

MULTIVARIABLE CALCULUS

OLIVER KNILL, MATH 21A

Lecture 1: Graphs, Contours, Slices

FUNCTION

A **function of two variables** $f(x, y)$ assigns to a point (x, y) a value $z = f(x, y)$. For $f(x, y) = \frac{yx^3}{3} - \frac{x}{3} - \frac{x^2}{2} - \frac{xy}{2}$ we get for example $f(1, 1) = -1$ or $f(-1, 2) = 1/6$.

GRAPH AND CONTOUR

Functions can be displayed by their **graphs** $z = f(x, y)$ in three dimensional space \mathbb{R}^3 .

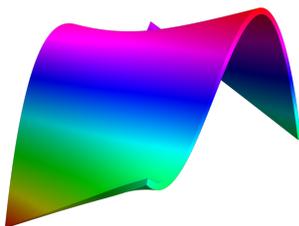


FIGURE 1.

The **contour map** $\{f(x, y) = c\}$ in the xy -coordinate plane shows various level curves.

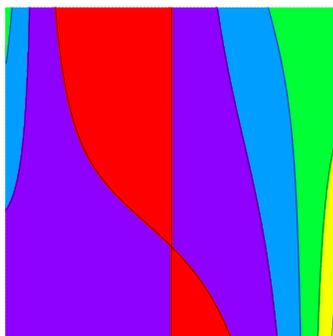


FIGURE 2.

TRACES

Intersections with planes parallel to coordinate planes are called **traces** or **slices**. The contour map shows a collection of z -traces.

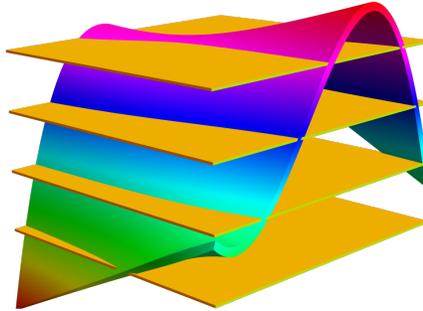


FIGURE 3.

Here we see x -traces, intersections with planes parallel to the x -axis.

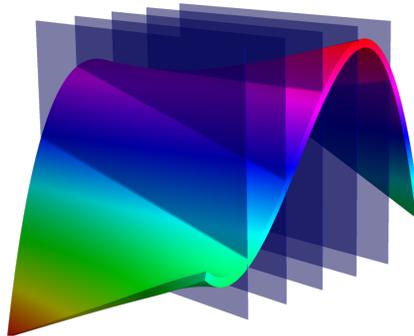


FIGURE 4.