

Lecture 7

Polar Integrals

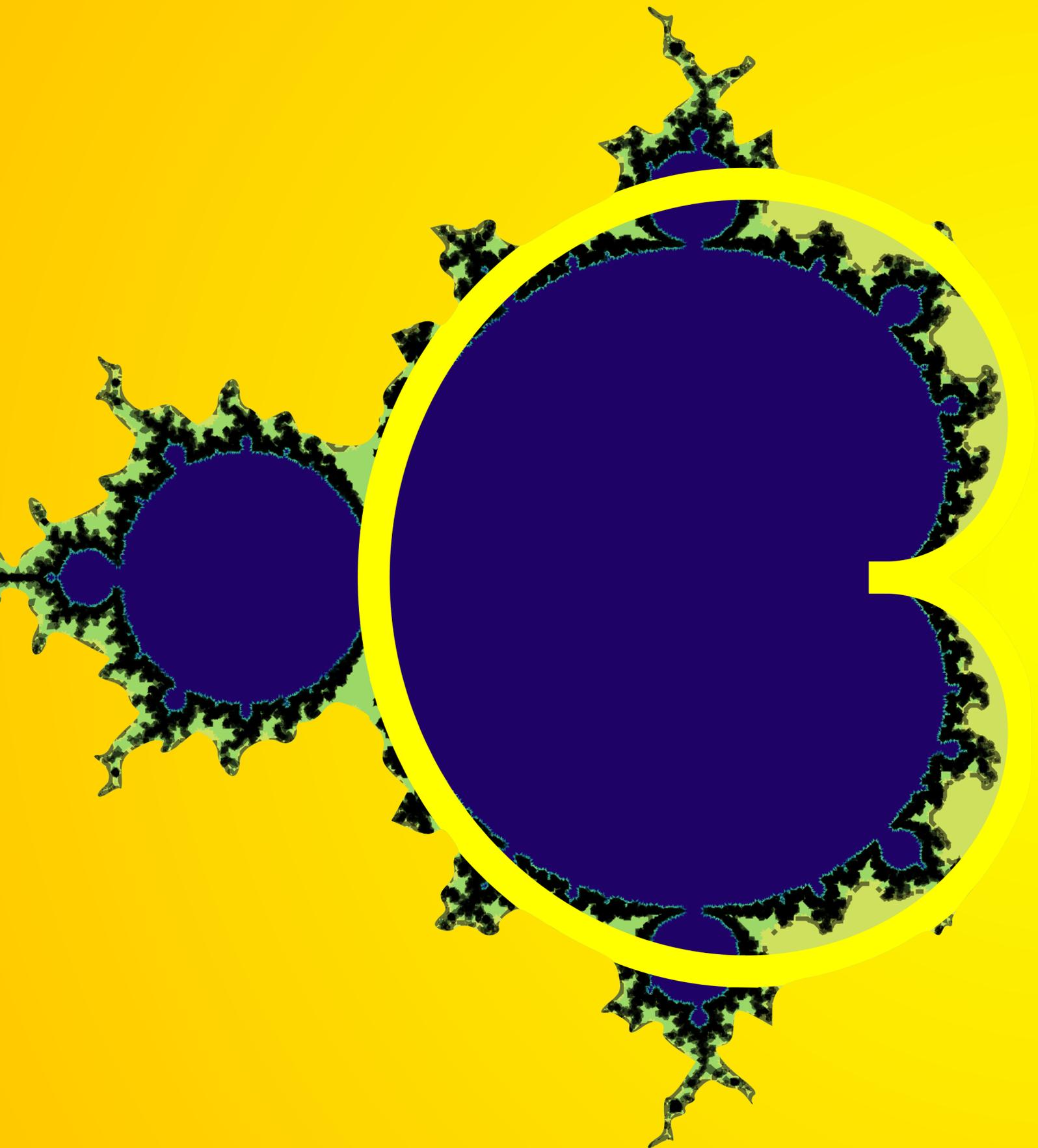


