



Lecture 5

Double Integrals

Table of Contents

1) Single Integral Review

2) Setting Up Double Integrals

3) Reduction to single integrals

4) Other ways to compute

5) Worksheet problems

*First about the
Mini exam*

MINI-EXAM PRACTICE 1

Name:	Student ID:
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As a member of Harvard College, I pledge that I will not give or receive assistance on this exam.

Signature: _____

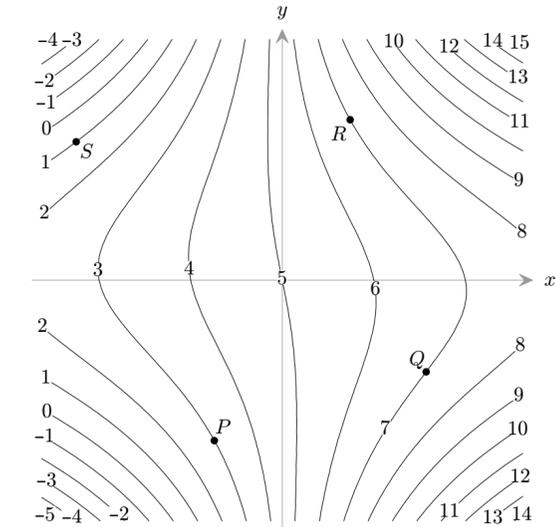
- You are allowed 75 minutes for this mini-exam. No notes, books, or electronic aids are allowed.
- Please show your work and explain your reasoning, and organize your work so that the reader can follow it. We'll be assessing your reasoning and explanation more than your final answer.
- The following formulas may be useful:
 - The surface area of a sphere of radius r is $4\pi r^2$.
 - The surface area of an open circular cone (like an ice cream cone) of radius r and height h is $\pi r\sqrt{r^2 + h^2}$.
 - The volume of a ball of radius r is $\frac{4}{3}\pi r^3$.
 - The volume of a solid cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.

Good luck!

Problem	Points	Score
1	9	
2	12	
3	8	
4	8	
Total	37	

1. (9 points)

Here's the contour map of a function $f(x, y)$. The value of f on each level set is labeled.



(a) Decide whether the following statements are true or false, and explain your reasoning briefly.

i. $f_x(P) \geq 0$

ii. $f_y(Q) \geq 0$

iii. $f_x(P) \geq f_x(Q)$

iv. $f_{yy}(S) \geq 0$

v. $f_{xy}(R) \geq 0$

(b) Can you find a point A at which $f_y(A) = 0$? If so, mark one on the contour plot above. Either way, explain your reasoning.

Thursday, 9/22/2022, 5:30 - 6:45 PM
(tomorrow in a week)

Content: PSets 0-4

Extra credit:

