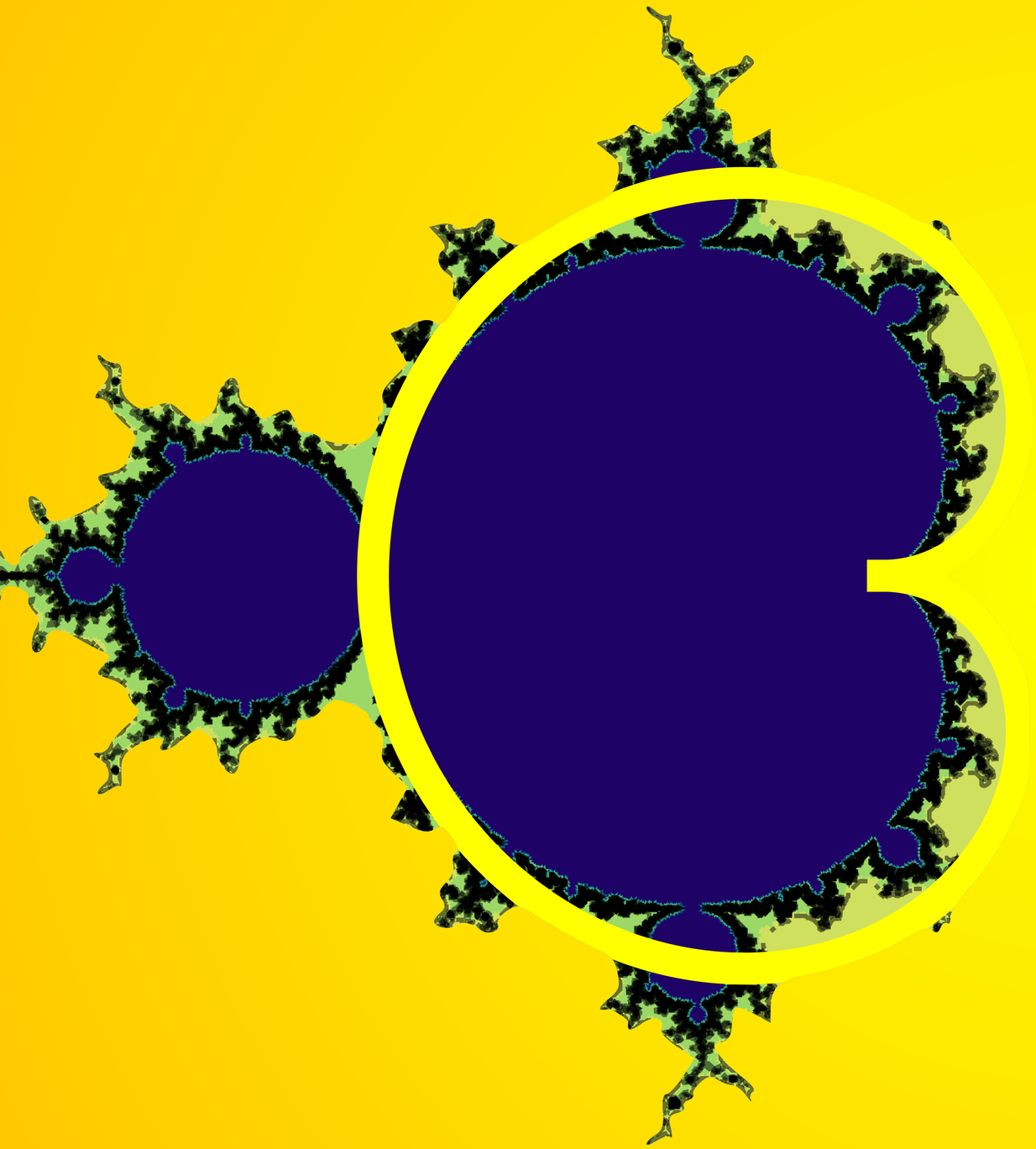

Lecture 6

Computing Integrals

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