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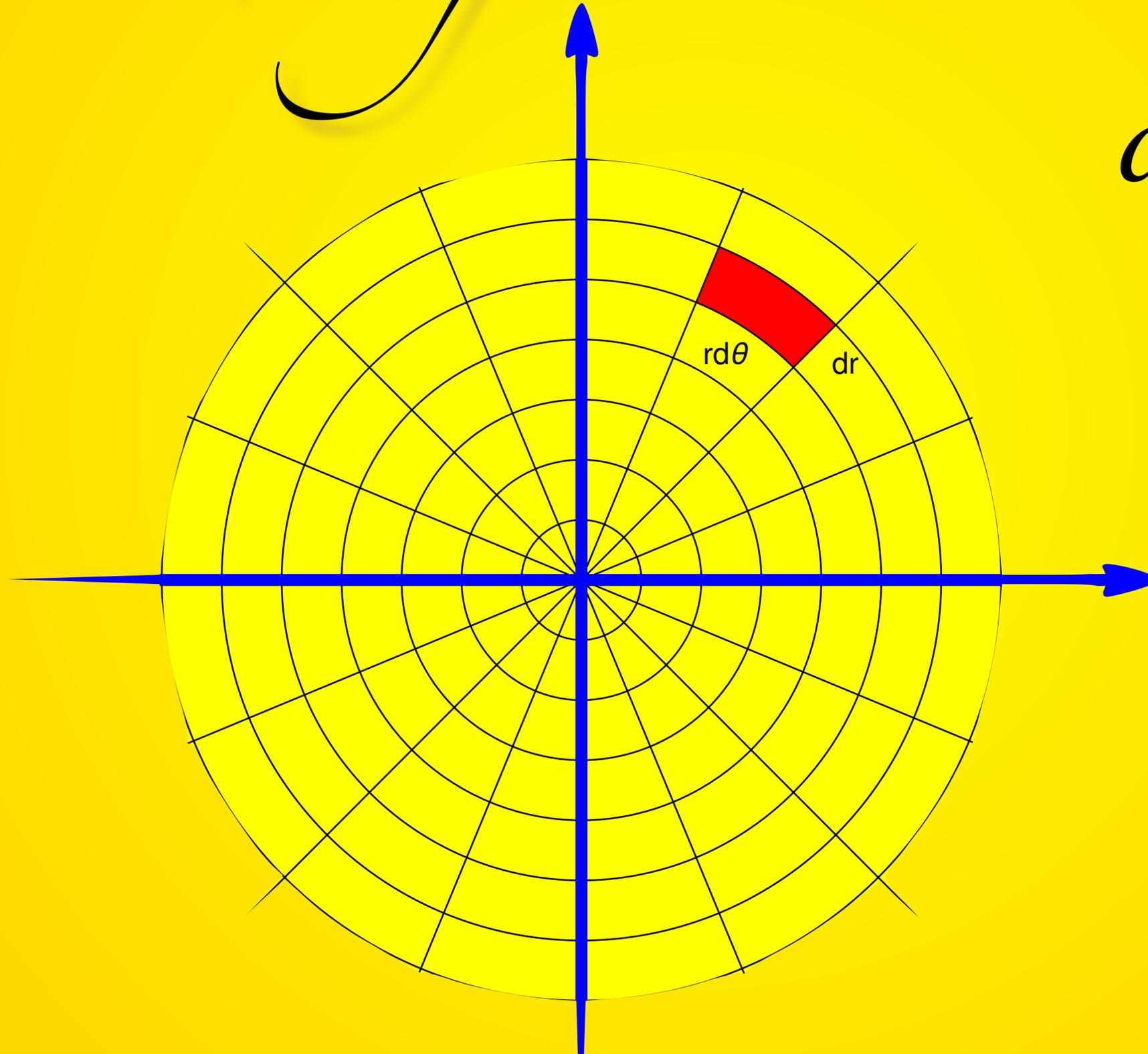
Integration factor

