MULTIVARIABLE CALCULUS

OLIVER KNILL, MATH 21A

Lecture 5: Double Integrals

Single Integral

In single variable calculus, the integral of a function $f(x)$ over an integral $[a, b]$ is a Riemann sum. The same is the case when integrating $f(x, y)$ over a region R

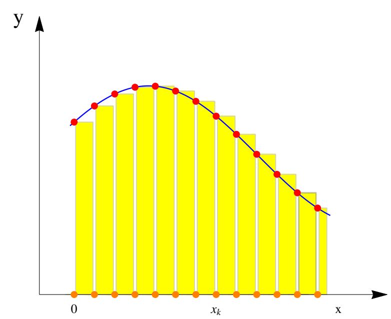


Figure 1. Riemann sum of a definite integral $\int_a^b f(x)\, dx$

Remember that

$$\boxed{\int_a^b 1\, dx \text{ is the length } b - a.}$$

Double Integrals

To sum over a region R in the xy-plane, we chop up the region into parts of area dA and sum up all the values $f(x, y)dA$ and write

$$\iint_R f(x, y)\, dA\ .$$

$$\boxed{\iint_R 1\, dA \text{ is the area of } R.}$$

Here is a challenge we do not know the answer yet

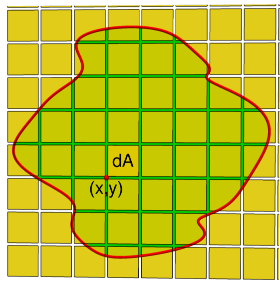


FIGURE 2. Integrating over a region is a limit of Riemann sums $\sum\sum_l f(x_k, y_k)dA_k$, where (x_k, y_k) are points in a chunk of area dA_k.

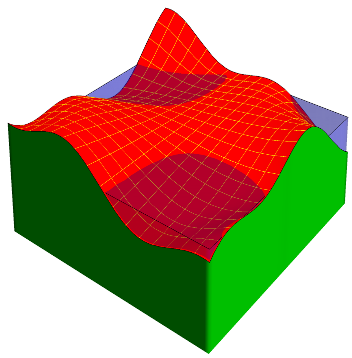
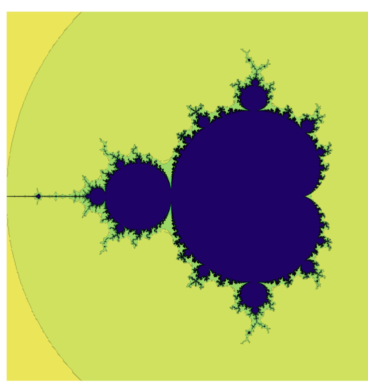


FIGURE 3. A double integral is a signed volume under the graph. A signed volume can also be negative.



What is the area of the Mandelbrot set? We know it is contained in the rectangle rectangle $x \in [-2, 1]$ and $y \in [-3/2, 3/2]$. By shooting randomly we can compute the area. This is called Monte Carlo computation. If you run the following code you can get numbers around 1.5.

```
M=Compile[{x,y},Module[{z=x+I y,k=0},
   While[Abs[z]<2.&&k<1000,z=N[z^2+x+I y];++k];Floor[k/1000]]];
9*Sum[M[-2+3 Random[],-1.5+3 Random[]],{1000000}]/1000000
```