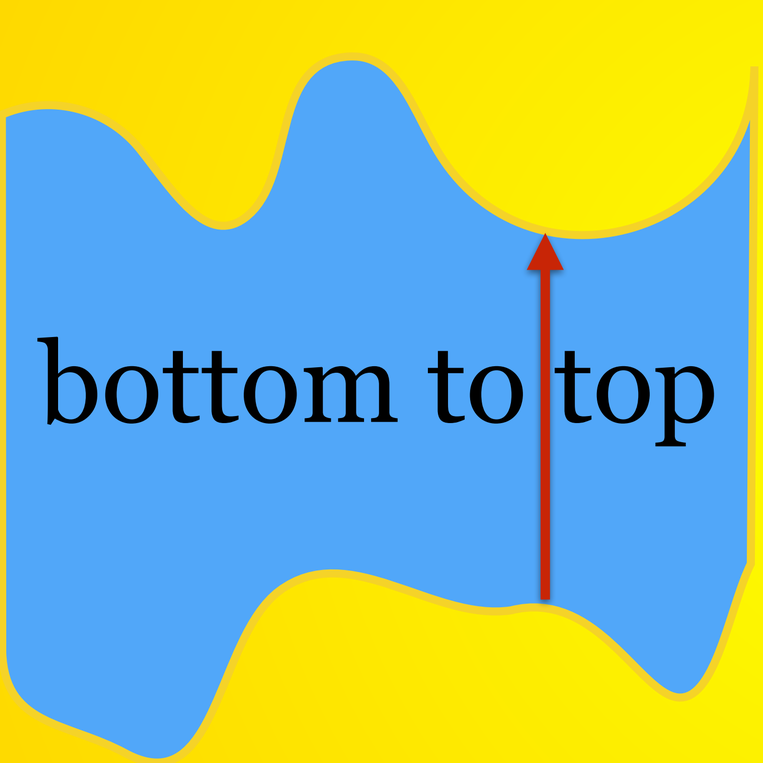
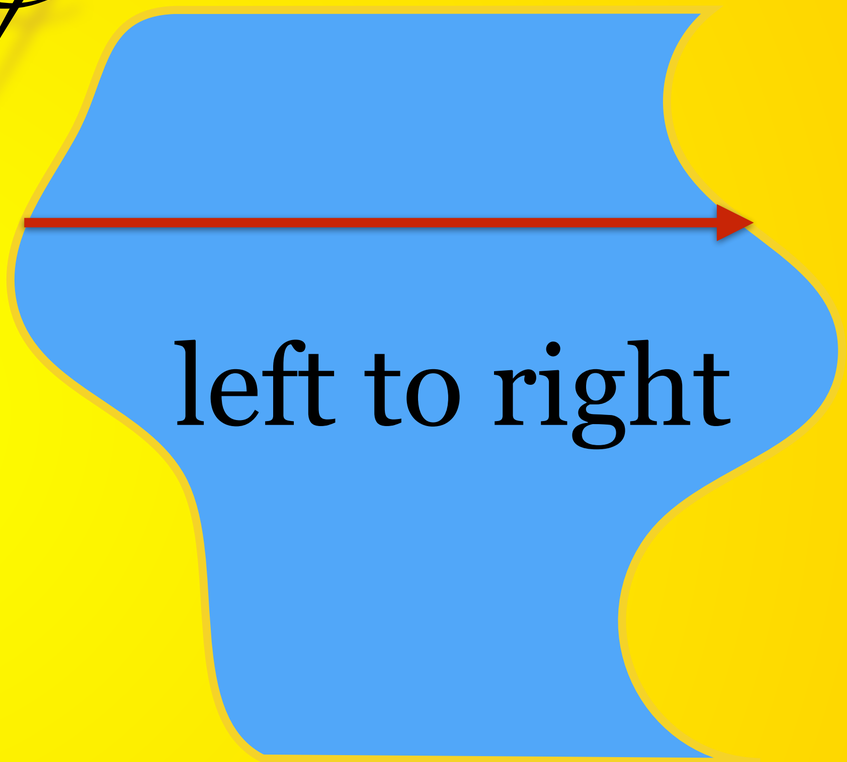
Basic Regions



