

Lecture 10

Spherical

Integrals

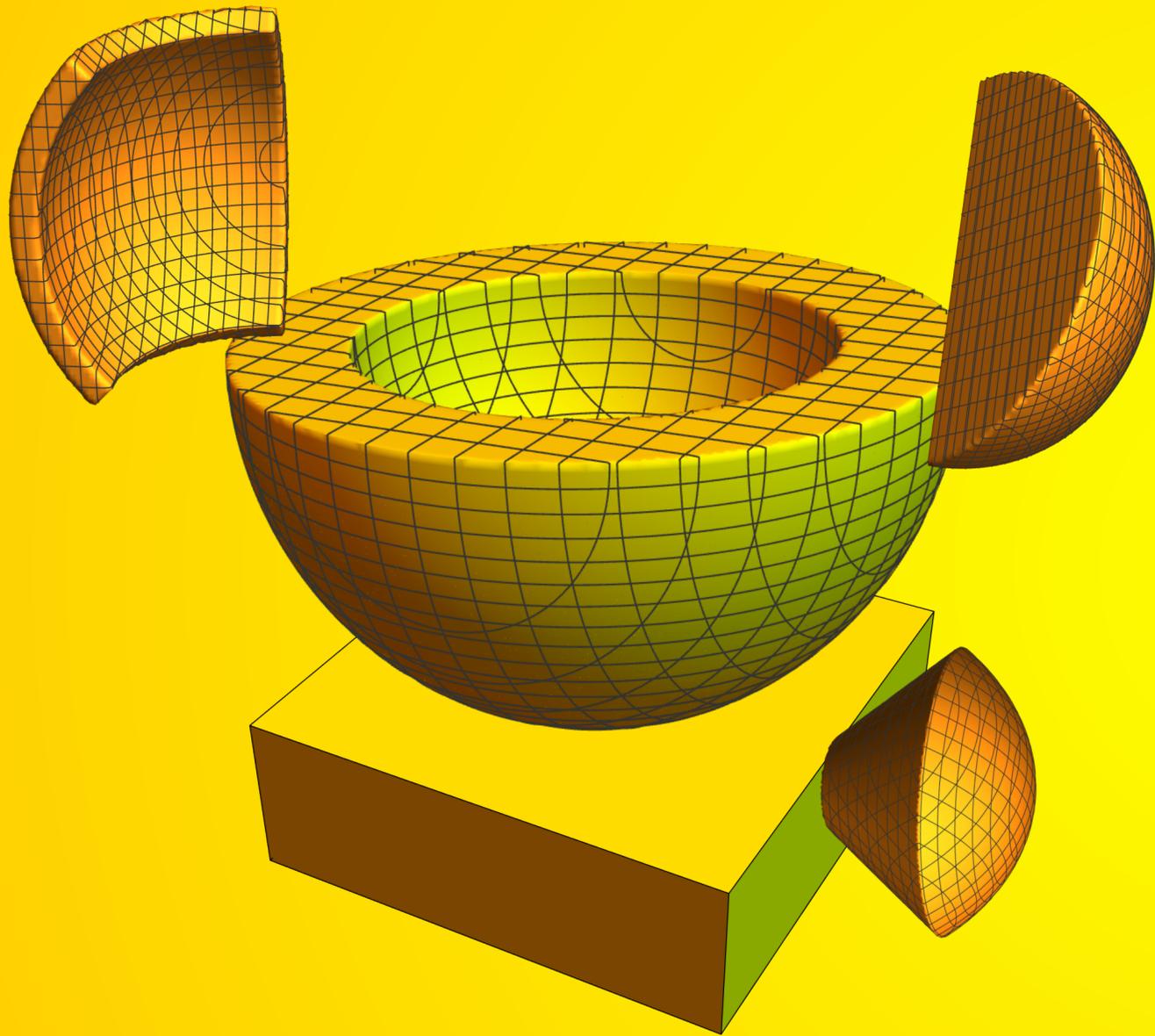


