

# Lecture 18

## Surface integrals

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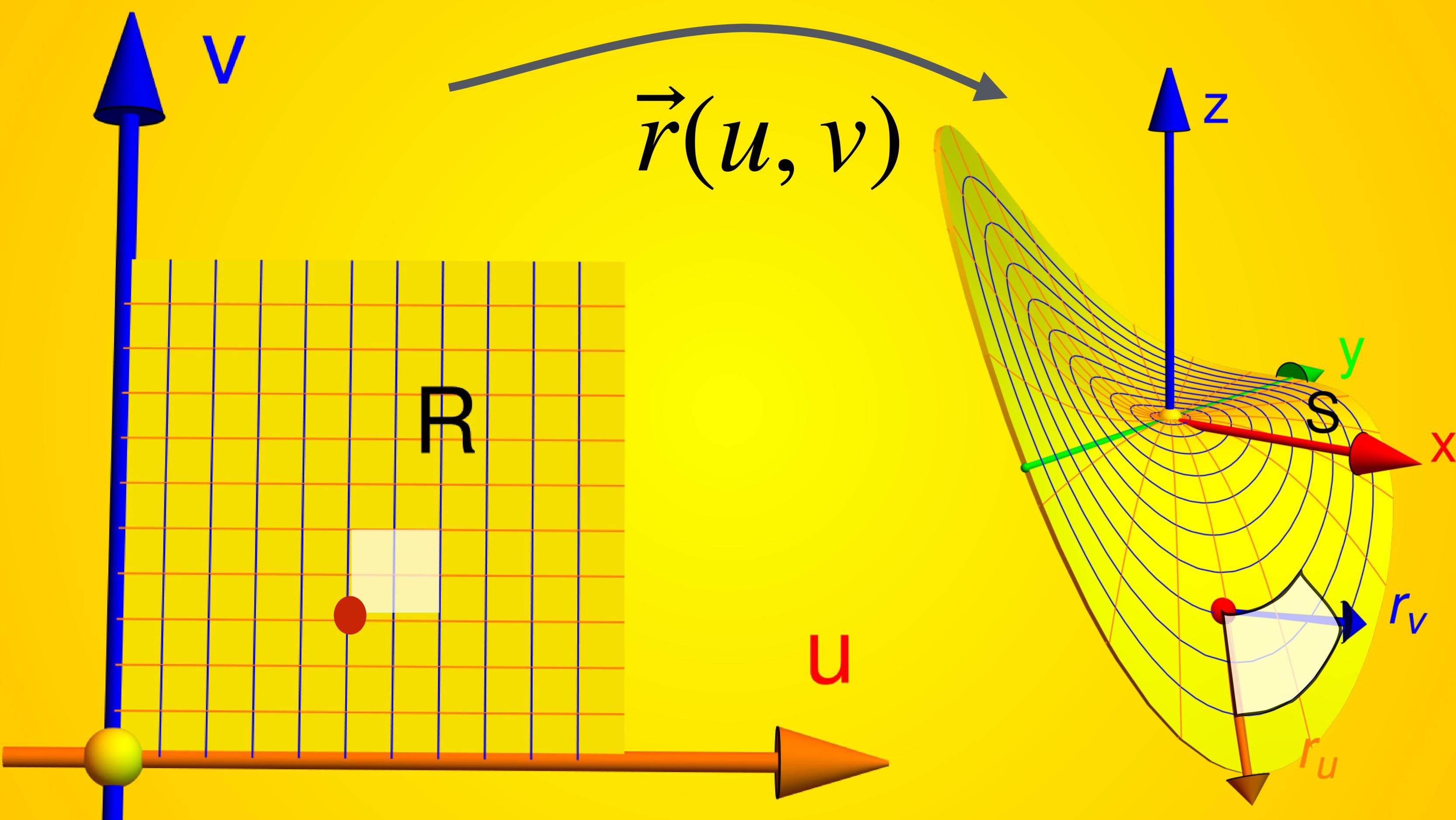
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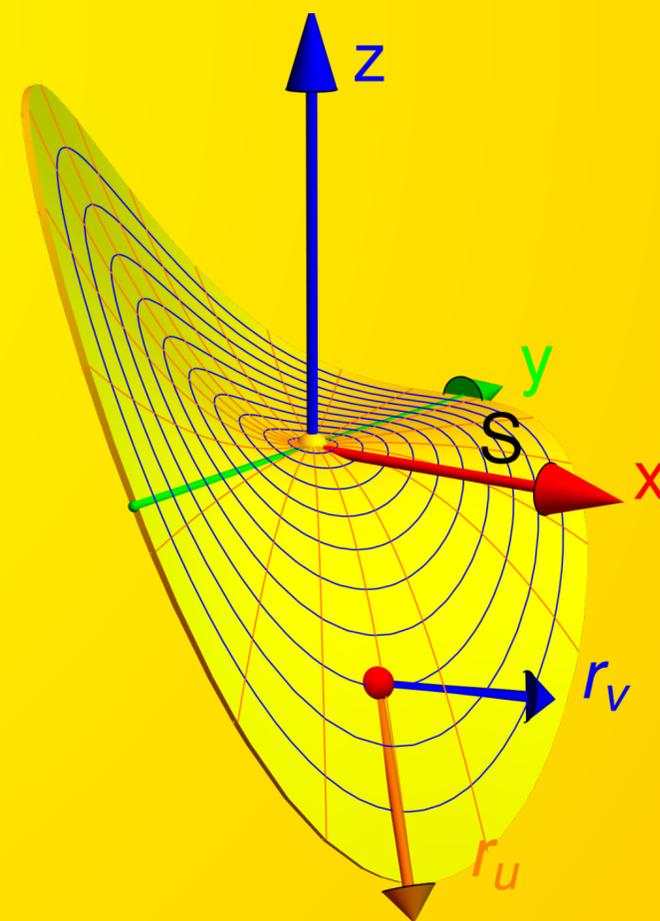