do Practice 1 until Friday 9/16

and grade Practice 1 until 9/19

Single integrals review

A

Compute $\int_0^{10} x^2 dx$

B

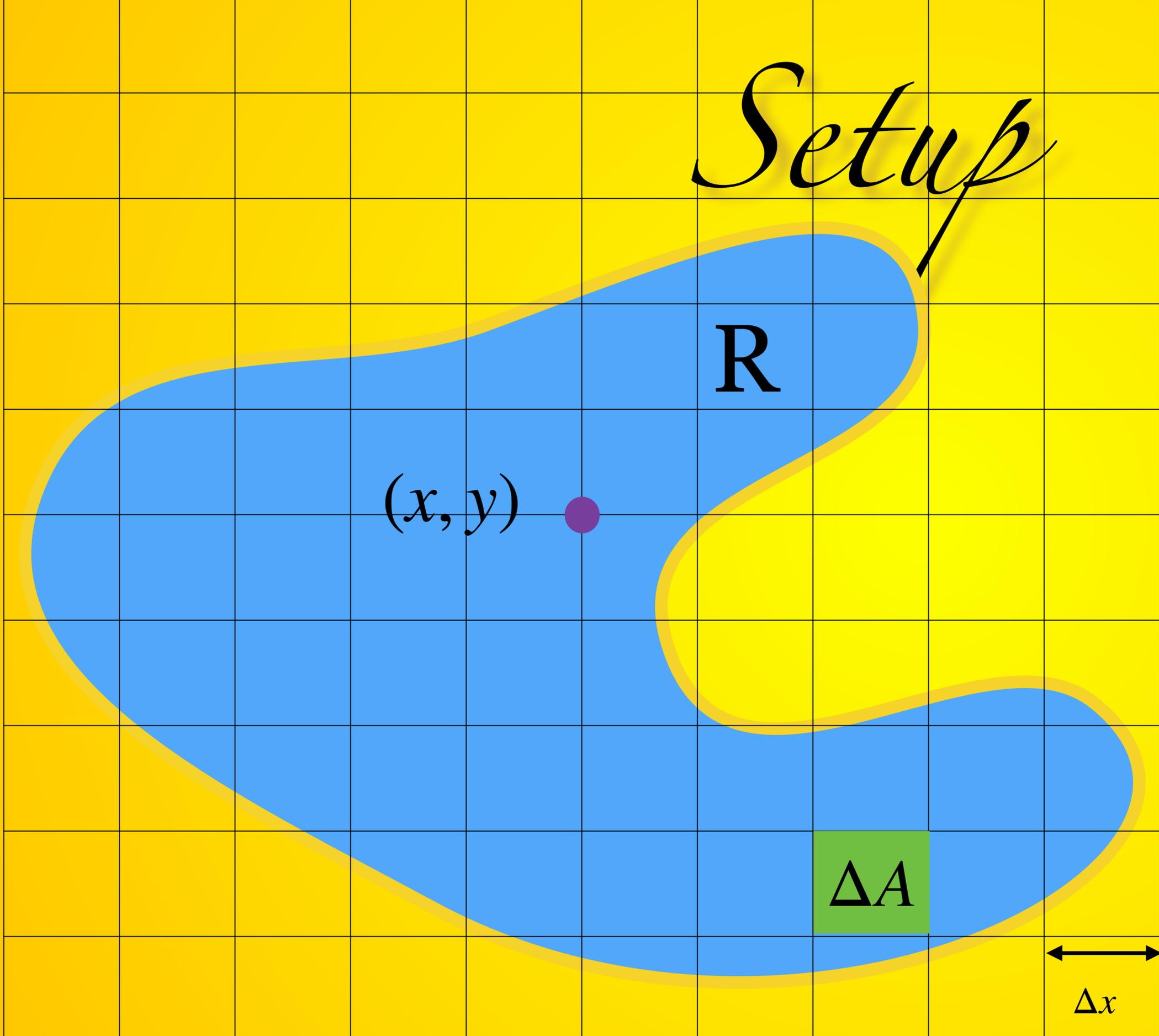