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1) The integration factor

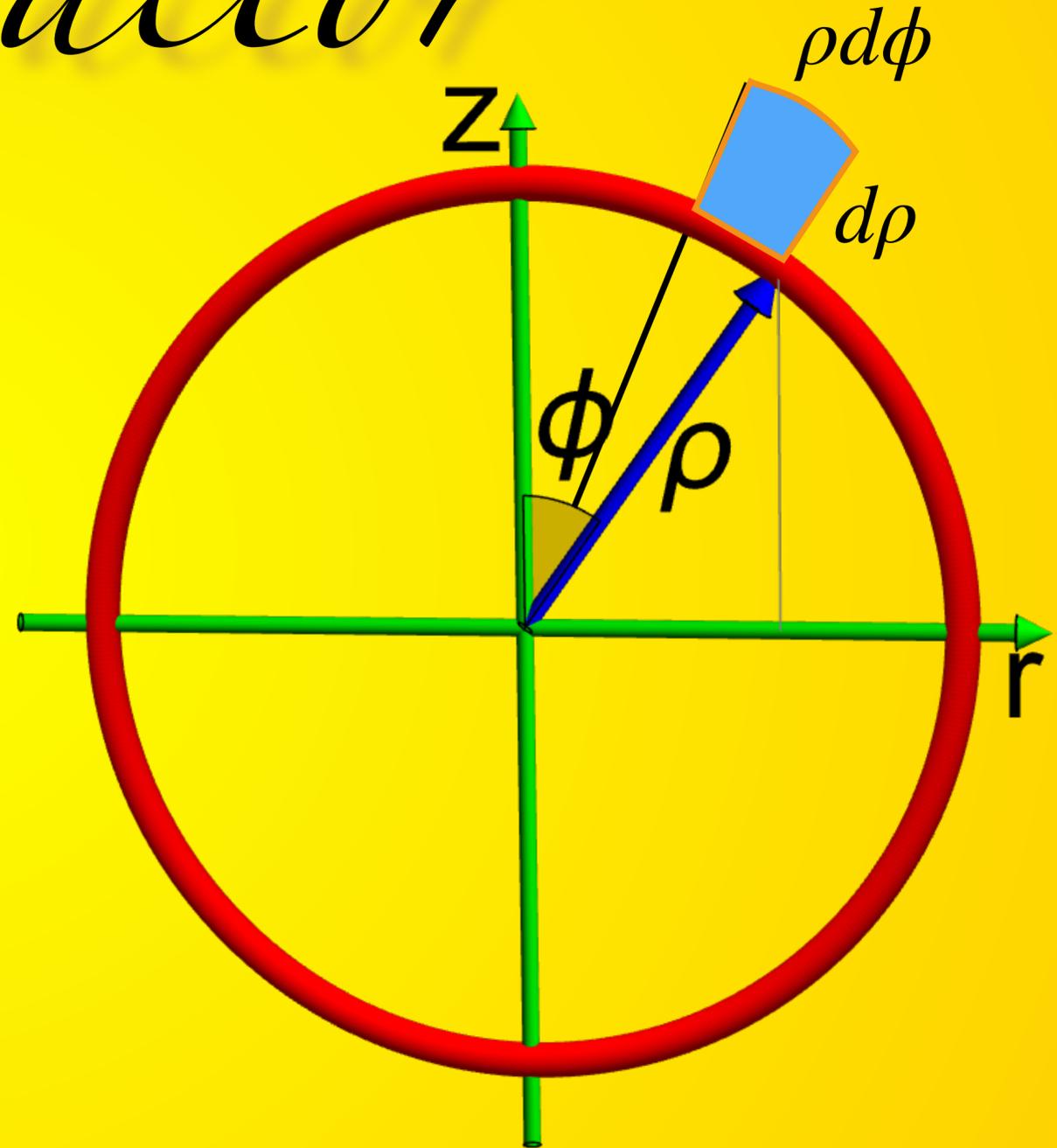
