OLIVER KNILL, MATH 21A

Lecture 2: Dimension and Cylindrical Coordinates

Dimension

The **circle** $x^2 + y^2 = 1$ in $\mathbb{R}^2$ is a one-dimensional object. The **disk** $x^2 + y^2 \leq 1$ in $\mathbb{R}^2$ is a two dimensional object. A **cylinder** $x^2 + y^2 = 1$ in $\mathbb{R}^3$ is a 2-dimensional object. It is a surface. The **solid cylinder** $x^2 + y^2 \leq 1$ is a 3-dimensional object. It is a **solid**. We need 3 parameters to locate a point in the solid cylinder.

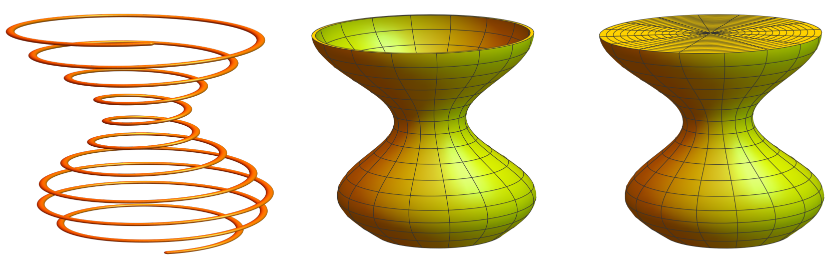

Figure 1.

Question: What is the dimension of the object $x^2 + y^2 = 0$ if the **ambient space** is $\mathbb{R}^2$? Answer: it is a point and so 0-dimensional. Question: What is the dimension of the object $x^2 + y^2 = 0$ if the **ambient space** is $\mathbb{R}^3$? Answer: it is the z-axes, a curve which is 1-dimensional.

Polar coordinates

A point (x, y) in the plane $\mathbb{R}^2$ can be located also by giving the distance $r = \sqrt{x^2 + y^2}$ to the origin $(0, 0)$ and by giving (as long as $(x, y) \neq (0, 0)$) the direction in the form of an angle $\theta \in [0, 2\pi)$. The point (x, y) can be recovered from the distance and angle using **trigonometry**:

$$\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta) \end{aligned}$$

The points with distance r from the origin form a circle $x^2 + y^2 = r^2$. Note that the circle is an object of dimension 1. We need only one parameter to describe the point. This parameter is the angle θ.

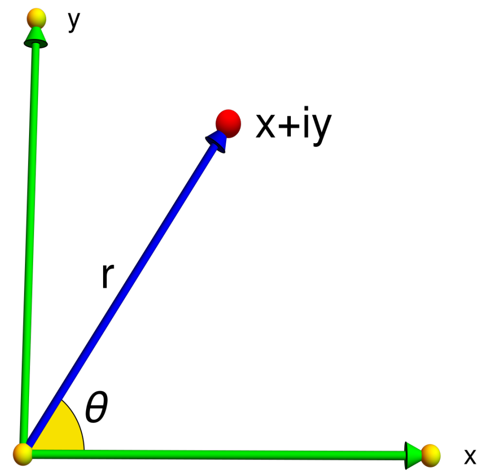

FIGURE 2.

CYLINDRICAL COORDINATES

A point (x, y, z) in space $\mathbb{R}^3$ can be described by the distance $r = \sqrt{x^2 + y^2}$ from the z-axes, the polar angle θ as well as the z coordinate z.

The points with distance r from the z-axes form a cylinder $x^2 + y^2 = r^2$. Note that this is now a **2-dimensional object**. We need two parameters to locate a point on the circle, the angle θ and the height z.

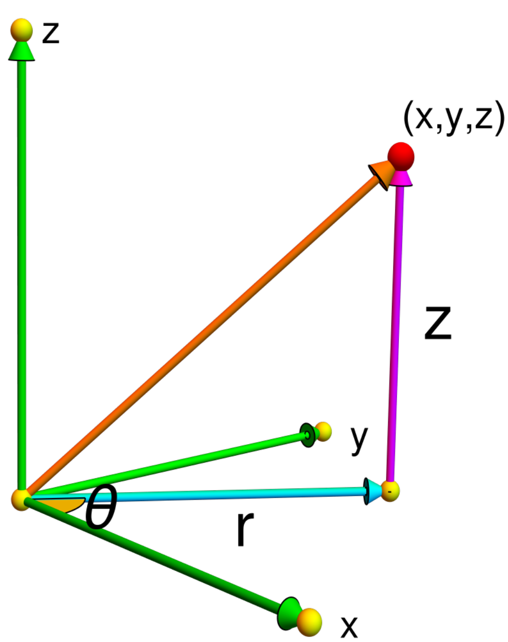

FIGURE 3.