Integration Factor

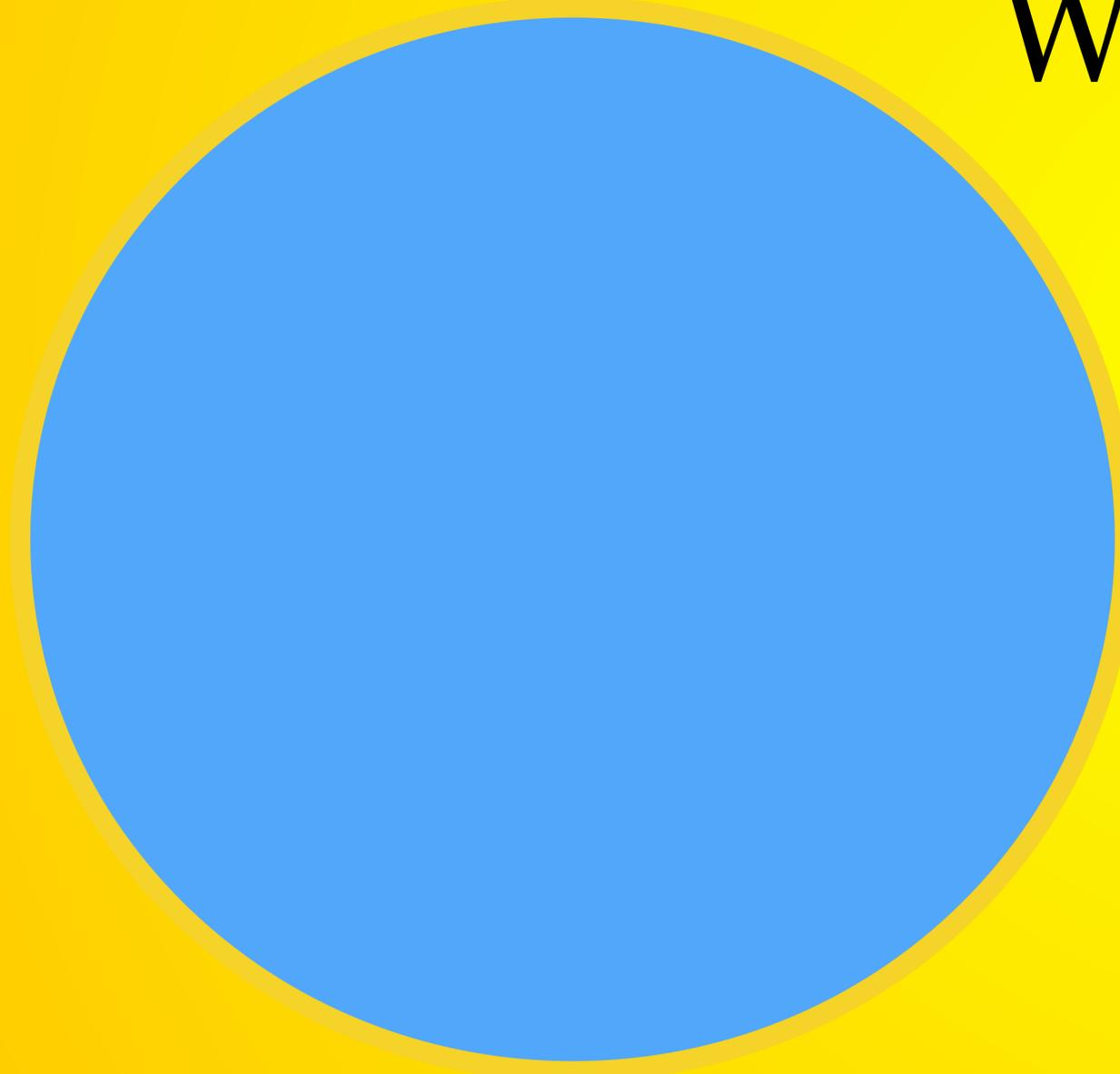
When converting to polar coordinates, include an integration factor r .