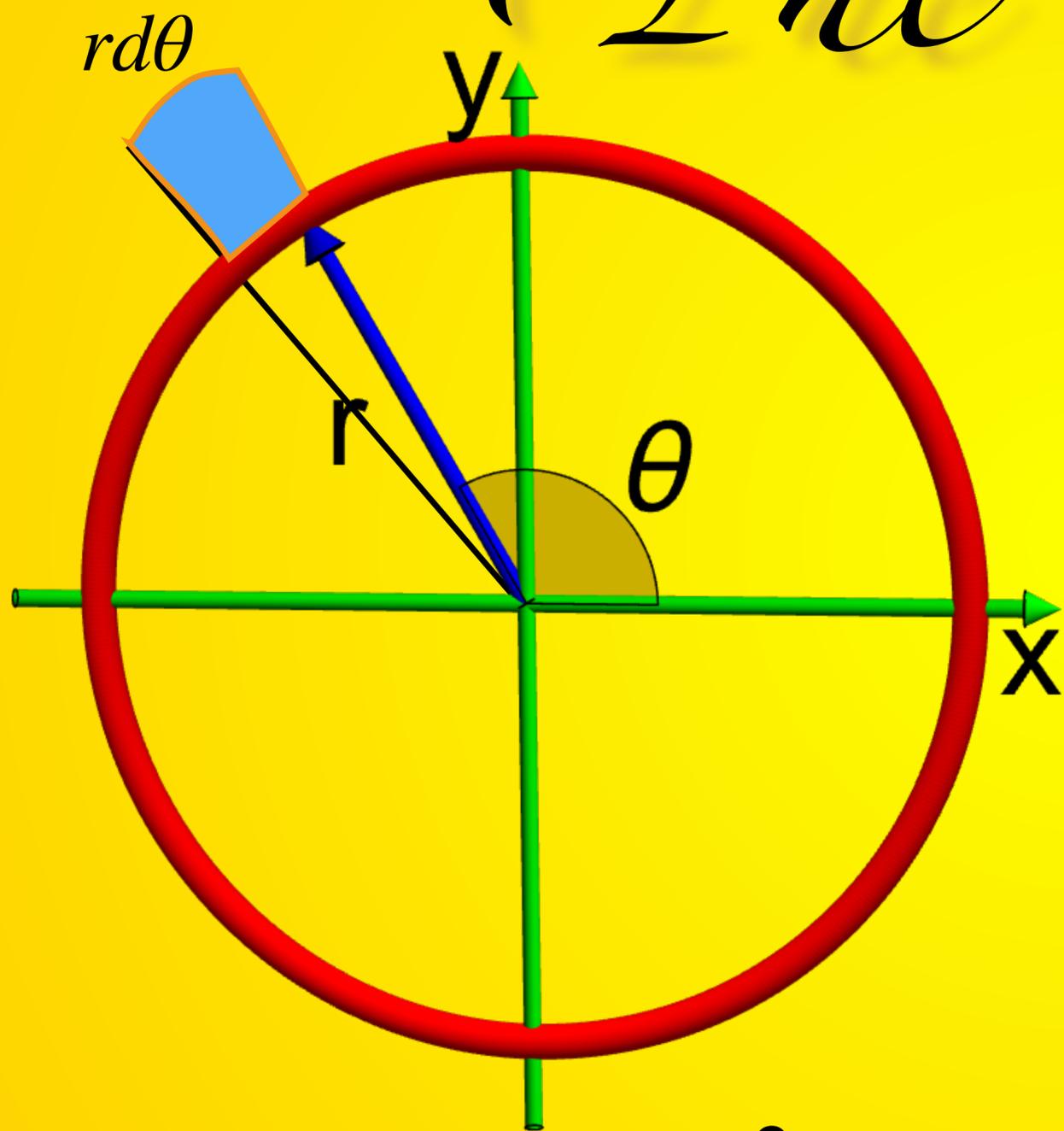
2) The sphere moment of inertia