$$\int_a^b \int_{g(x)}^{h(x)} f(x,y)\, dydx$$



$$\int_a^b \int_{g(y)}^{h(y)} f(x,y)\, dxdy$$

Reduce to single integral

How to do it

Make a picture

Decide how to slice

Write down outer integral

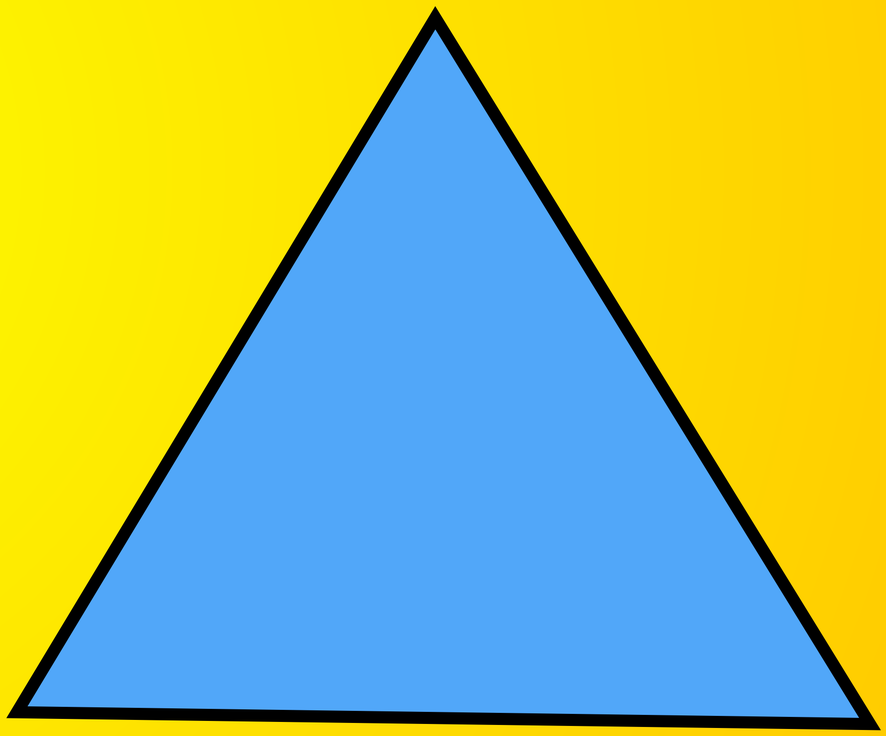
Write down inner integral