Argument

$$dA = r dr d\theta$$



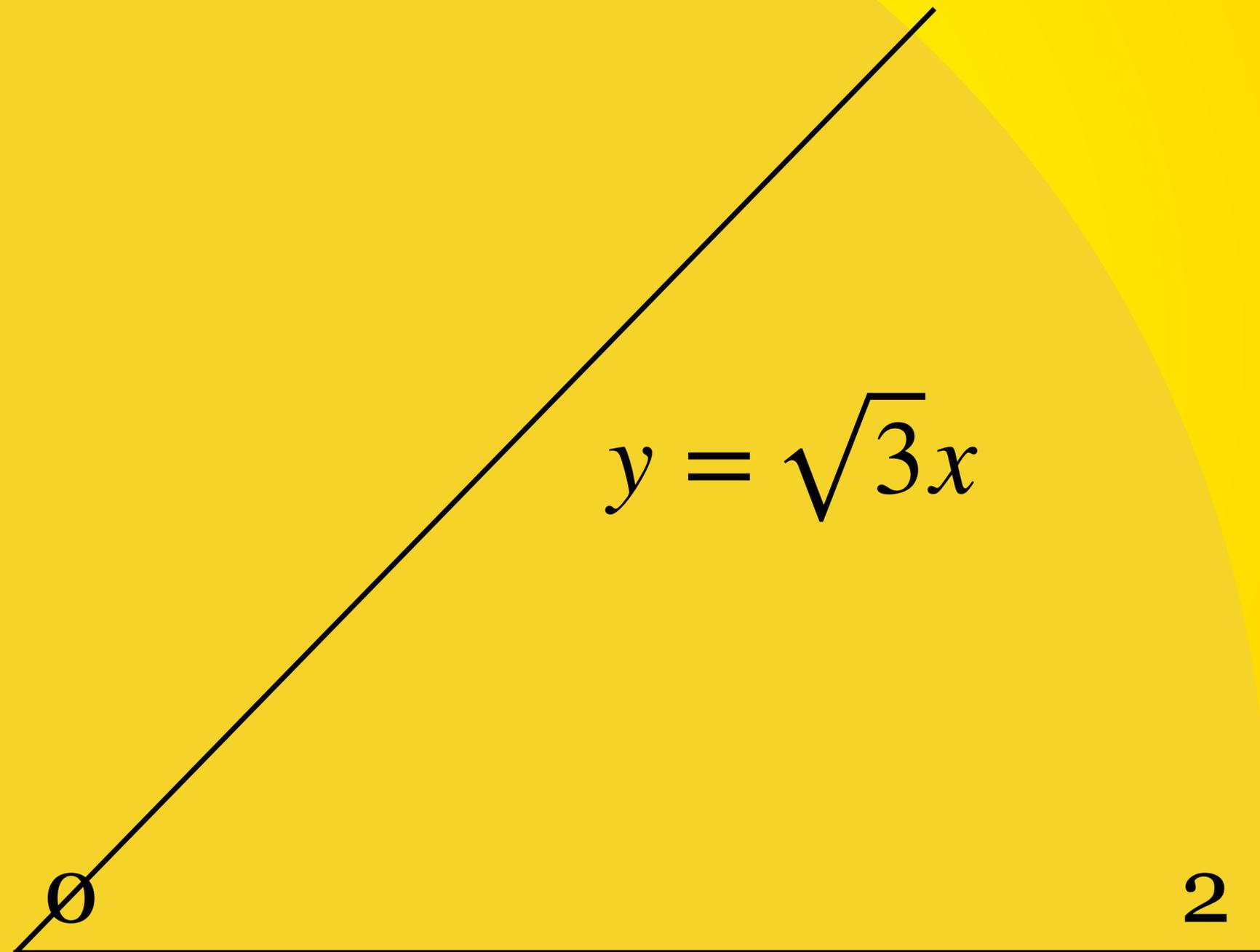
Examples



What is $\iint_R x^2 + y^2 dA$

if R is the unit disk?