What is $\int_0^1 \sqrt{1-x^2} dx$

C

What is $\int_0^{2\pi} \sin(3x)\cos(17x) dx$

Double Integrals

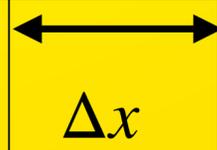
Setup

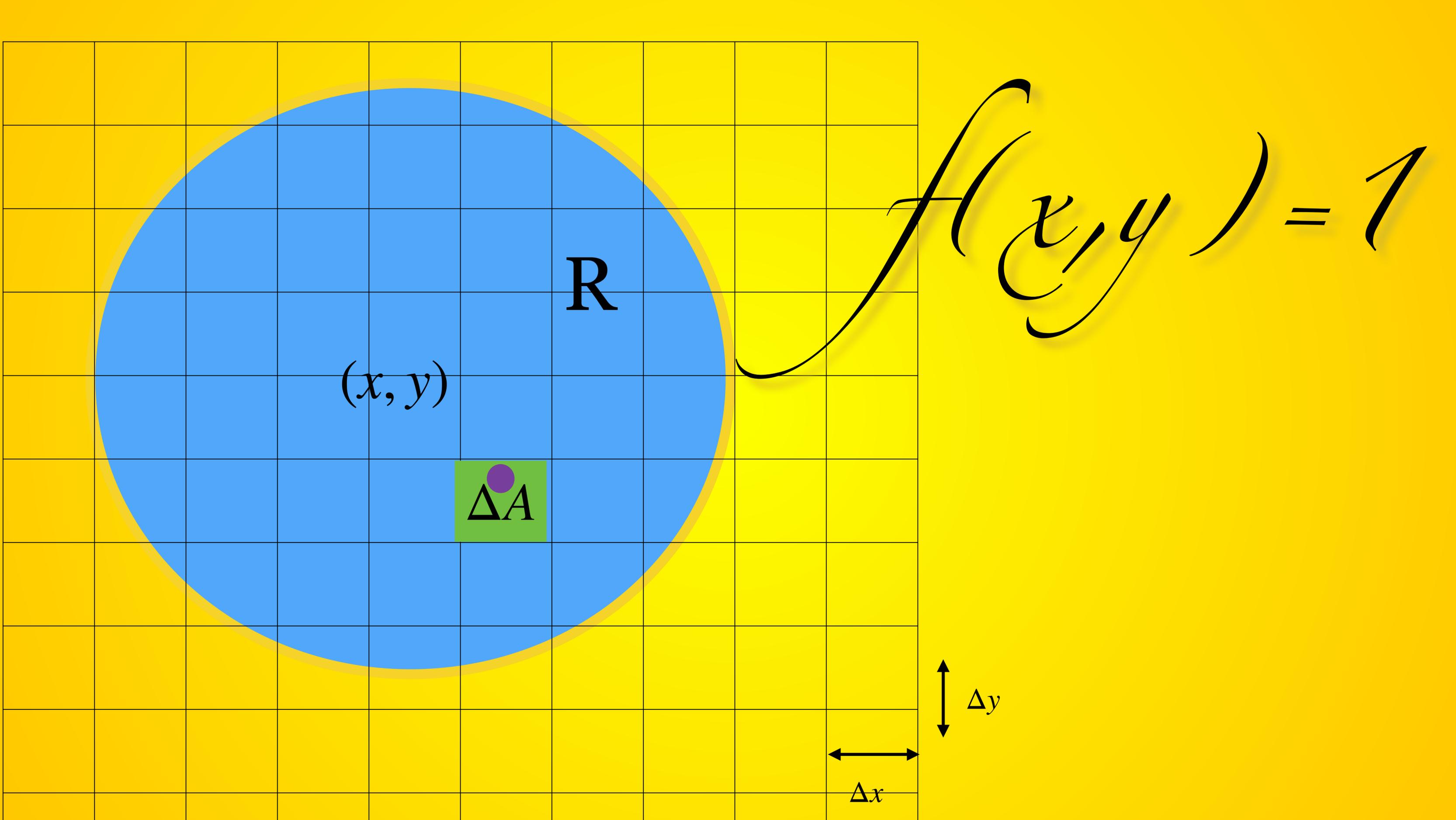


$$\sum_{(x,y)} f(x, y) \Delta A$$

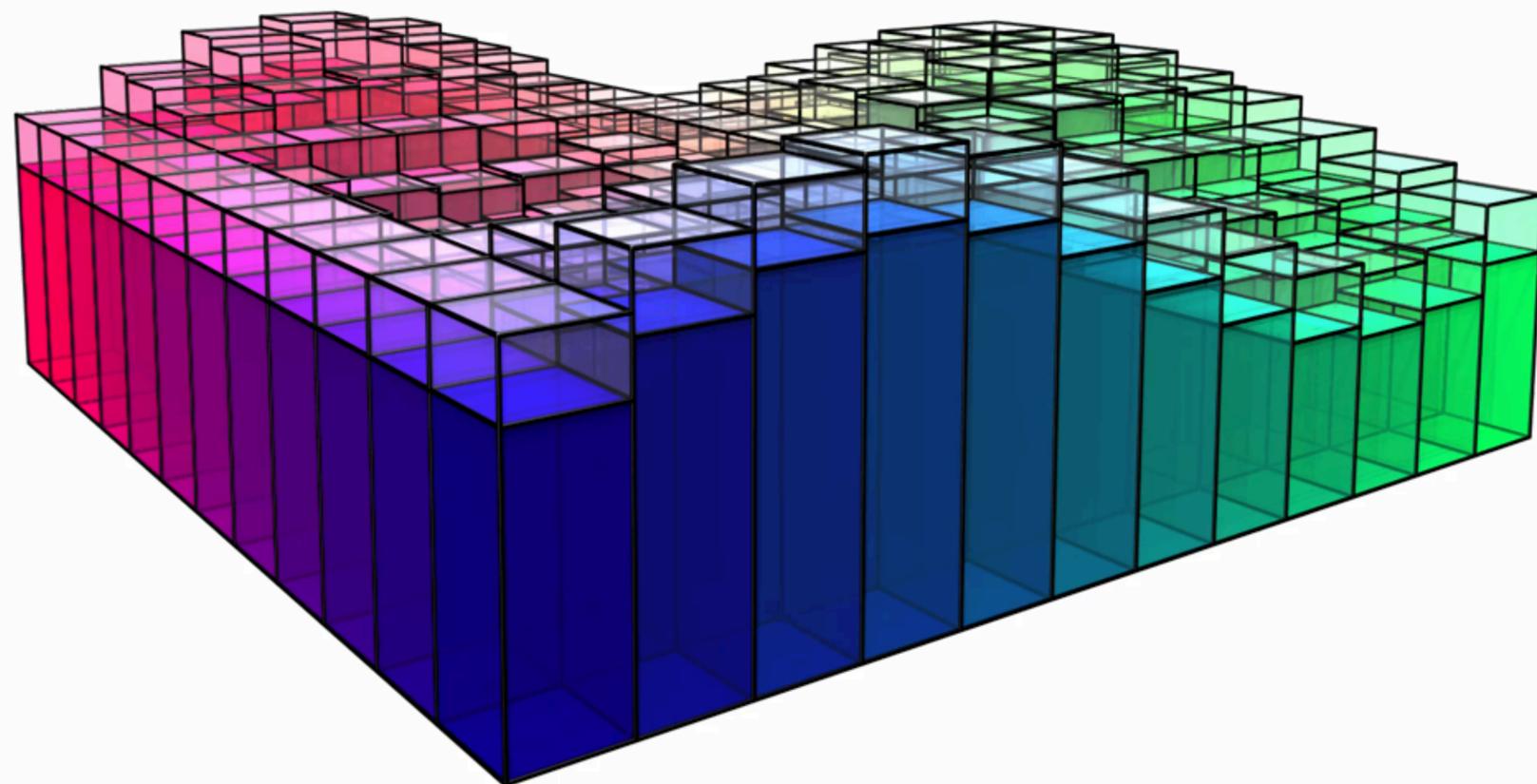


