



Lecture 4

Partial Derivatives

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Partial Derivatives

$$f(x, y) = \sin(xy) + e^y + \ln(x) + 1$$

A

Compute $f_x(x, y)$ and $f_y(x, y)$

B

Compute $f_{xx}(x, y)$ and $f_{yy}(x, y)$

C

Compute $f_{xy}(x, y)$ and $f_{yx}(x, y)$

Tougher Questions

A

What happens if $f_x(x, y) = 0$ for all x, y ?

B

What happens if $f_y(x, y) = 0$ for all x, y ?

C

Assume $f_x(x, y) = f_y(x, y) = 0$ everywhere. What does this say about f ?

Meaning



Mt Pisgah

X 346

HIGH

Andrews Hill

WINCHESTER COUNTRY CLUB

HUTCHINSON

ARLINGTON

ST

CHURCH

B.M.

462

WEDGE LAKE

Lake

Mystic

Upper

VALLEY

Whipple Hill

374

Turkey Hill

RD

MYSTIC

RIDGE

PK

B

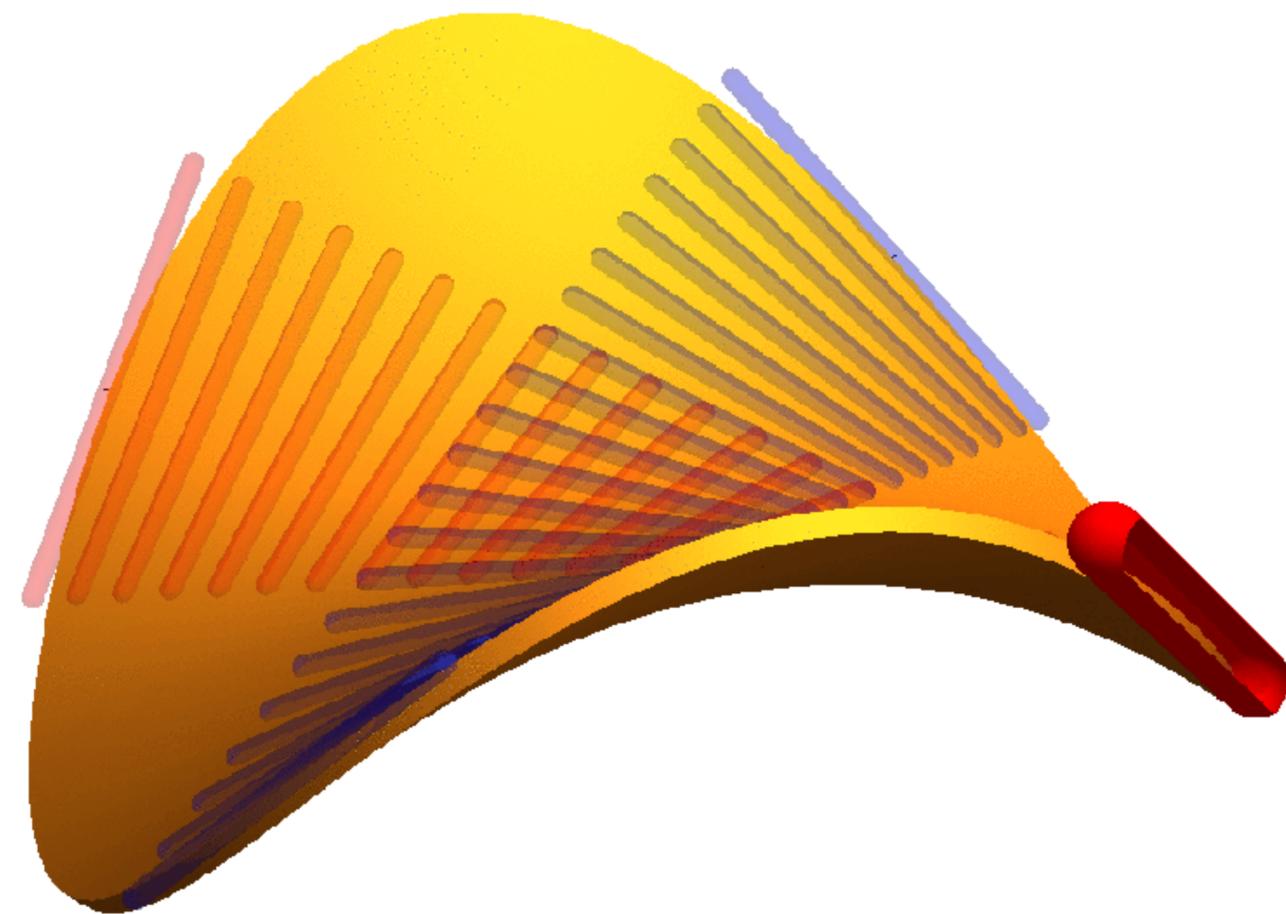
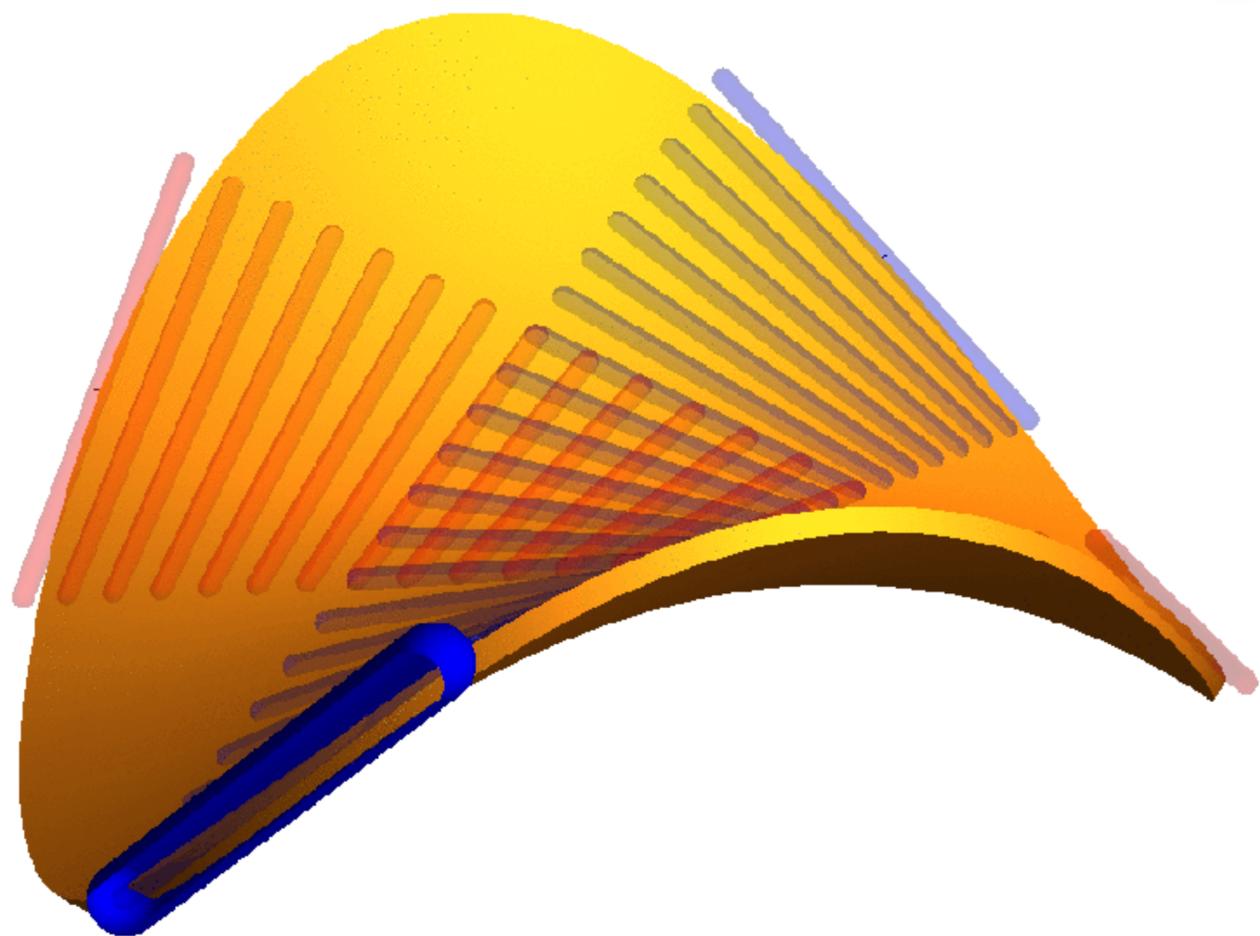
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Clairaut's Theorem