Start evaluating inner integral

Examples

A

Integrate $f(x,y)=y$ over R bound by $y=0, y=1+x, y=1-x$



A

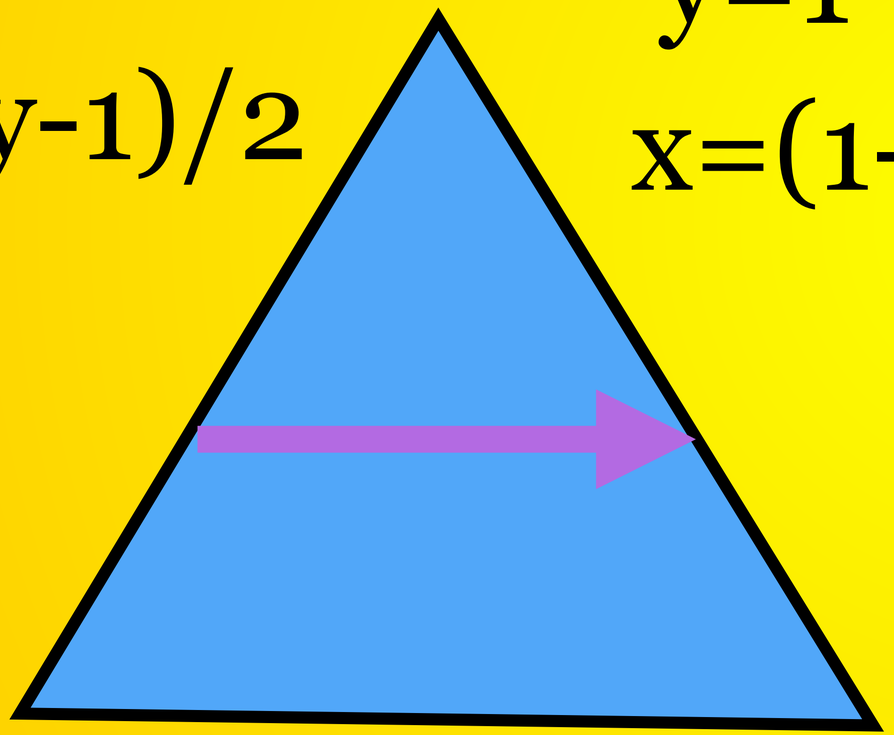
Slice Horizontally

$y=1+2x$

$x=(y-1)/2$

$y=1-2x$

$x=(1-y)/2$

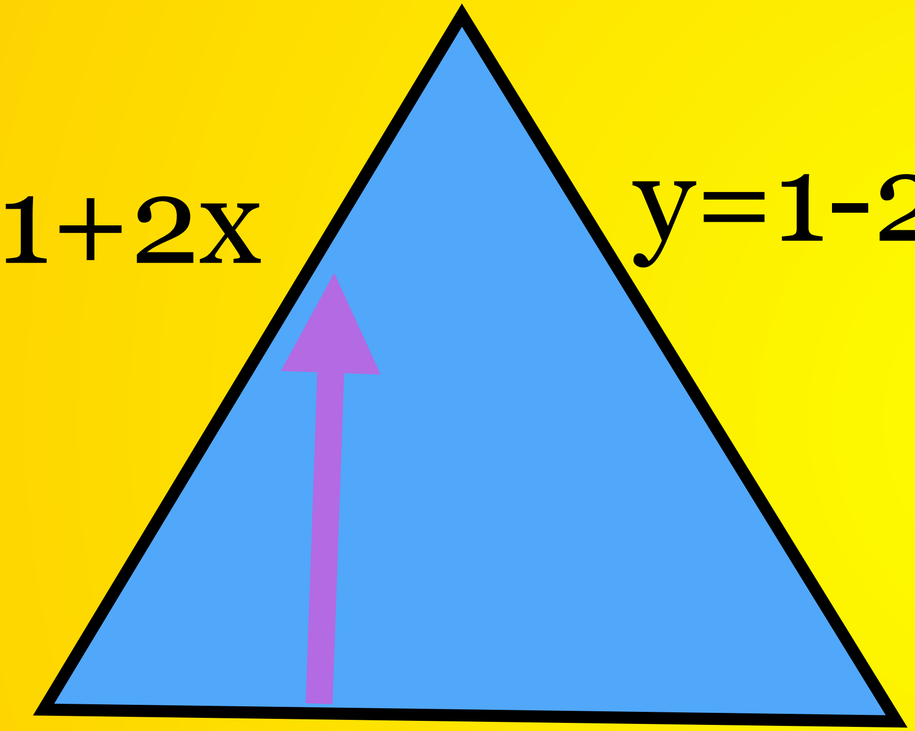


$y=0$