$$\iint_R f(x, y) dA$$



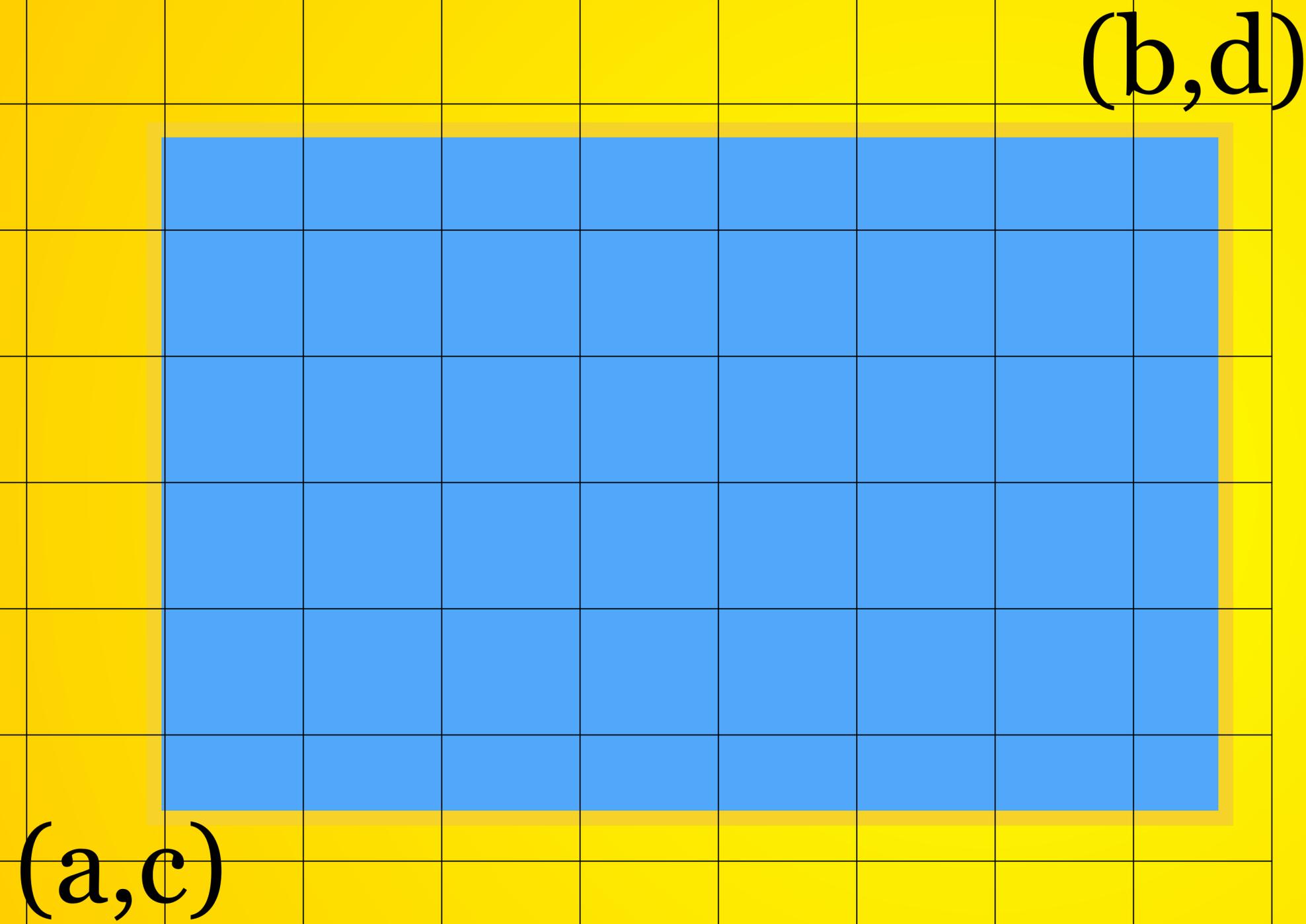


Volume Interpretation



Worksheet

Order of Summation



$$\int_a^b \int_c^d f(x, y) \, dy \, dx$$