If f has continuous second derivatives then

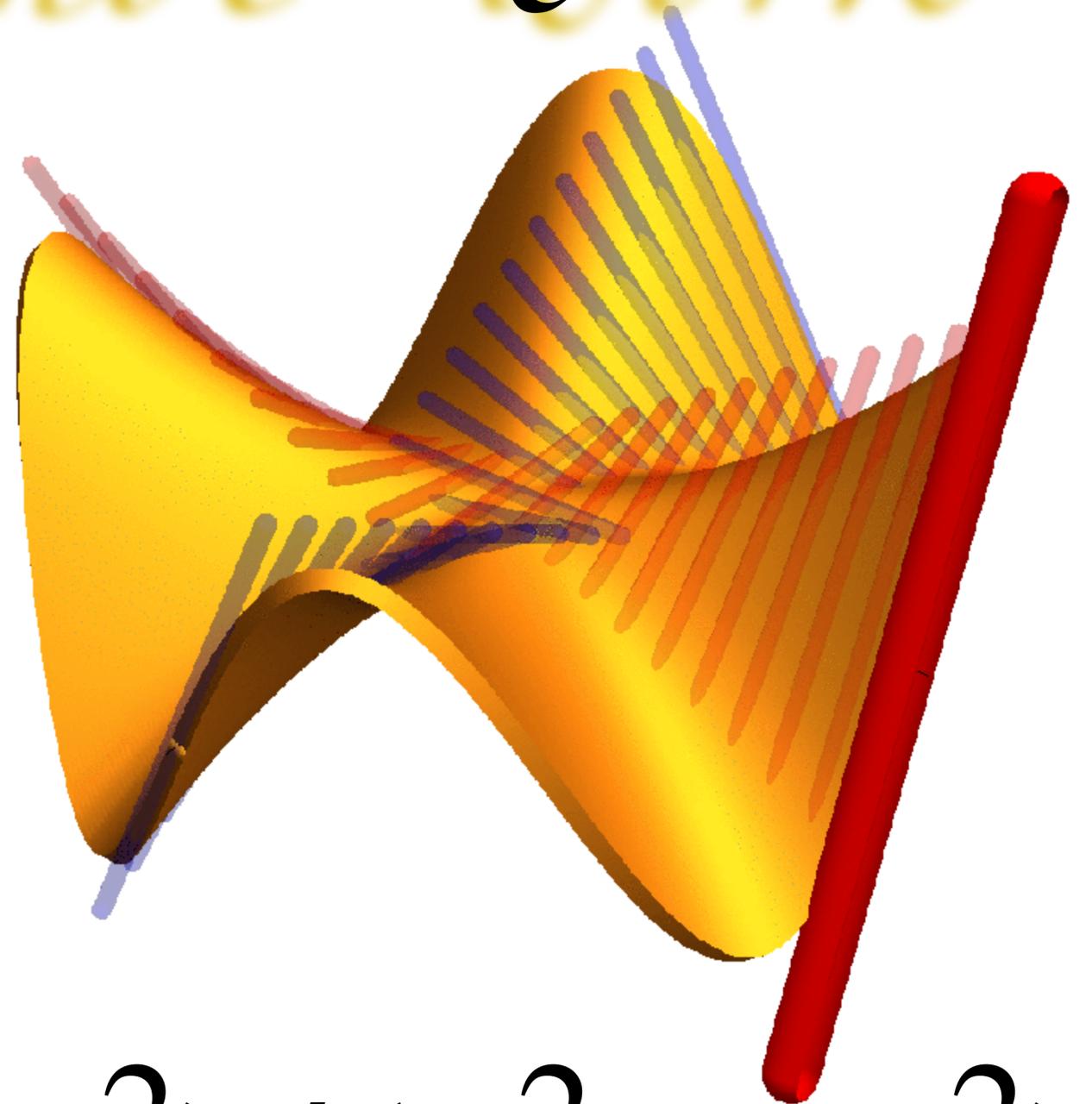