$$\int_0^1 \int_{(y-1)/2}^{(1-y)/2} y\,dx\,dy$$

A

Slice Vertically



We need to split

$$\int_{-1/2}^{0}\int_{0}^{2x} y\,dx\,dy$$

$$+\int_{0}^{1/2}\int_{0}^{1-2x} y\,dx\,dy$$

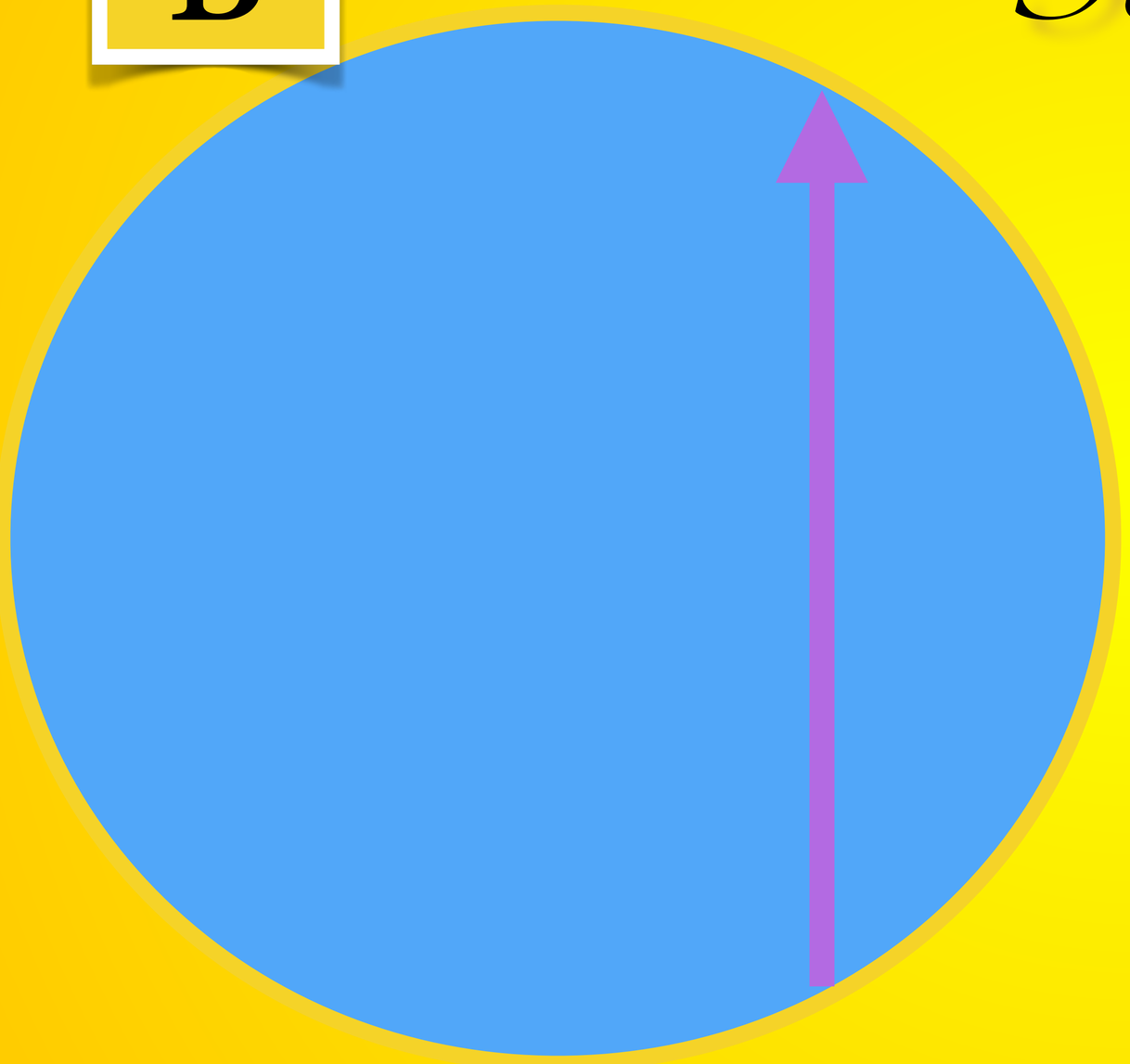
B

What is $\iint_R 1/\sqrt{1-x^2}\, dA$

if R is the unit disk?