Problem 4



From Statistics

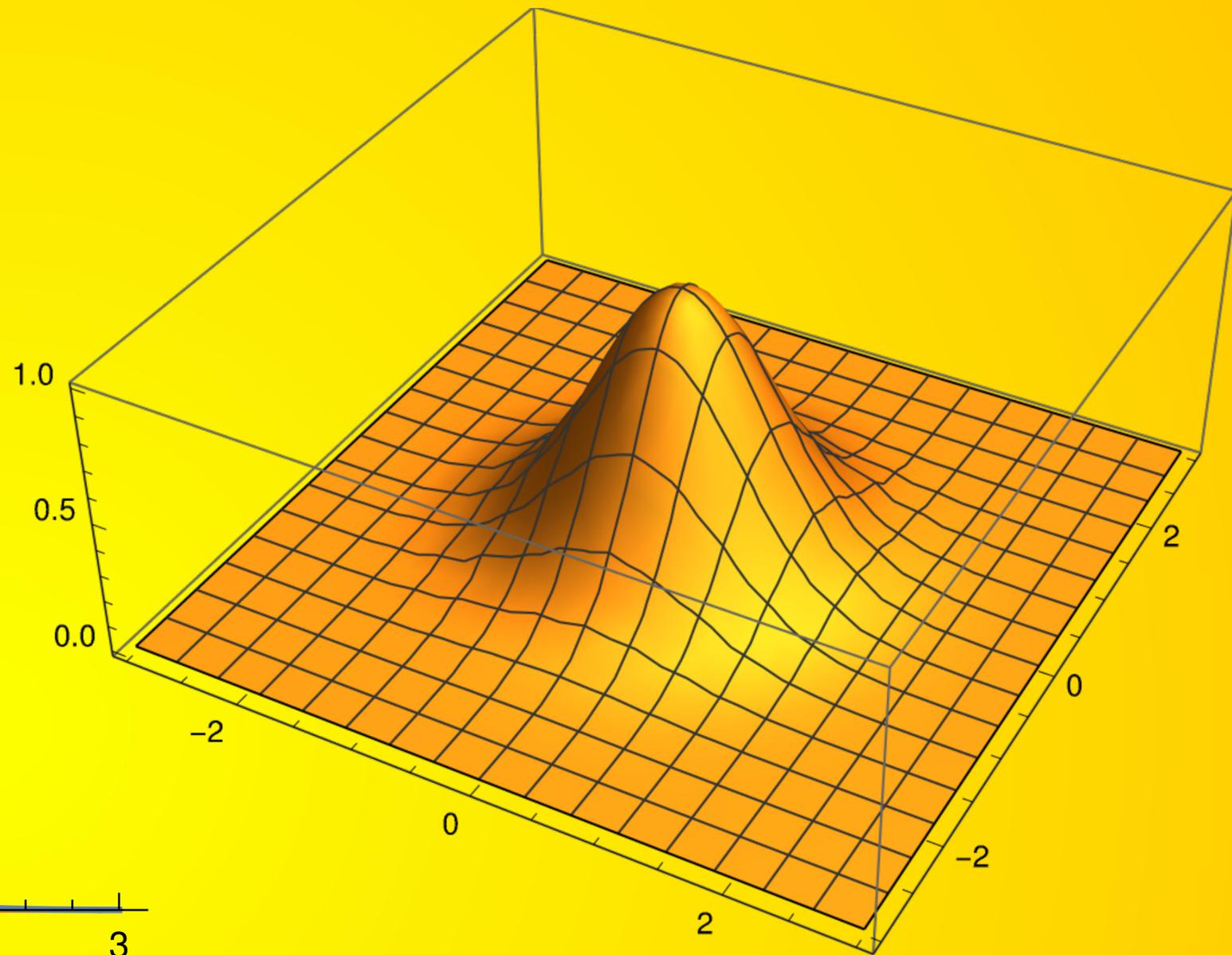