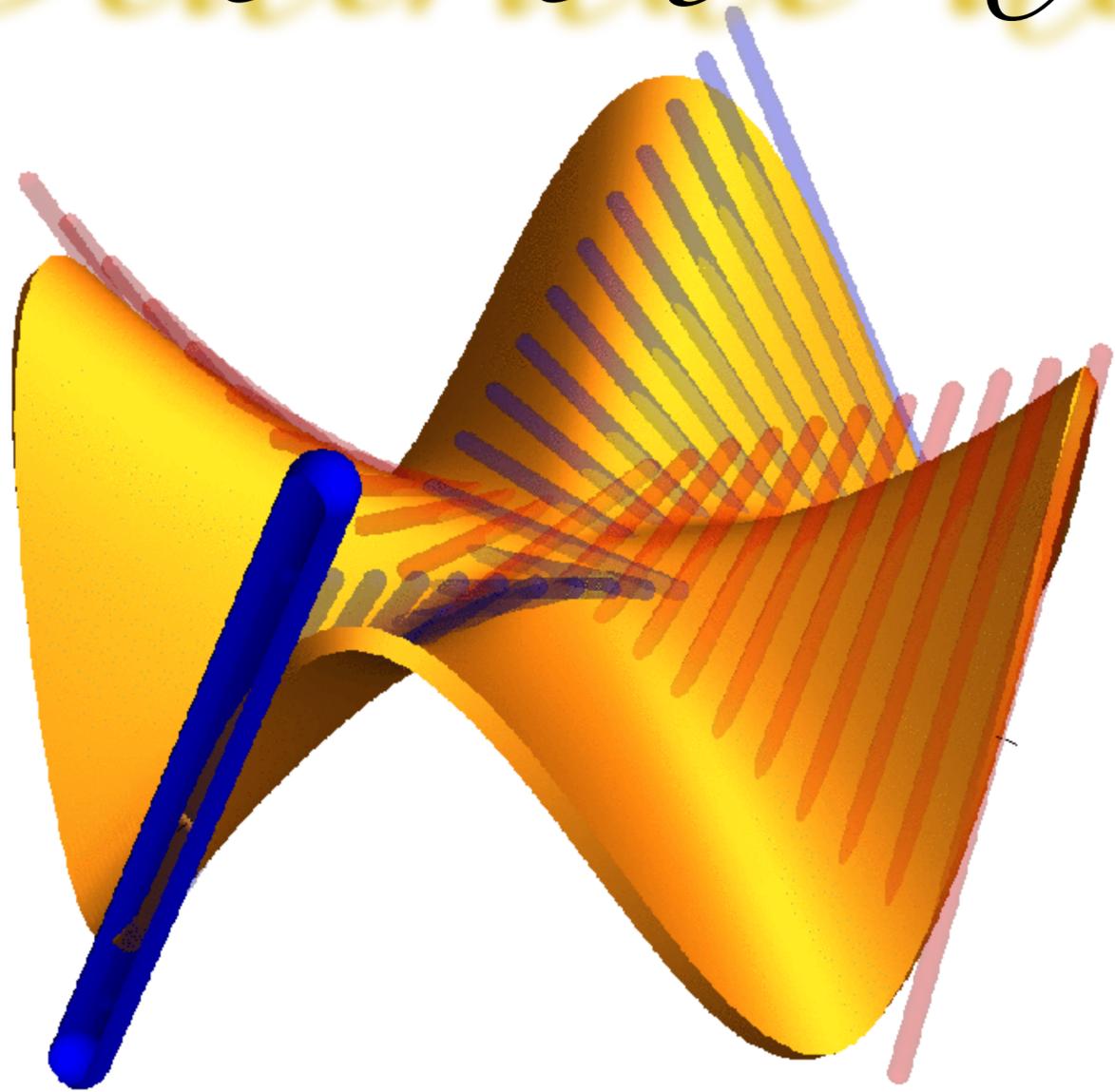
$$f_{xy} = f_{yx}$$