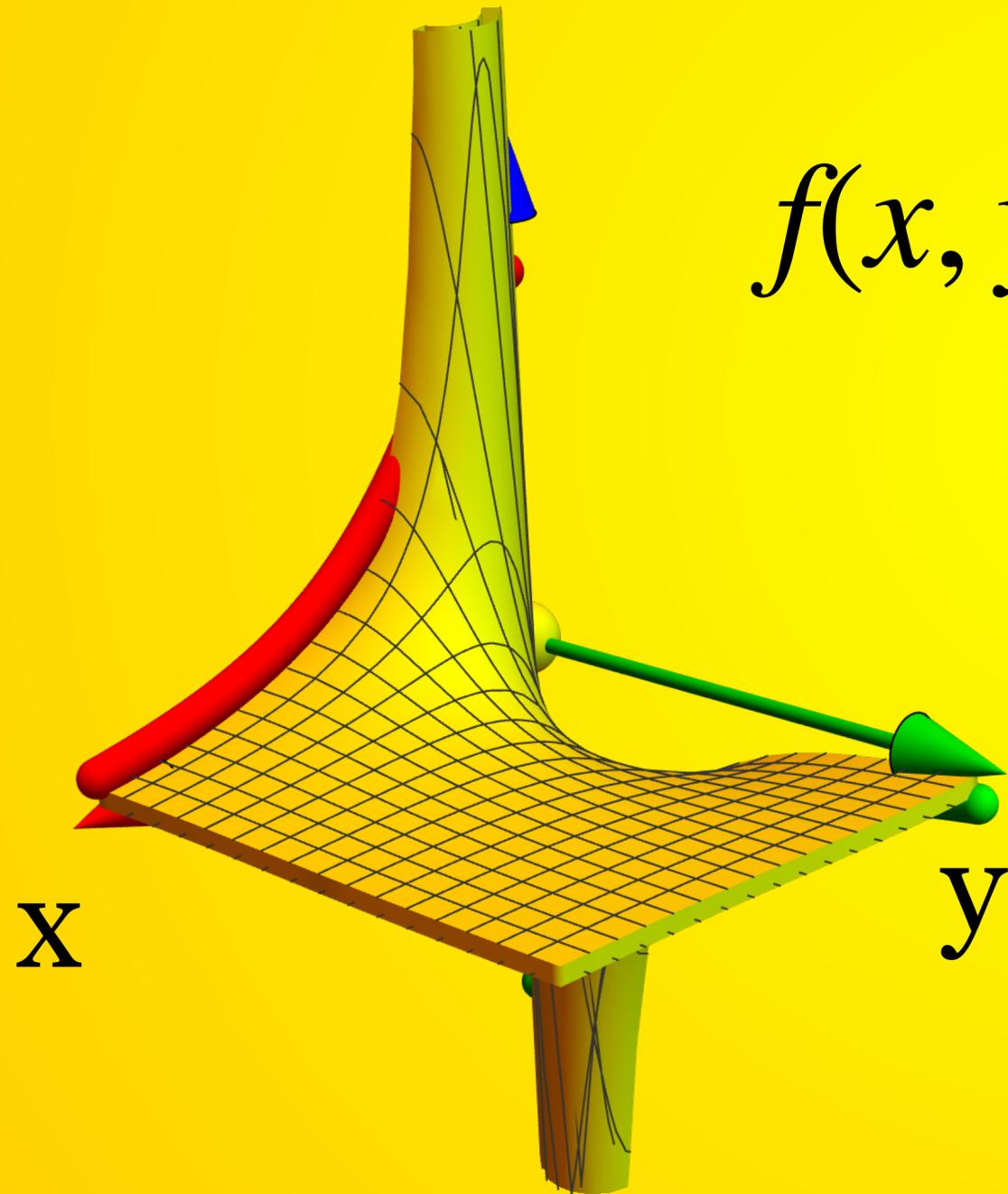
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$$\int_c^d \int_a^b f(x, y) \, dx \, dy$$