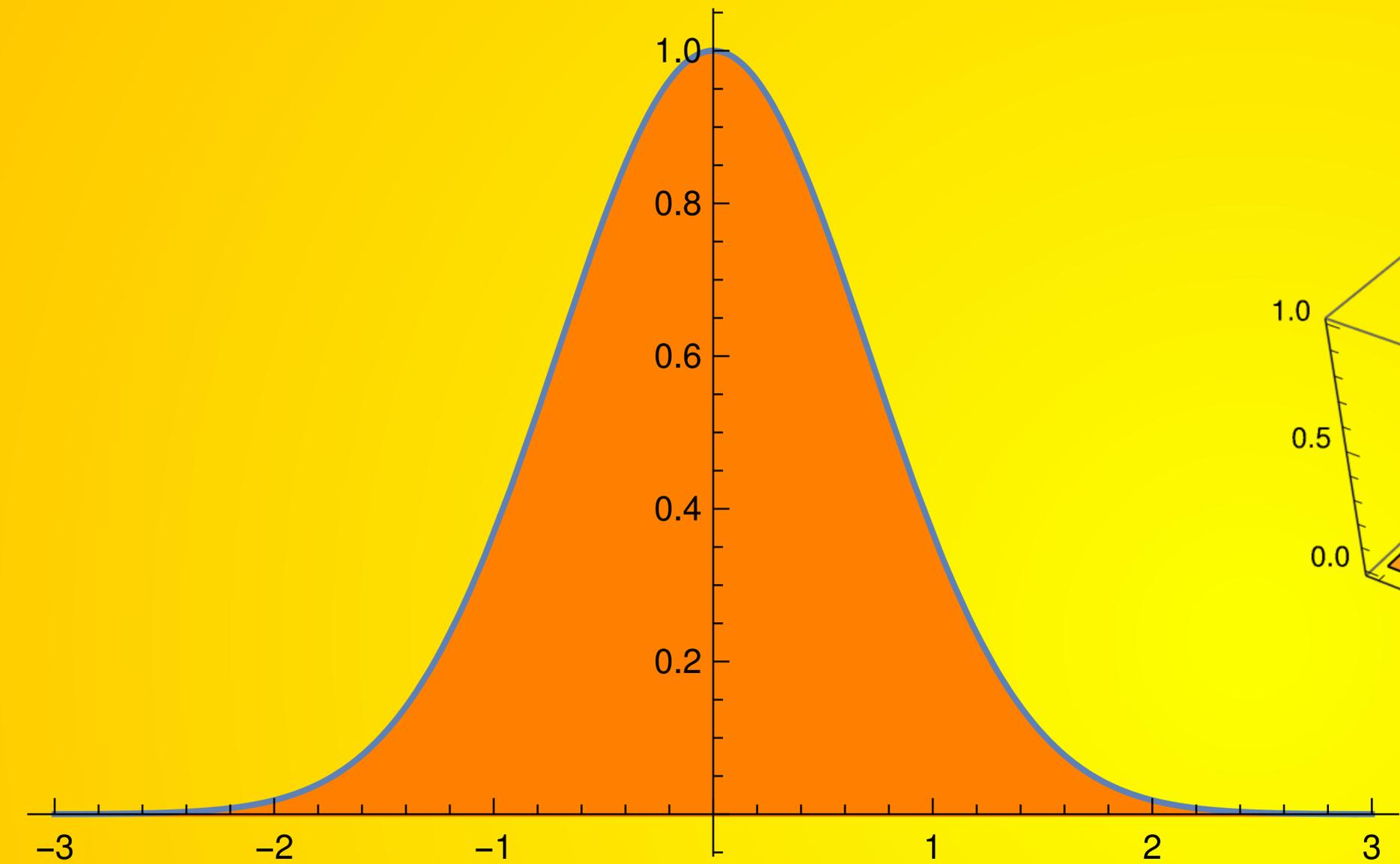
$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