Clairaut Works



$$f(x, y) = 2xy$$

Clairaut does not work



$$f(x, y) = 2xy(y^2 - x^2)/(x^2 + y^2)$$

Map Problem



Mt
Gilboa

Turkey
Hill

SYMMES
HOSPITAL

BRATTLE STA

2
A

Arlington Heights

ARLINGTON

Brook

MT PLEASANT
CEMETERY

Lower
Mystic

Upper
Mystic

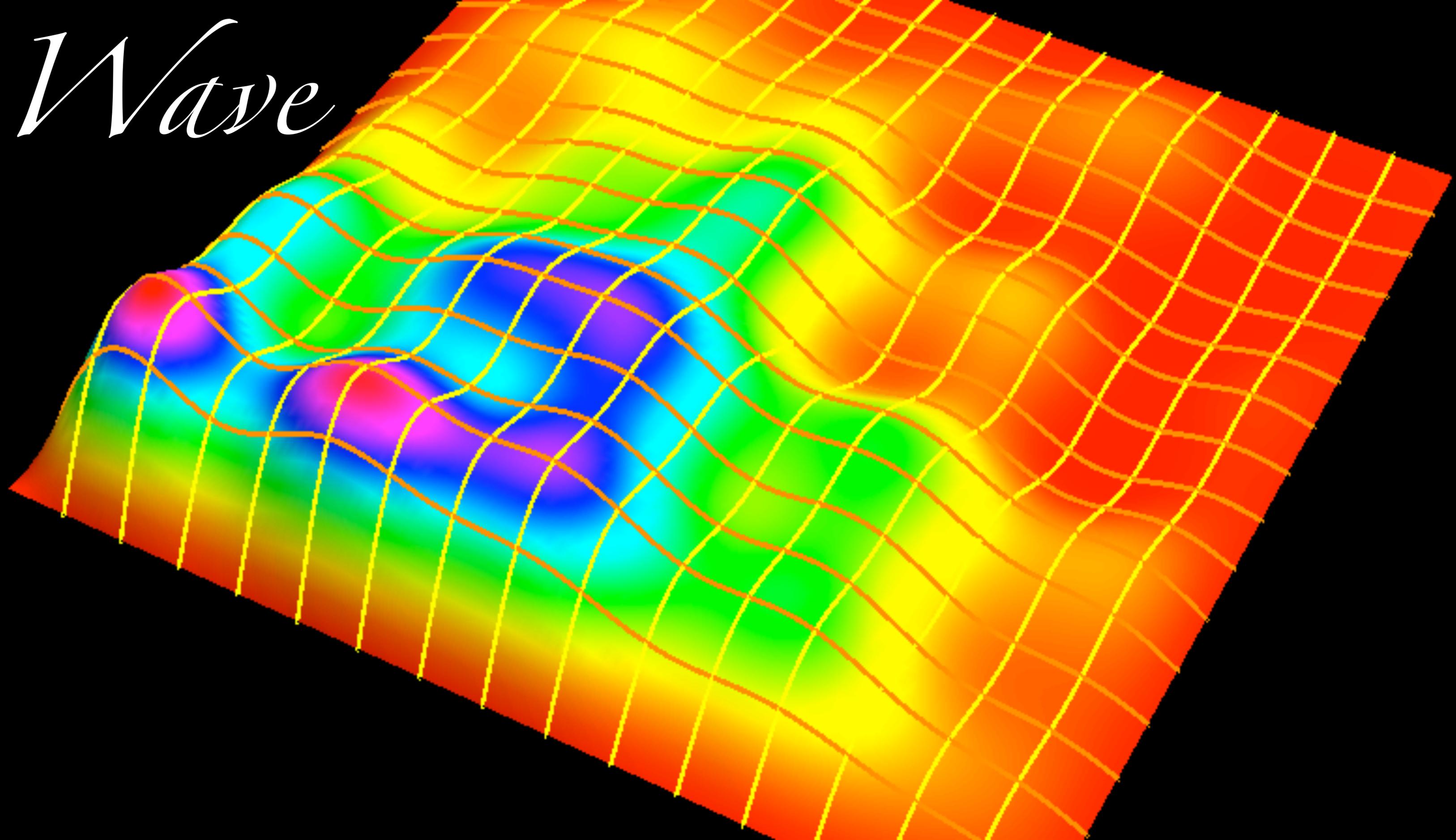
RINES
SANATORIUM

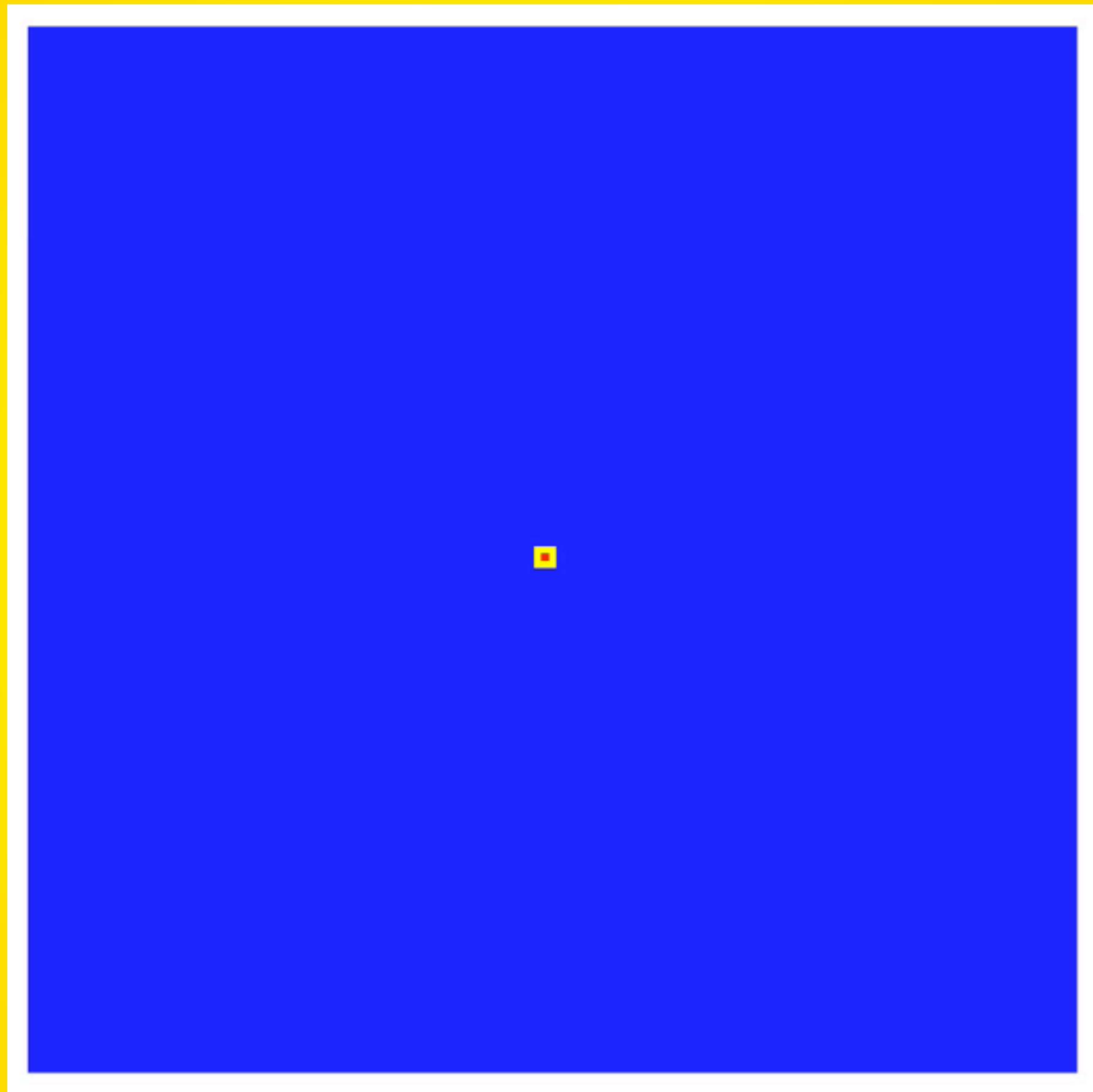
Arlington

Heat

$$f_t = f_{xx}$$

Wave



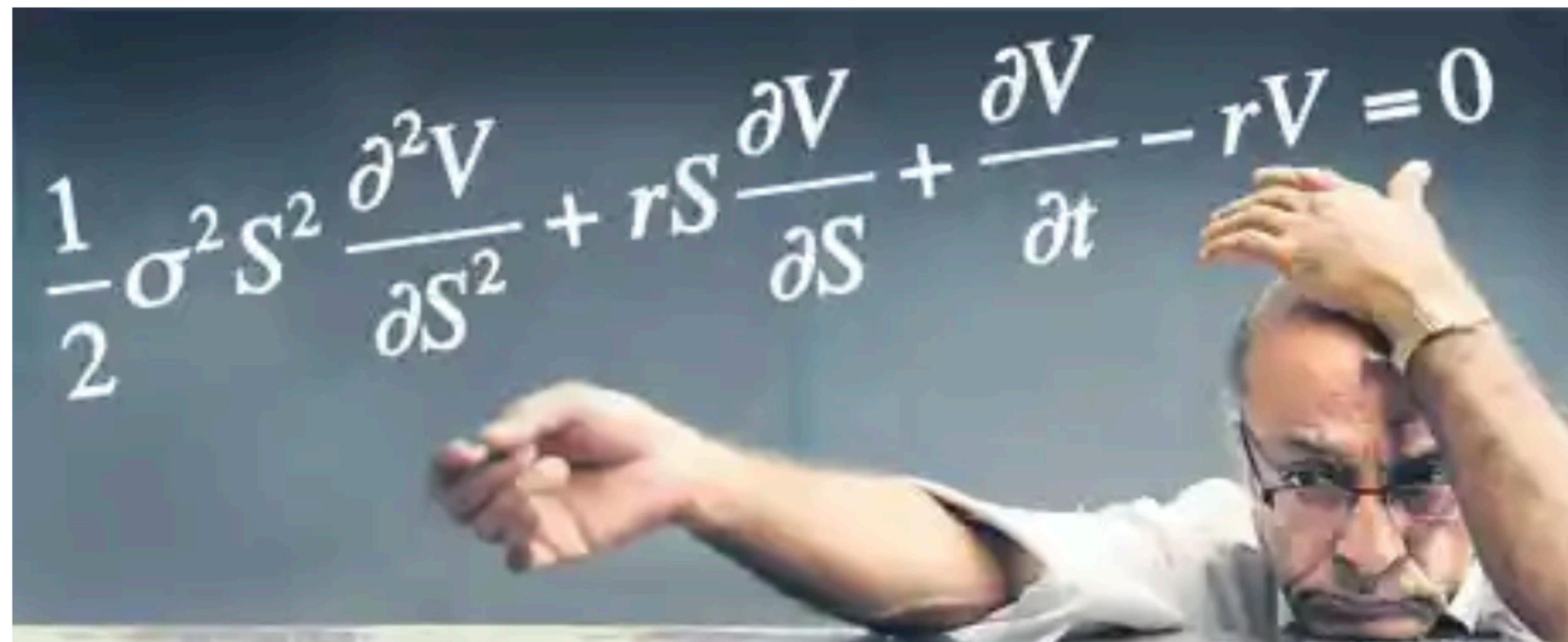


The mathematical equation that caused the banks to crash

The Black-Scholes equation was the mathematical justification for the trading that plunged the world's banks into catastrophe

Ian Stewart

Sat 11 Feb 2012 19.05 EST



In the Black-Scholes equation, the symbols represent these variables: σ = volatility of returns of



Get some Cash

MILLENNIUM PROBLEMS

Margaretta Taylor
Lobby



Homework due Wednesday

PROBLEM SET 4 - PARTIAL DERIVATIVES

You should be able to:

- Estimate the value or determine the sign of a partial derivative of a function of two variables given a graph or contour map.
- Interpret partial derivatives in context.
- Calculate partial derivatives of functions of multiple variables from their formulas and use them to approximate function values.
- State and apply Clairaut's Theorem.

As a reminder, you should always show your work / include some explanation of your reasoning, unless a problem specifically says that isn't necessary.

