OLIVER KNILL, MATH 21A

Lecture 17: Surfaces

Parametric Surfaces

With the four classes of parametrized surfaces **spheres**, **planes**, **graphs** and **surfaces of revolution**, one can cover a lot of surfaces. Here are examples of each of the four classes. Can you identify and visualize each of them?

$\vec{r}(u,v) = \langle u, v, u^2 - v^2 \rangle$	$\vec{r}(u,v) = \langle u+v, u-v, 3 \rangle$
$\vec{r}(u,v) = \langle \sin(v)\cos(u), \sin(v)\sin(u), \cos(u) \rangle$	$\vec{r}(u,v) = \langle v\cos(u), v\sin(u), v \rangle$

Recall that any function

$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$$

from a region R in $\mathbb{R}^2$ to space $\mathbb{R}^3$ defines a **surface** S and that the **grid curves** help to plot them. Computer graphics by default include the grid lines to visualize the surface better.

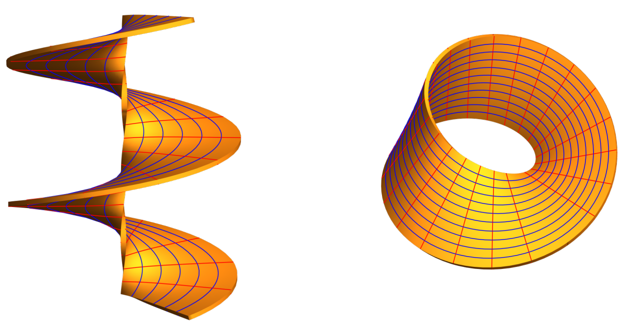

FIGURE 1. A helicoid $\vec{r}(t,s) = \langle s\cos(t), s\sin(t), t \rangle$ and the Moebius strip $\vec{r}(u,v) = \langle (2+v\cos(u/2))\cos(u), (2+v\cos(u/2))\sin(u), v\sin(u/2) \rangle$.

Fun with surfaces

Here is some art work which have appeared for Summer Math 21a exams over the years. In the summer of 2022, we have taken a floor $\vec{r}(x,y) = \langle x, y, 1 \rangle$, a pole $\vec{r}(\theta, z) = \langle (1+0.1\sin(10z))\cos(\theta), (1+0.1\sin(10z))\sin(\theta), z \rangle$ a Yoga chair $\vec{r}(\theta,\phi) = \langle \cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi)/3 \rangle$ an umbrella $\vec{r}(x,y) = \langle x, y, 10-|x|-|y| \rangle$ and a lazy chair $\vec{r}(u,v) = \langle u, v, u^2 - v^2 \rangle$. In the summer of 2019, we have build a cool iced macciato with cream and a cherry on top. It contains a cup surface in the form of a cone $x^2 + y^2 = z^2/25$, a waffle lemon slize $x = 4 - \sin(yz/5)$ a cherry sphere, a cookie straw in the form of a cylinder and a coaster in the form of a plane. From 2014 is a monkey riding the Monkey saddle $\langle u, v, vu^2 - v^3 \rangle$.

FIGURE 2. A picnick scene, a cool iced macchiato and a monkey riding the monkey saddle.

In the summer of 2018, we built a Candy chess piece. Unlike with other chess games, the figures are edible. The **queen** in our candy game is made of different parts. The head $\langle 3\cos(\theta)\sin(\phi), 3\sin(\theta)\sin(\phi), 10{+}3\cos(\phi) \rangle$, the neck $\langle \cos(\theta), \sin(\theta), z \rangle$, the crown $\langle x, y, \sin(xy){+}13 \rangle$, the skirt $\langle z\cos(\theta), z\sin(\theta), z{-}1 \rangle$ and the shoulder $\langle \cos(\theta\sin(\phi), 5\sin(\theta\sin(\phi), \cos(\phi)) \rangle$.

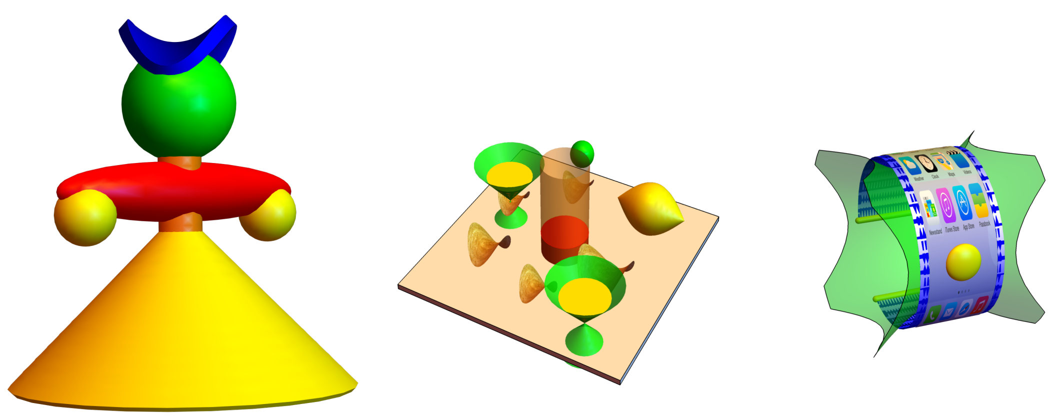

FIGURE 3. An sweet queen in an edible chess game from 2018 and a summer refreshment from summer 2016. And an iWatch from 2014.

In 2014, in anticipation of the iWatch, we crated our dream watch using among other surface the following one $\vec{r}(u,v) = \langle \sqrt{v^2+1}\cos(u), \sqrt{v^2+1}\sin(u), v \rangle$. Can you see what it is?

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