Problem

Show that $\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx = \sqrt{2\pi} \sigma$

Hint: First show that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)/2\sigma^2} dx dy = 2\pi \sigma^2$$



$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\iint_{\mathbb{R}^2} e^{-x^2-y^2} dA = \pi$$

Problem 7

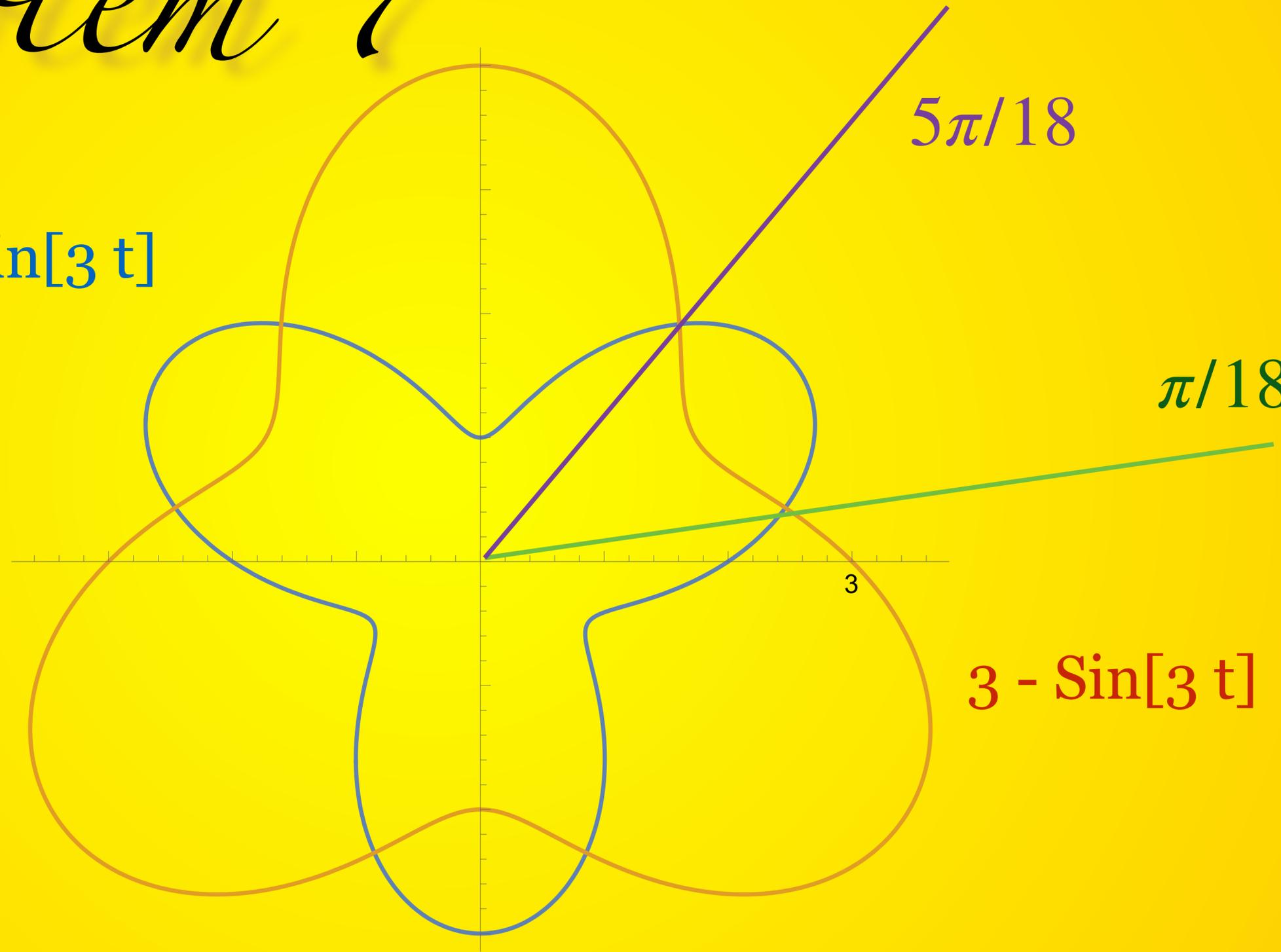