Fubini



Fubini Counter example

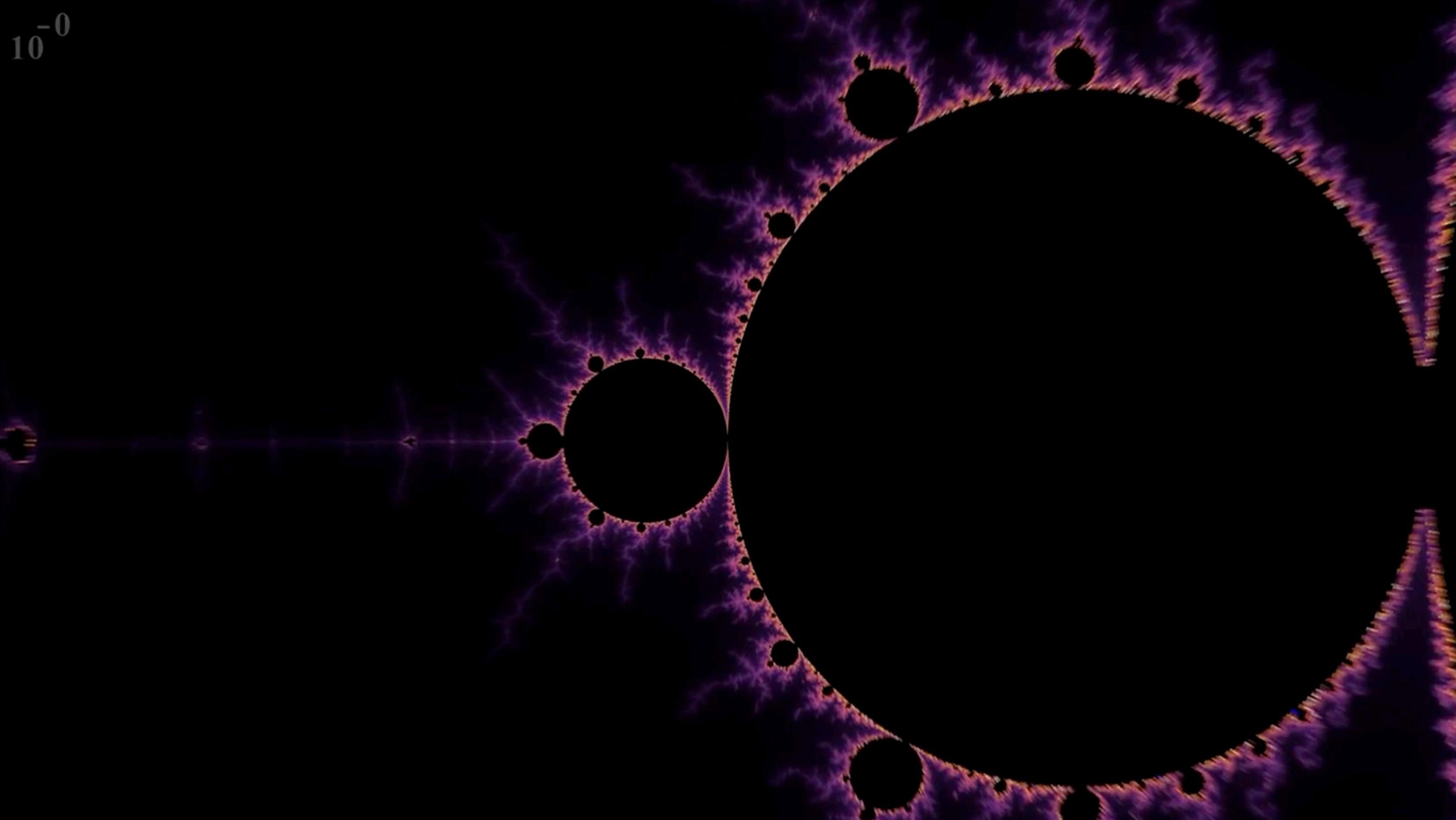


$$f(x, y) = \frac{(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\frac{d}{dx} \frac{-x}{x^2 + y^2} = f(x, y)$$

