

MULTIVARIABLE CALCULUS

OLIVER KNILL, MATH 21A

Lecture 3: Spherical Coordinates

SPHERICAL COORDINATES

A point (x, y, z) in space \mathbb{R}^3 can be fixed also by giving the distance $\rho = \sqrt{x^2 + y^2 + z^2}$ to the origin $(0, 0, 0)$ by giving the polar angle of (x, y) as before $\theta \in [0, 2\pi)$ and a by providing a third angle $\phi \in [0, \pi]$, the angle to the positive z -axis. Because $r = \rho \sin(\phi)$, we have

$$x = \rho \sin(\phi) \cos(\theta)$$

$$y = \rho \sin(\phi) \sin(\theta)$$

$$z = \rho \cos(\phi)$$

The points with distance ρ from the origin form a sphere $x^2 + y^2 + z^2 = \rho^2$.

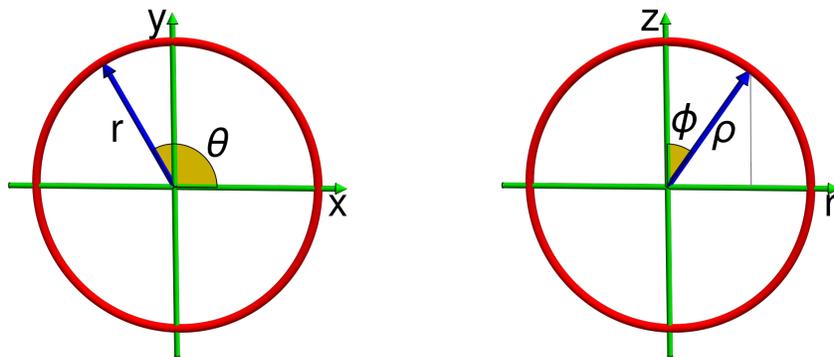


FIGURE 1. The key picture to get spherical coordinates

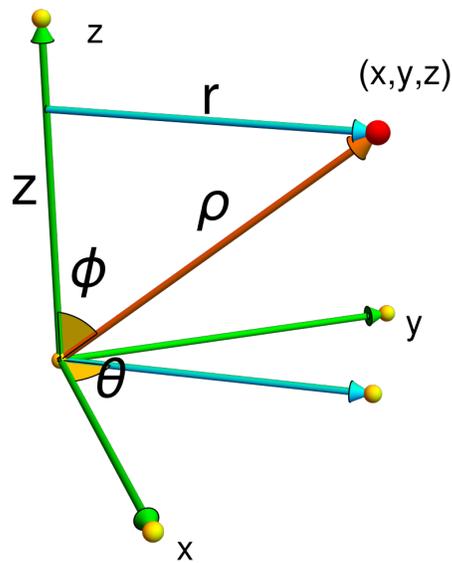


FIGURE 2.

SOME QUESTIONS

Make sure you make sense about the following questions. We will look at that in class.

- Why is the angle ϕ chosen on $[0, \pi]$ and not $[0, 2\pi)$? (ambiguity)
- Does every point have a unique θ angle in $[0, 2\pi)$ attached? (no)
- What sets do you get with $\rho = c$? (a sphere or a point)
- What sets do you get with $\phi = c$? (a half cone or a half line)
- What sets do you get by setting $\theta = c$? (always a half plane)

SOME OBJECTS

Make sure you make sense about the following questions. We will look at that in class.

- What is $\rho = 1$?
- What is $\phi = 1$?
- What is $\theta = 1$?
- What is $\rho = \sin(\phi)$?