- Now that you've participated in a few classes and completed a few problem sets, it's a good time to reflect on how things are going overall. Please fill out [this short survey](#), which is intended to help you reflect on your learning so far.
- Verify that Clairaut's Theorem holds for $f(x, y) = e^{x \sin y}$ by finding both partial derivatives f_{xy} and f_{yx} .
- Open this [GeoGebra applet](#). It shows the graph of a function $f(x, y)$, together with a point that you can move around using the sliders. Initially, the point is at $(3, -2, f(3, -2))$.

- Click the appropriate checkbox to see $f_x(3, -2)$. Is $f_x(3, -2)$ positive, negative, or 0?
- Click the appropriate checkbox to see $f_y(3, -2)$. Is $f_y(3, -2)$ positive, negative, or 0?
- Keep the x -coordinate of the point at 3, but use the slider to change the y -coordinate to 0. Is $f_y(3, 0)$ positive, negative, or 0?
- Keep the y -coordinate of the point at 0, but use the slider to change the x -coordinate to 2.5. Now, increase the x -coordinate from 2.5 to 3.5 (while leaving the y -coordinate constant). As you increase x from 2.5 to 3.5, is f_y increasing, decreasing, or staying constant?
- Using your answer to (d), which of the following is a valid conclusion?

- $f_{yx}(3, 0) < 0$
- $f_{yx}(3, 0) > 0$
- $f_{yy}(3, 0) < 0$
- $f_{yy}(3, 0) > 0$

- Is $\frac{\partial^2 f}{\partial y^2}(3, -2)$ positive or negative?

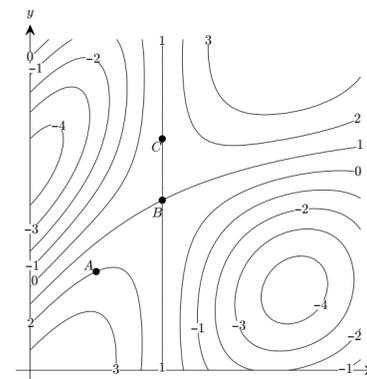
- Interpreting partial derivatives.

- The price P in dollars to purchase a used car is a function of its original cost C in dollars and its age A in years.
 - What are the units of $\frac{\partial P}{\partial C}$ and $\frac{\partial P}{\partial A}$?
 - Do you expect $\frac{\partial P}{\partial C}$ to be positive or negative? Why?
 - Do you expect $\frac{\partial P}{\partial A}$ to be positive or negative? Why?

- As you've undoubtedly experienced, if you're standing outside, both the temperature and the wind speed affect how cold you feel. The wind-chill index gives the perceived temperature as a function of actual temperature and wind speed.

The wind-chill index is modeled by the function $W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}$ where T is the temperature in $^{\circ}\text{C}$ and v is the wind speed in km/hour.⁽¹⁾ When $T = -15^{\circ}\text{C}$ and $v = 30$ km/hour, by how much would you expect the apparent temperature W to drop if the actual temperature decreases by 0.1°C ? What if the wind speed increases by 2 km/hour?

- Here's the contour map of a function $f(x, y)$.



Decide whether each of the following is positive, negative, or zero, and explain your reasoning briefly.

- | | | |
|--------------|-----------------|-----------------|
| (a) $f_y(B)$ | (c) $f_x(B)$ | (e) $f_{xx}(B)$ |
| (b) $f_x(A)$ | (d) $f_{yy}(A)$ | (f) $f_{xy}(C)$ |

- Next class, we'll begin talking about how to integrate multivariable functions. To evaluate such integrals, you'll need to be very comfortable evaluating single-variable integrals using methods like integration by substitution and integration by parts. If you haven't already, we recommend doing some

⁽¹⁾See <https://www.nytimes.com/2017/09/14/science/maurice-blustein-who-modernized-the-wind-chill-index-dies-at-76.html> for an explanation of how two scientists came up with this formula. (The formula in that article looks different because it uses temperature in $^{\circ}\text{F}$ and wind speed in mph.)

practice by working through the last problem on the "Single-Variable Calculus Review" handout. (You don't need to turn anything in.)

- Just a reminder: the material on this problem set is the last material that will be covered on the mini-exam.

For next class, read the first two sections of the "Double Integrals" handout, "Review of the single-variable definite integral" and "The double integral".

THE END