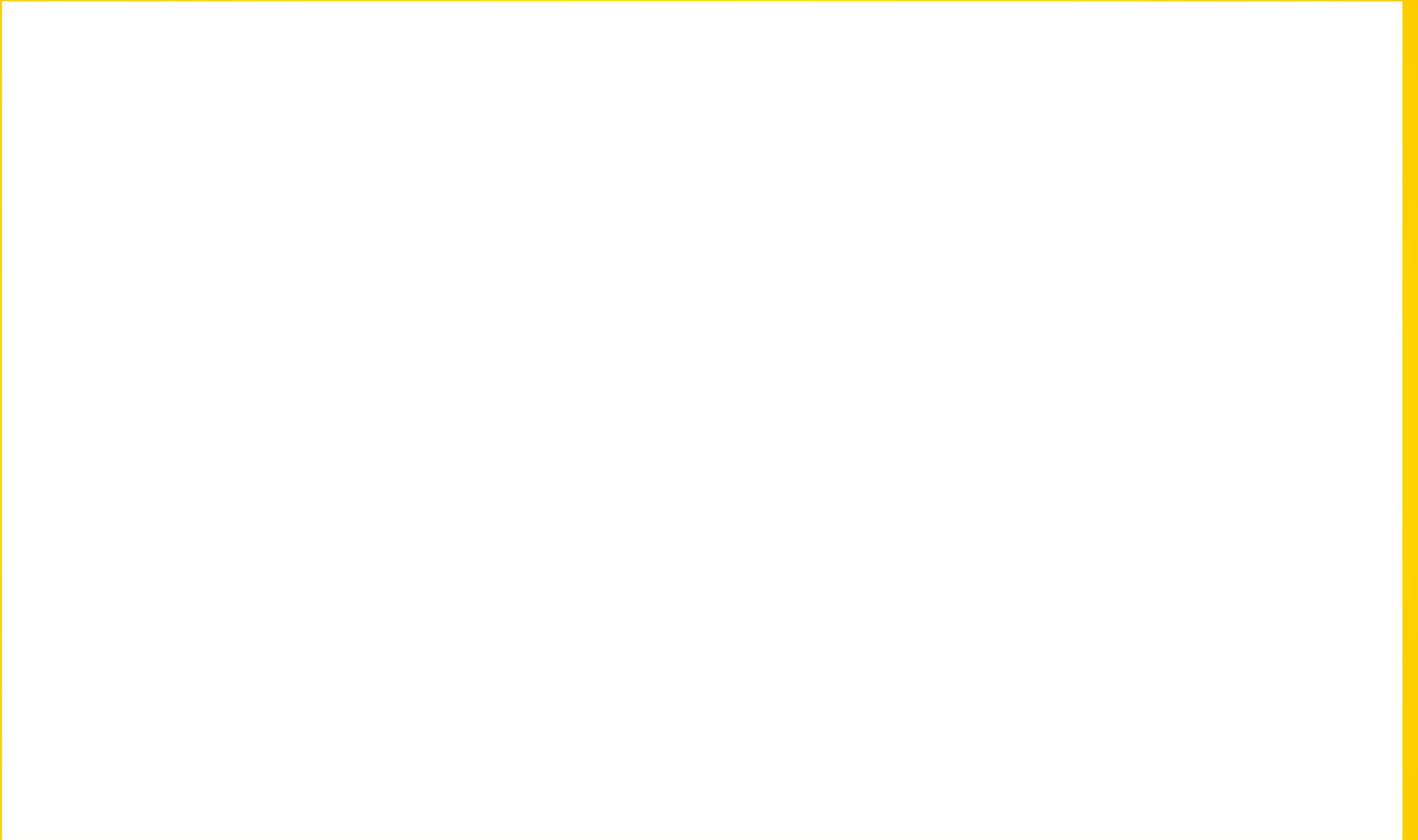
$$\frac{d}{dy} \frac{y}{x^2 + y^2} = f(x, y)$$

Area of Mandelbrot?



Homework

due Friday



THE END