x^2y + \cos^6(y), xy^2, 2xyz + e^{\sin(x)}]$ through the outwards oriented solid bound by $x = 0, y = 0, z = 0$, and $x + 2y + z = 2$.

3 Compute the volume of the solid enclosed by the surface parametrized by $\vec{r}(u, v) = [\sin^5(v) \cos^5(u), \sin^5(v) \sin^5(u), \cos^5(v)]$ with $0 \leq u \leq 2\pi$ and $0 \leq v \leq \pi$. Use $F = [0, 0, z]$ in order not to have to compute too much. You then only need to compute the last component of $r_u \times r_v$! You can then use Mathematica to compute the integral. If you get a negative sign, you might have to blame the orientation.
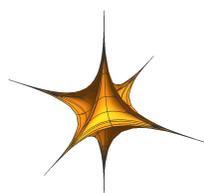
4 Find $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = [-x + \sin(y^5) + \cos(e^z), 55y + \sin(z) + \sin(z), z + \sin(x) + e^y]$ and S is the boundary of the Escher stair solid displayed in the picture. The right picture shows the same figure from an other angle leading to the illusion. Each brick is a cube of unit length 1.
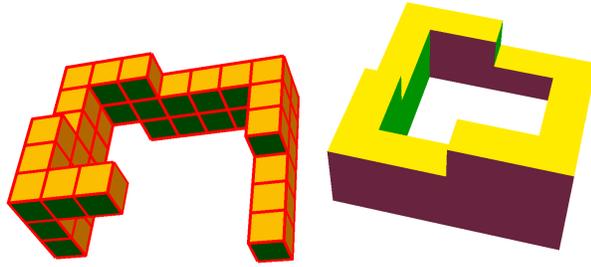
5 a) Use one of the integral theorems to evaluate $\int\int_S \text{curl}(\vec{F}) \cdot d\vec{S}$, where $\vec{F}(x,y,z) = [x^2yz, yz^2, z^3e^{xy}]$, where is the part of upwards oriented surface $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$.

b) Use one of the integral theorems to compute the line integral of $\vec{F}(x,y,z) = [x^3, y^5, 2z]$ along the path $\vec{r}(t) = [\cos(t) + t^{21}\sin(17t), \sin(t) + \sin(20t), t]$ from $t = 0$ to $t = 10\pi$.

Main points

Divergence Theorem. $\iiint_E \text{div}(\vec{F})\, dV = \iint_S \vec{F} \cdot d\vec{S}$. All integral theorems are incarnations of **the fundamental theorem of multivariable Calculus**

$$\int_G dF = \int_{\delta G} F$$

where dF is a **derivative** of F and δG is the **boundary** of G.