3) The Octahedron

4) The Sphere

5) Worksheet problems

The Factor



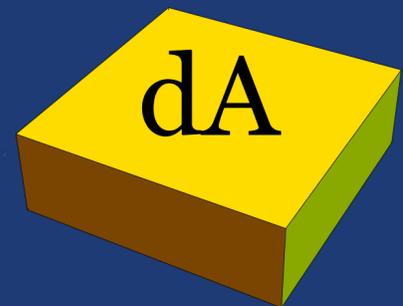
$$\rho^2 \sin(\phi) d\rho d\theta d\phi$$

$$\phi = 0$$


$$dA \quad \rho d\phi$$

$$\rho^2 \sin(\phi) d\theta$$

$$\phi = \frac{\pi}{2}$$


$$dA \quad d\rho$$

$$dA = \rho^2 \sin(\phi) d\theta d\phi$$

$$dV = dA d\rho$$

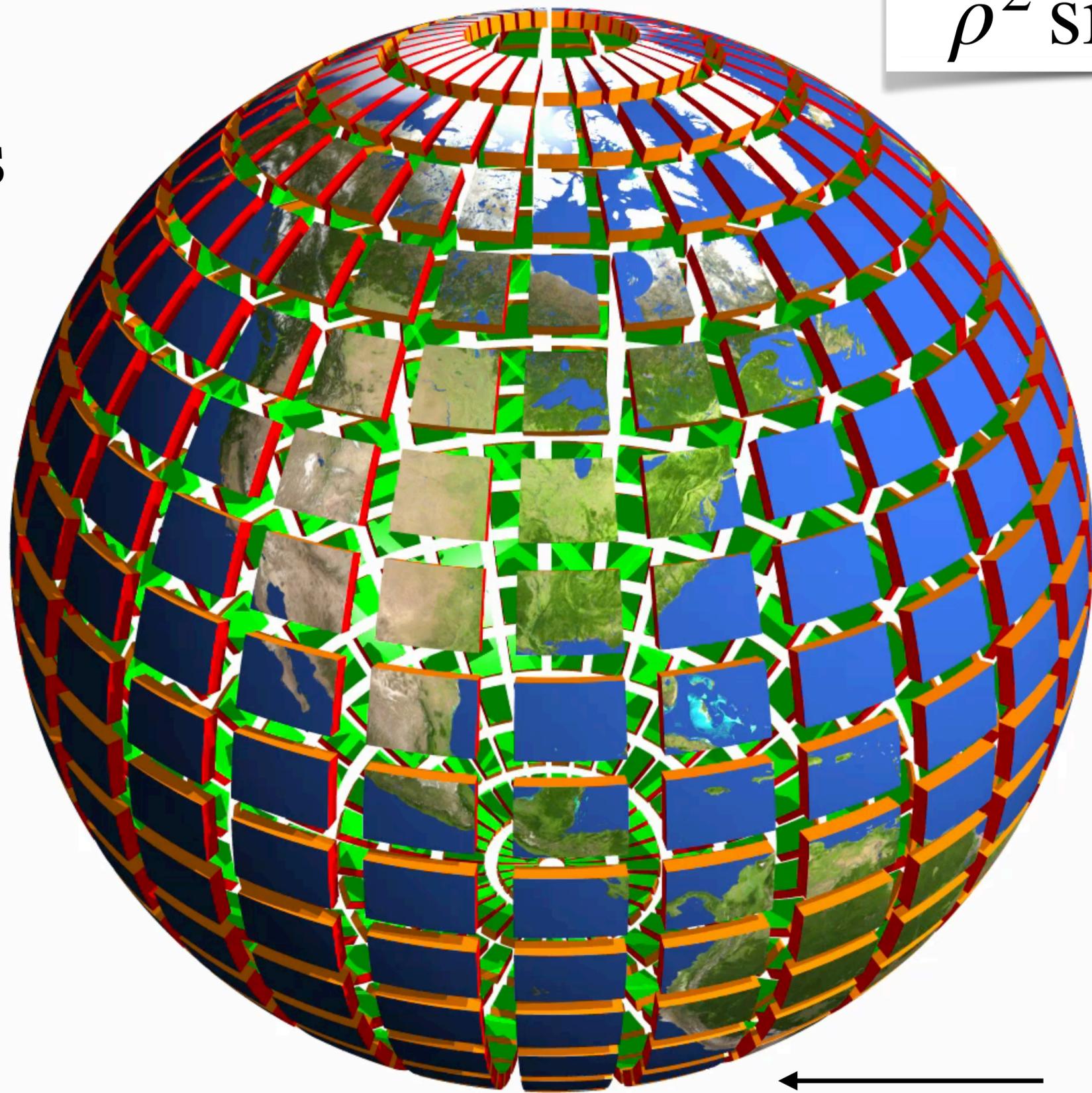
$$\phi = \pi$$

$$\theta = 0$$

$$\rho^2 \sin(\phi) d\rho d\theta d\phi$$

area of
sphere grows
like ρ^2

distortion
factor