B

Slice Vertically

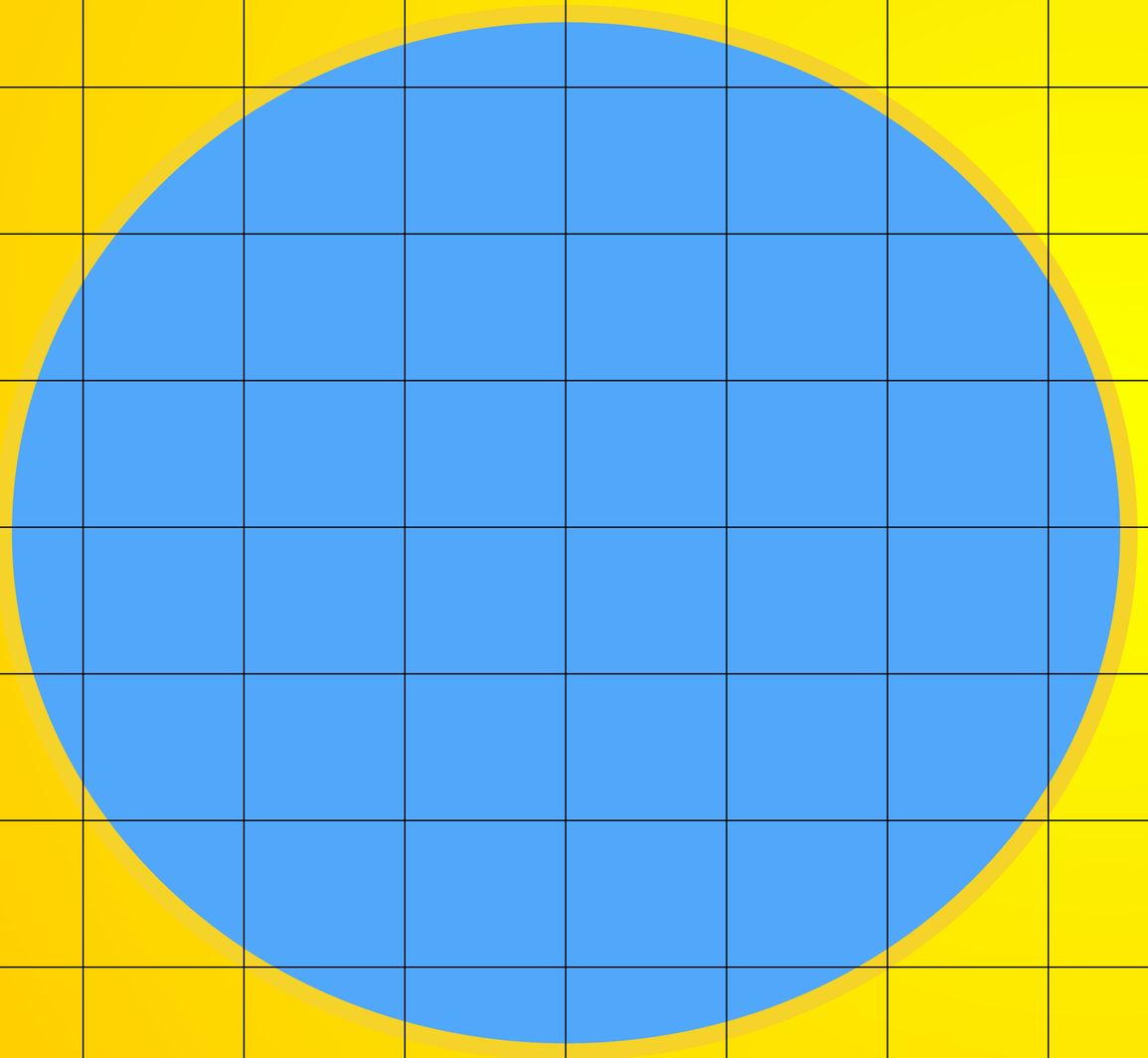


$$\iint_R 1/\sqrt{1-x^2}\,dydx$$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2}}\,dydx$$

$$\int_{-1}^{1} 2\,dx = 4$$

c



Apropos:

Disk

Area

$$2\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

-> Monday

EXTRA CREDIT

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

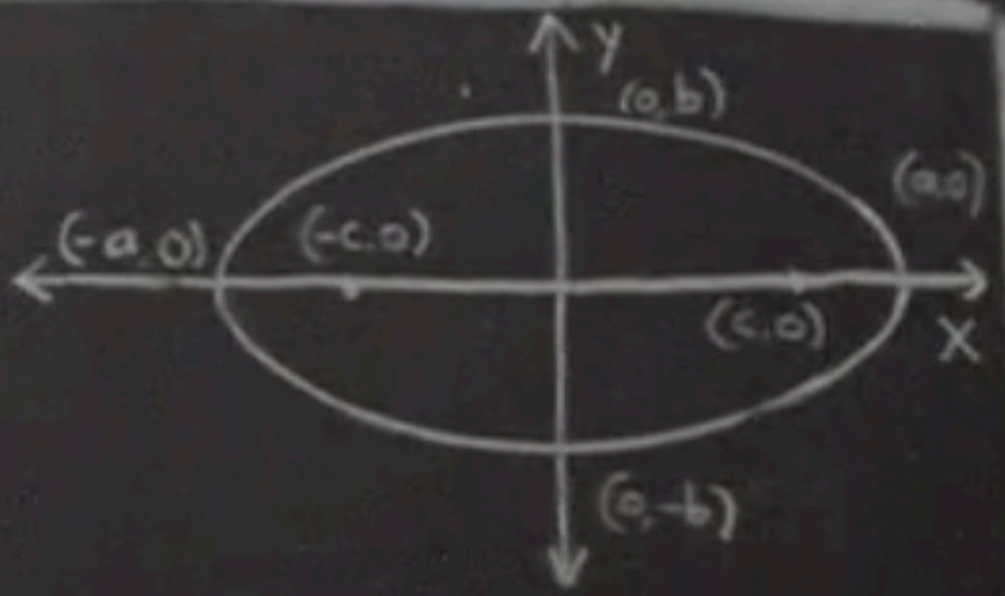
$e \to 0 \Rightarrow a \to b \to r$

$e \to 0 \Rightarrow c \wedge -c \to 0$

$A_{\odot} = \pi r^2$

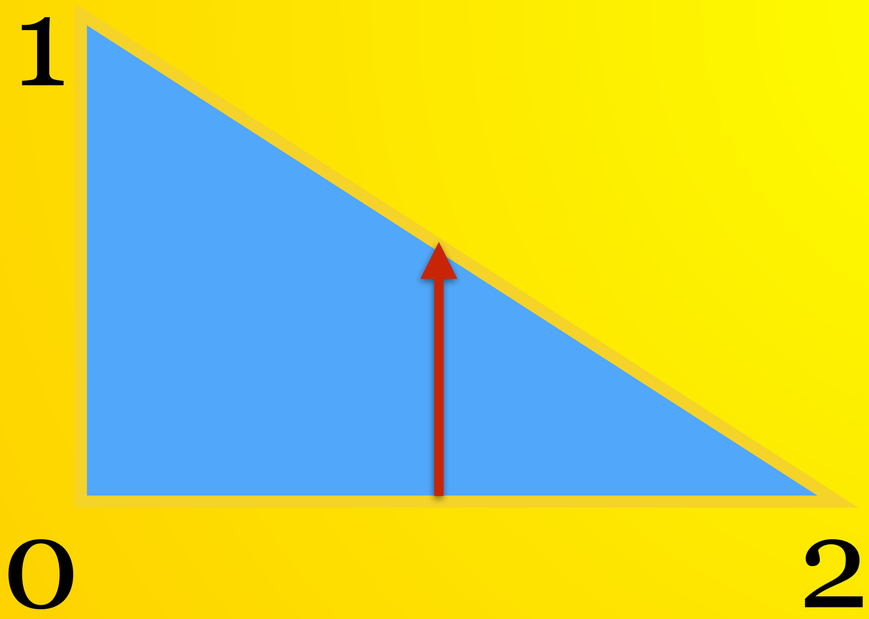
TO PROVE: $A_E = \pi ab$

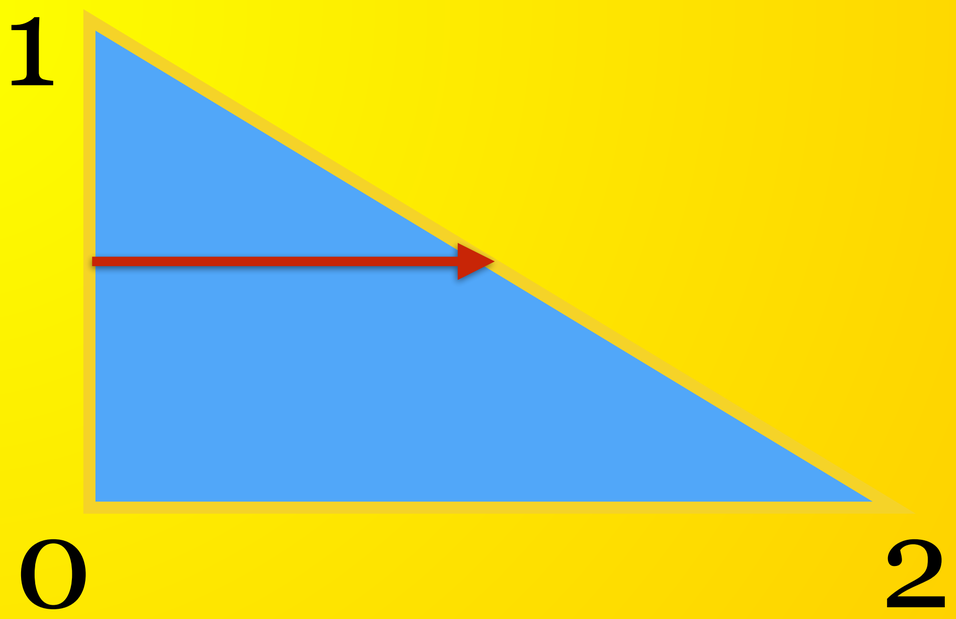
$$\left(A_E = \int_{-a}^{a} (y_2 - y_1)\, dx\right)$$




Rushmore

