MULTIVARIABLE CALCULUS

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Lecture 35: Overview

The theorems

The fundamental theorem of line integrals, Stokes theorem and the divergence theorem all generalize the fundamental theorem of calculus in three dimensions

Theorem: $\int_C \text{grad}(f) \cdot d\vec{r} = f(B) - f(A)$

Theorem: $\int\int_S \text{curl}(\vec{F}) \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$

Theorem: $\int\int\int_E \text{div}(\vec{F})\ dV = \int\int_S \vec{F} \cdot d\vec{S}$

In two dimensions, there were only two theorems, the fundamental theorem of line integrals and Green's theorem

Theorem: $\int_C \text{grad}(f) \cdot d\vec{r} = f(B) - f(A)$

Theorem: $\int\int_S \text{curl}(\vec{F}) dA = \int_C \vec{F} \cdot d\vec{r}$

And in one dimension, there is only one theorem, the fundamental theorem of calculus.

Theorem: $\int_A^B f'(x)\ dx = f(B) - f(A)$

All of these theorems actually are the same. There is a formalism, where we define a derivative of a "field" F and a boundary dG of a "geometry". Then the general Stokes theorem is

Theorem: $\int_G dF = \int_{dG} F$

It tells that if we integrate the derivative of a field over a geometry, we get the same than integrating the field over the boundary of the geometry.

The magic is that we can do that in arbitrary dimensions, where we have no cross product any more.

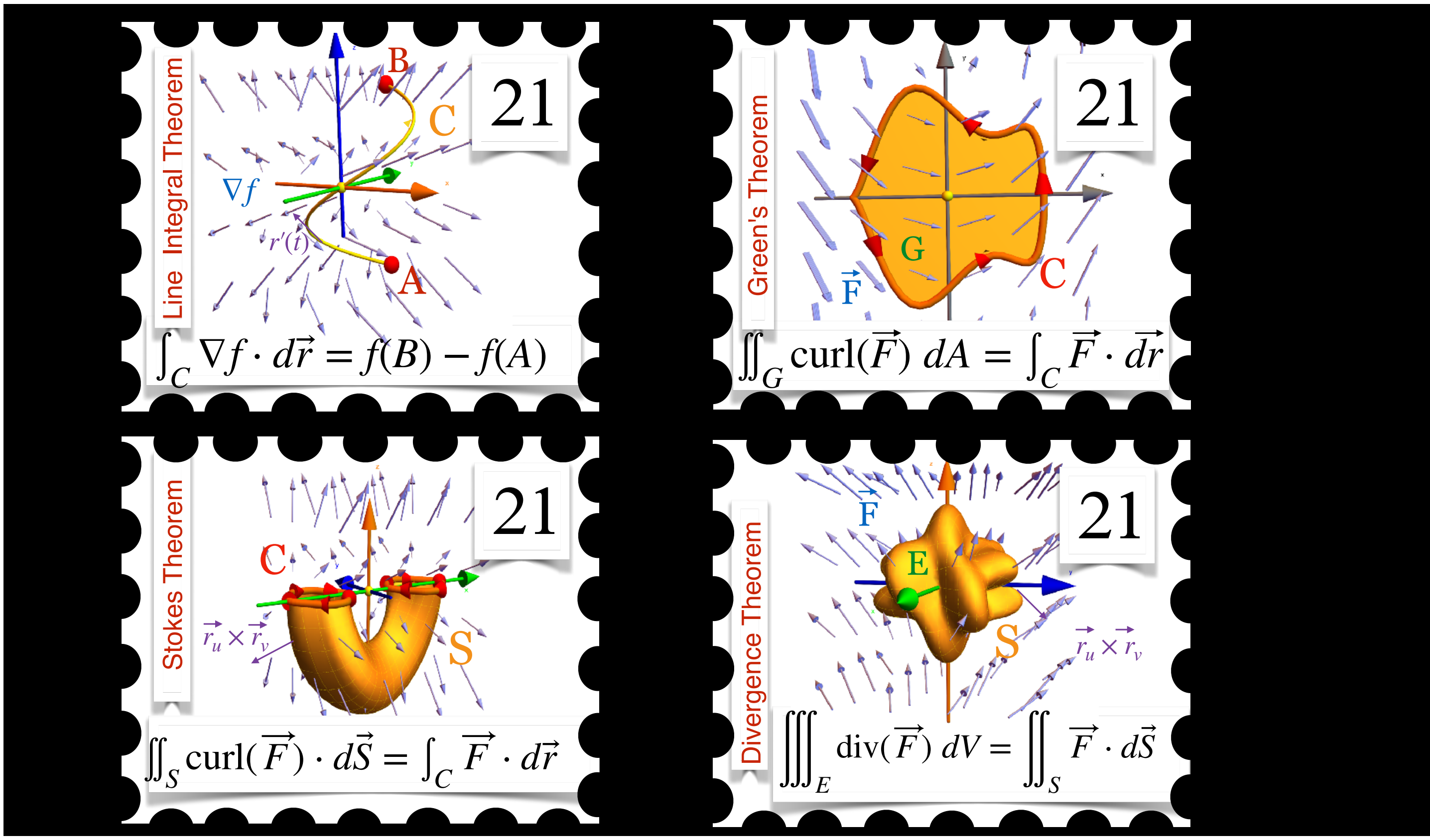

FIGURE 1. Math 21a stamp collection with integral theorems.

34.1. Integral theorems deal with **geometries** G and **fields** F. **Integration** pairs them in the form of **Stokes theorem**

$$\int_G dF = \int_{dG} F$$

which involves the **boundary** dG of G and the **exterior derivative** dF of F. One can classify the theorems by looking at the dimension n of the underlying space and the dimension m of the object G. In dimension n, there are n derivatives, n integrals and n theorems.