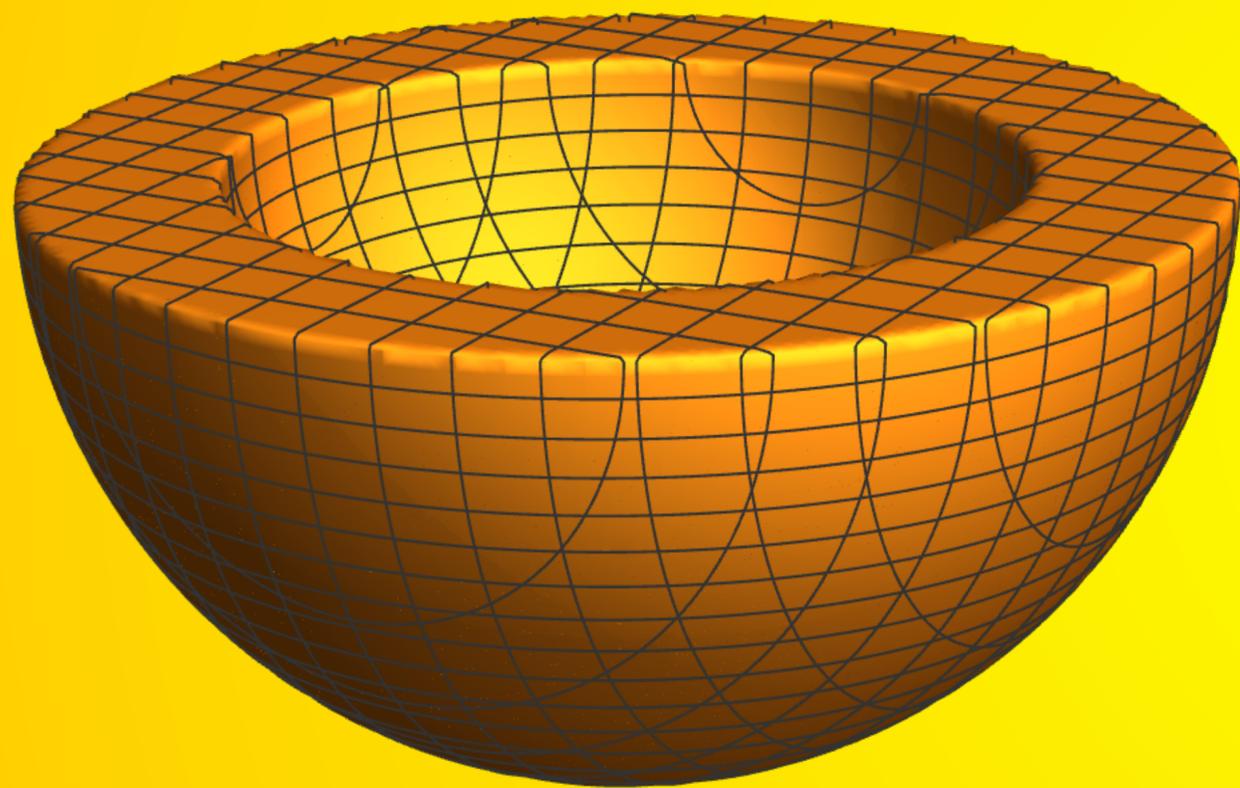


$\sin(\phi)$
large

$\sin(\phi)$
small

Some Solids

Avocado

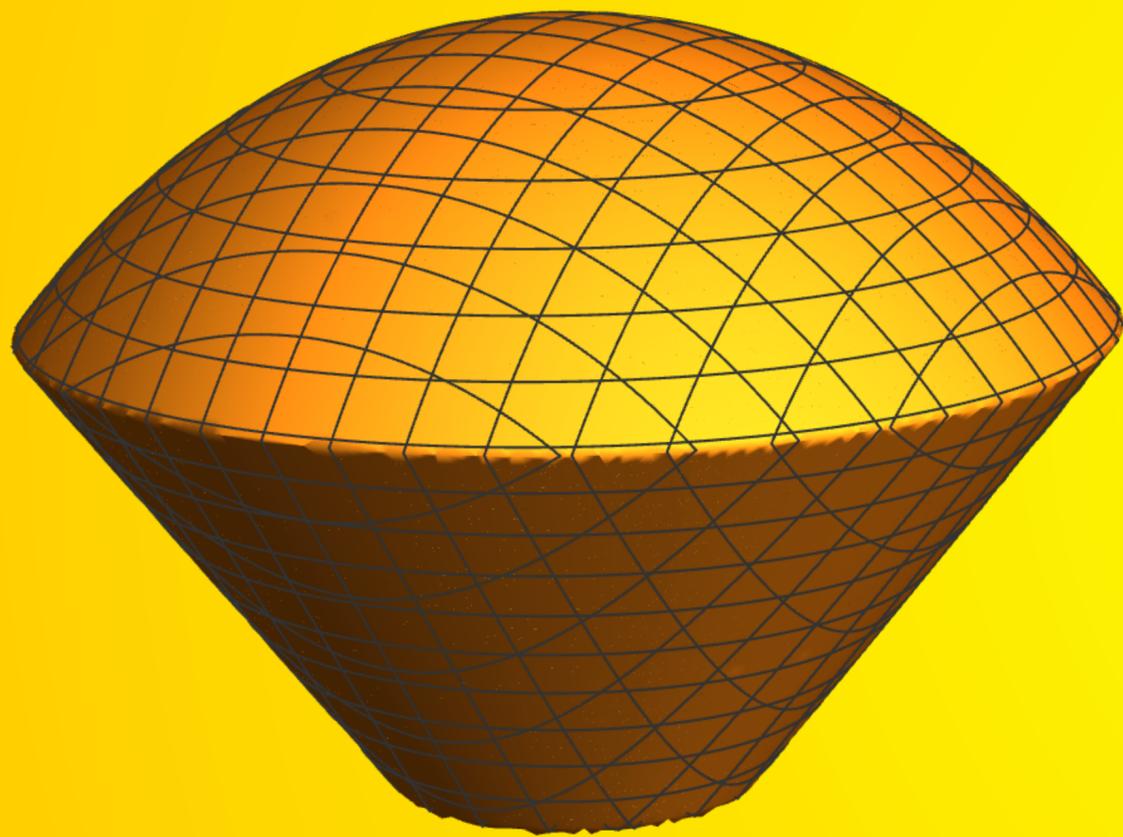


$$\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_2^3 f(x, y, z) \rho^2 \sin(\phi) d\rho d\phi d\theta$$