Change order





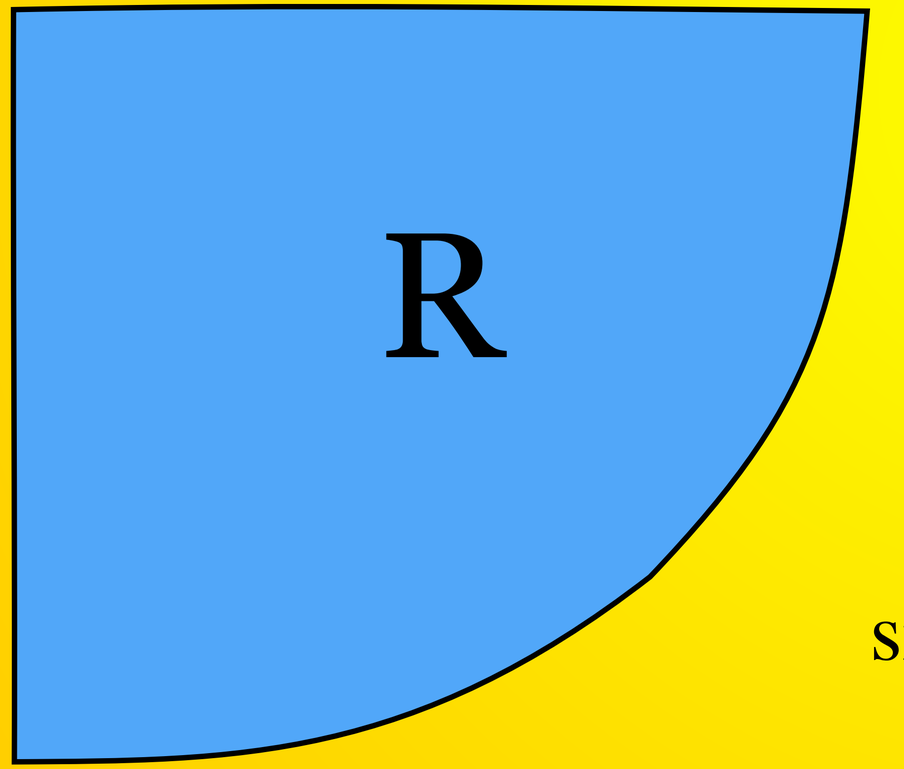
Worksheet Problem 6

$$\int_0^1 \int_{-\sqrt{1-x^2}}^0 2x \cos(y - y^3/3)\, dydx$$

We can not get started here! What to do?

$$\int_0^1 \int_{-\sqrt{1-x^2}}^0 2x\cos(y-\frac{y^3}{3})\,dydx$$

Change the order:



$$\int_{-1}^0 \int_0^{\sqrt{1-y^2}} 2x\cos(y-\frac{y^3}{3})\,dxdy$$

$$\int_{-1}^0 (1-y^2)\cos(y-\frac{y^3}{3})\,dy$$

$$\sin(y-y^3/3)\,|_{-1}^0 = \sin(2/3)$$

Homework due Monday

THE END