$$2 + \sin[3 t]$$

$$5\pi/18$$

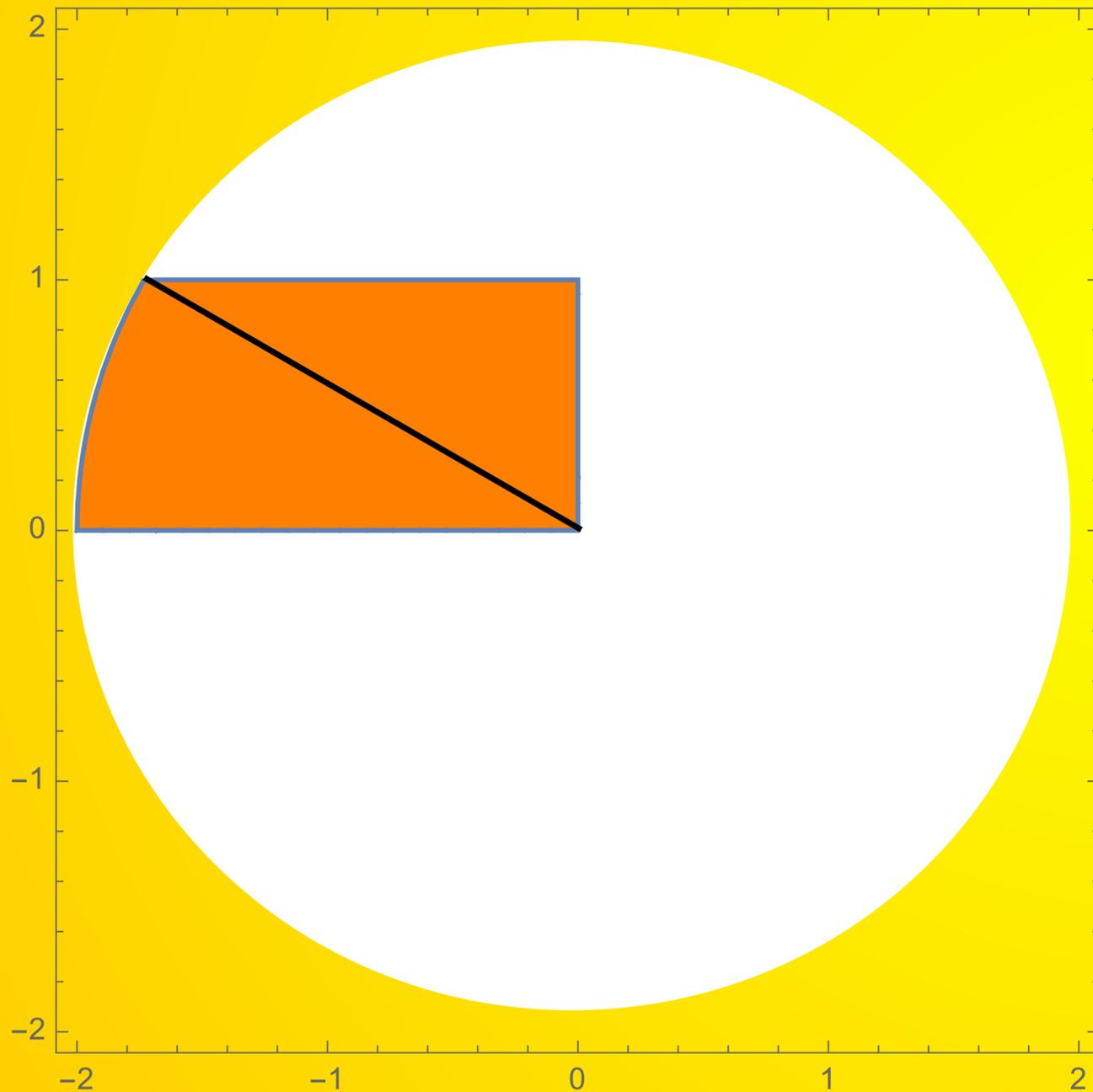
$$\pi/18$$

$$3 - \sin[3 t]$$

3



Problem 8



(Either slice horizontally and use cartesian coordinates (still involving square roots)
Or then split into two parts
The triangle can be done in Cartesian,
the sector in polar coordinates.)

Homework due Wednesday

THE END