$$4 < x^2 + y^2 + z^2 < 9$$

$$z < 0$$

Muffin

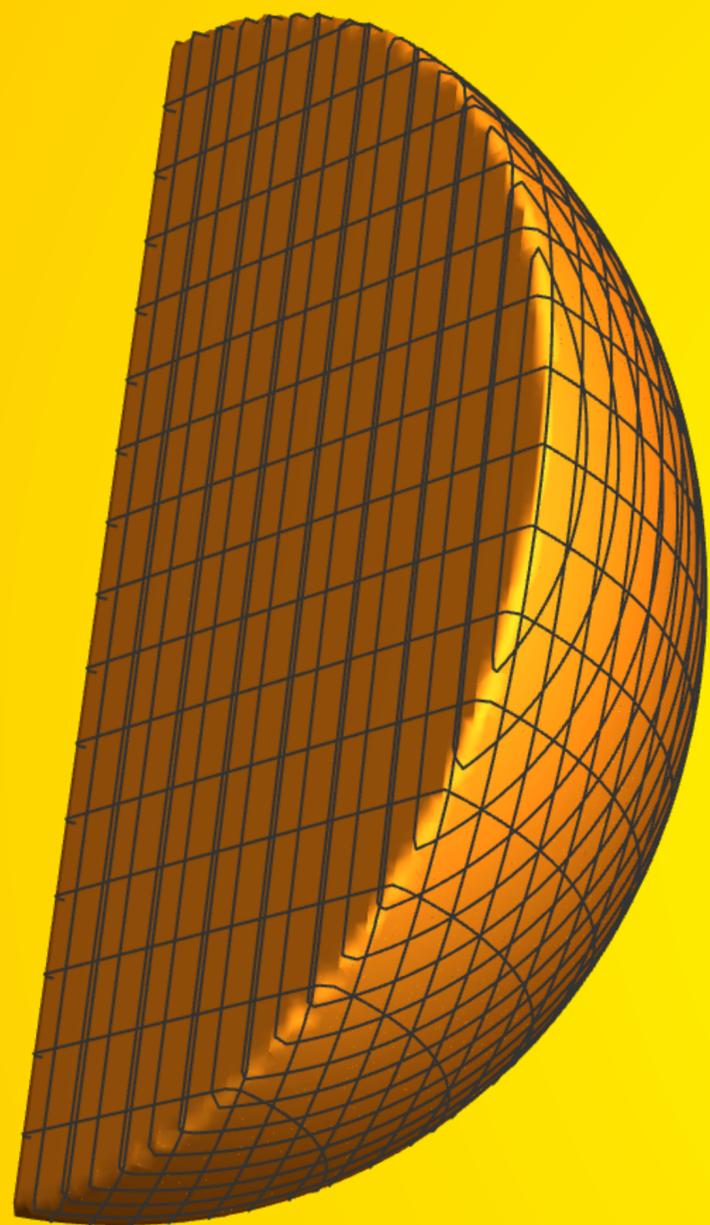


$$\int_0^{2\pi} \int_0^{\pi/4} \int_1^3 f(x, y, z) \rho^2 \sin(\phi) d\rho d\phi d\theta$$

$$1 < x^2 + y^2 + z^2 < 9$$

$$x^2 + y^2 < z^2$$

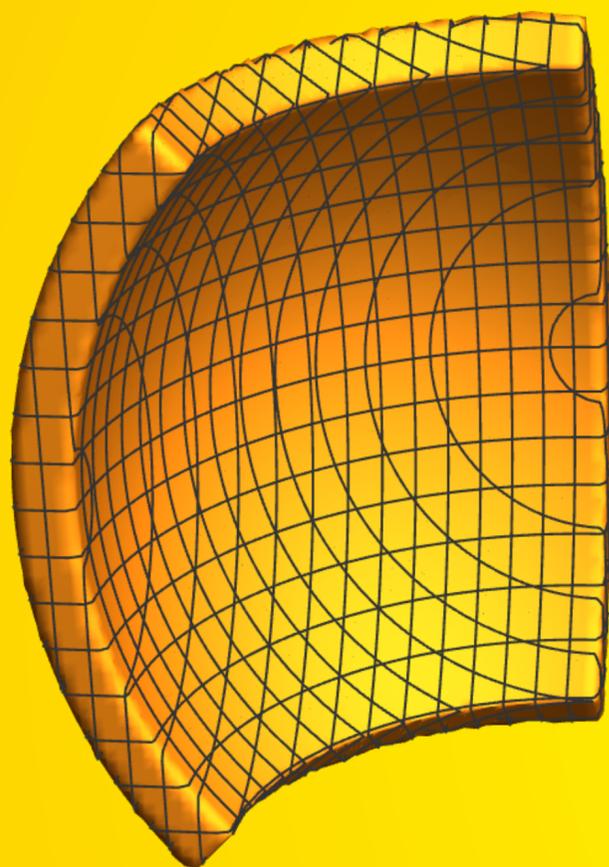
Cheese



$$\int_0^{\pi/3} \int_0^{\pi} \int_0^2 f(x, y, z) \rho^2 \sin(\phi) d\rho d\phi d\theta$$

$$x^2 + y^2 + z^2 < 4, \quad 0 < \theta < \frac{\pi}{3}$$

Orange Peel



$$\int_{\pi/2}^{\pi} \int_{\pi/4}^{3\pi/4} \int_{\sqrt{7}}^3 f(x, y, z) \rho^2 \sin(\phi) d\rho d\phi d\theta$$

$$7 < x^2 + y^2 + z^2 < 9, \quad \frac{\pi}{2} < \theta < \pi$$
$$x^2 + y^2 > z^2,$$

Worksheet

Homework due Wednesday

THE END