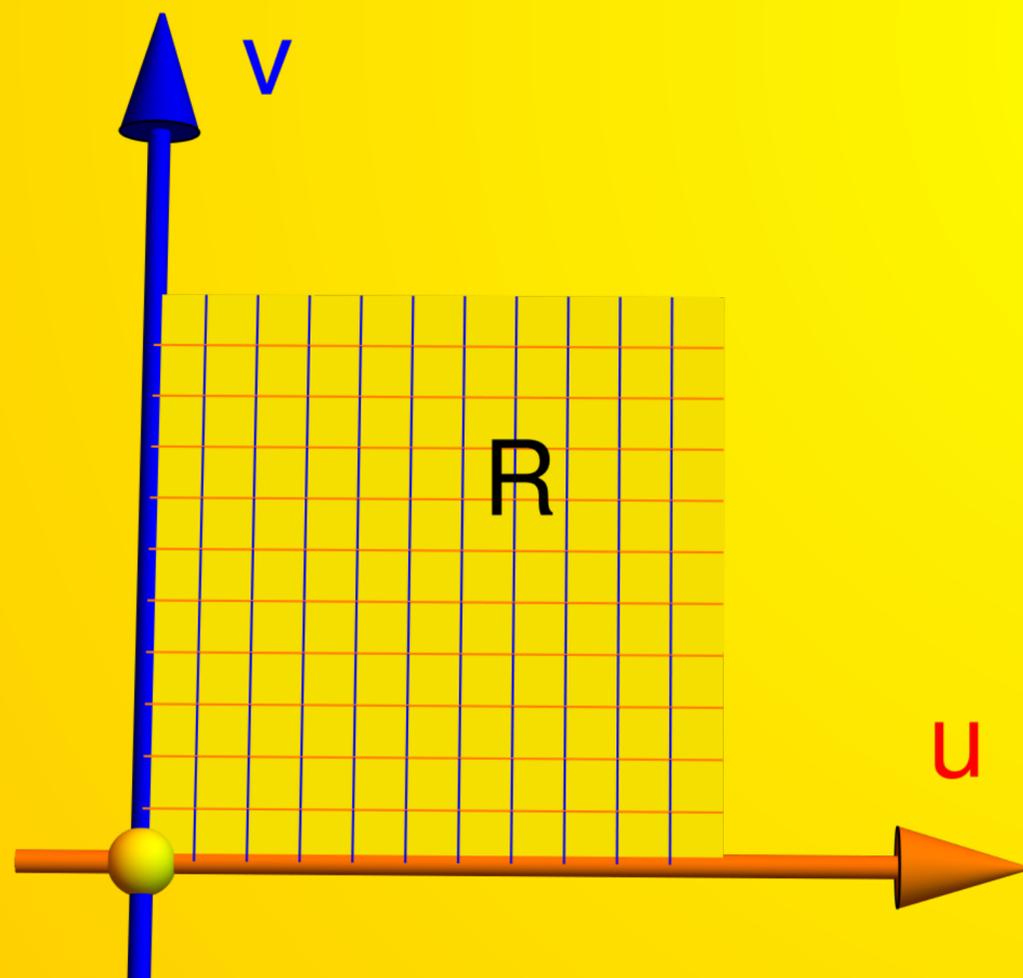
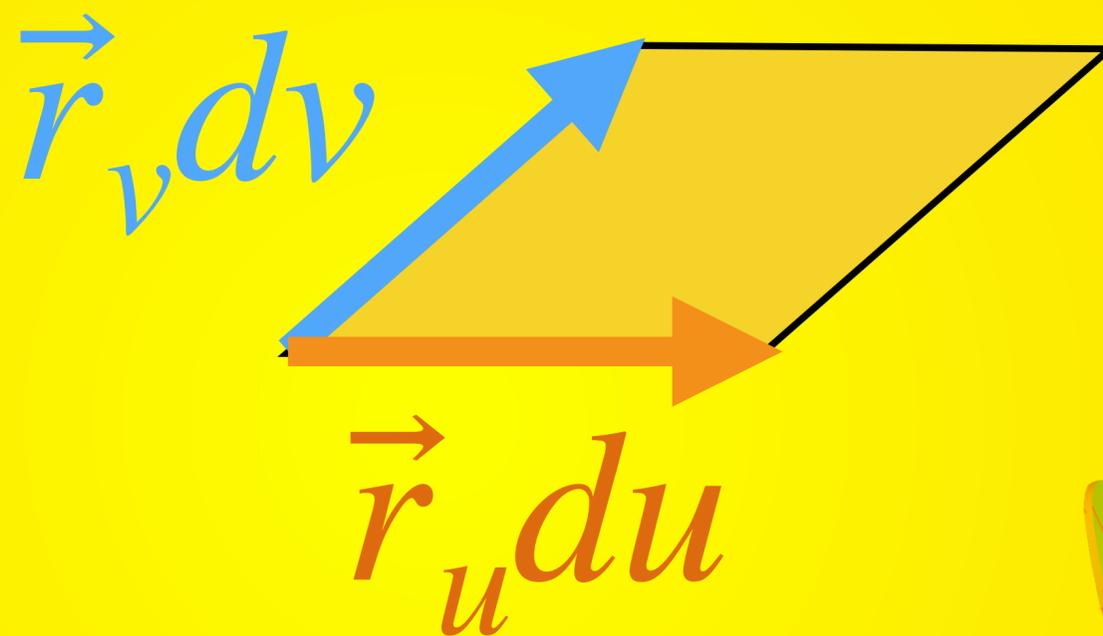
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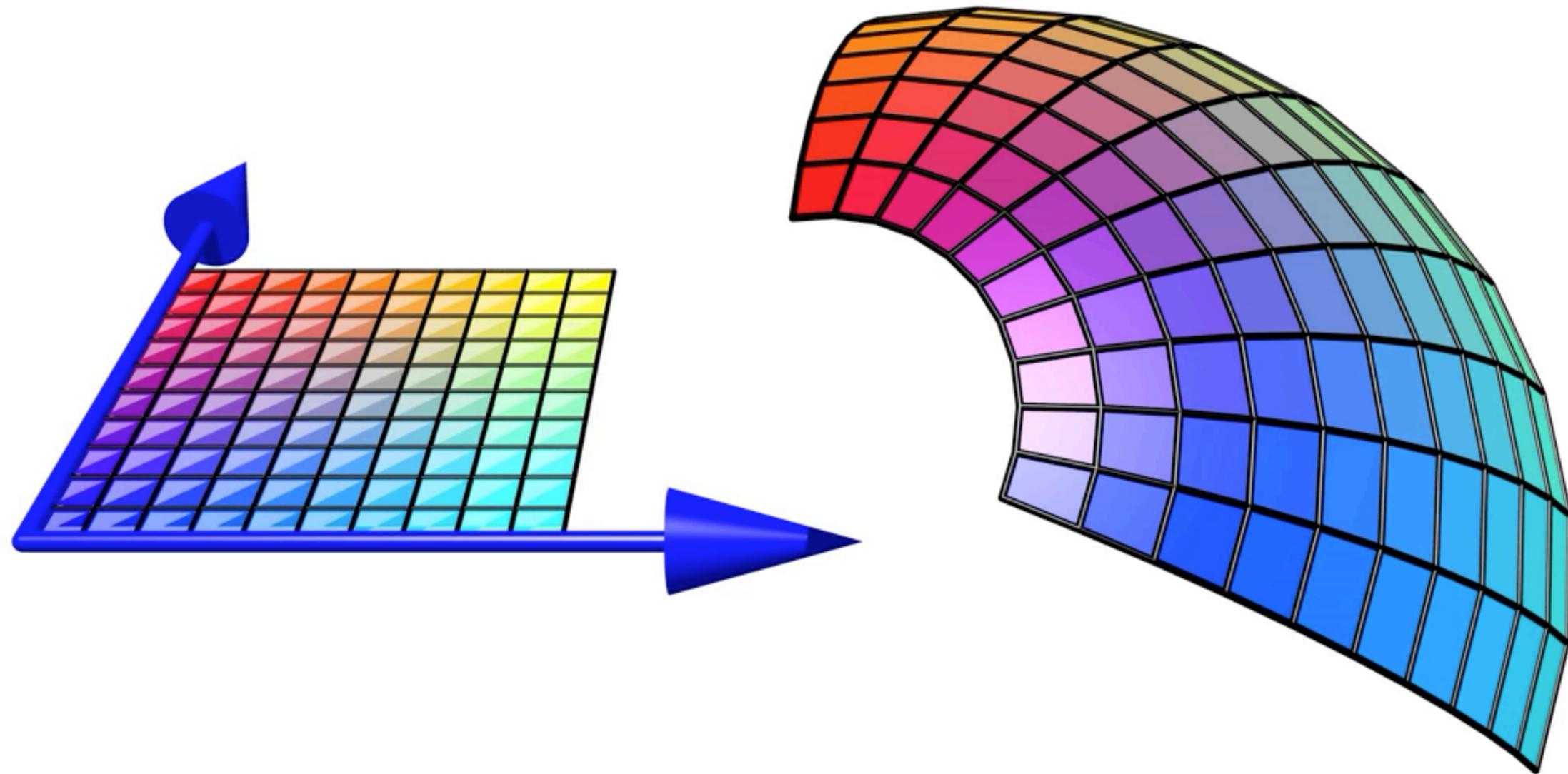
*The formula*



$$dS = |\vec{r}_u \times \vec{r}_v| \, du \, dv$$



# *Parametrization*



2000

from youtube "Why povray is great 2019"

# 2006 Math 21a

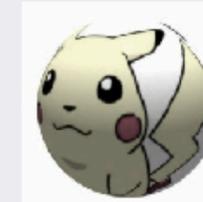


## Marble book

Mathematics Math21a, Spring 2006

Multivariable Calculus

Oliver Knill, SciCtr 434, [knill.harvard.edu](http://knill.harvard.edu)



[Exhibit: table of content](#)



marblebook

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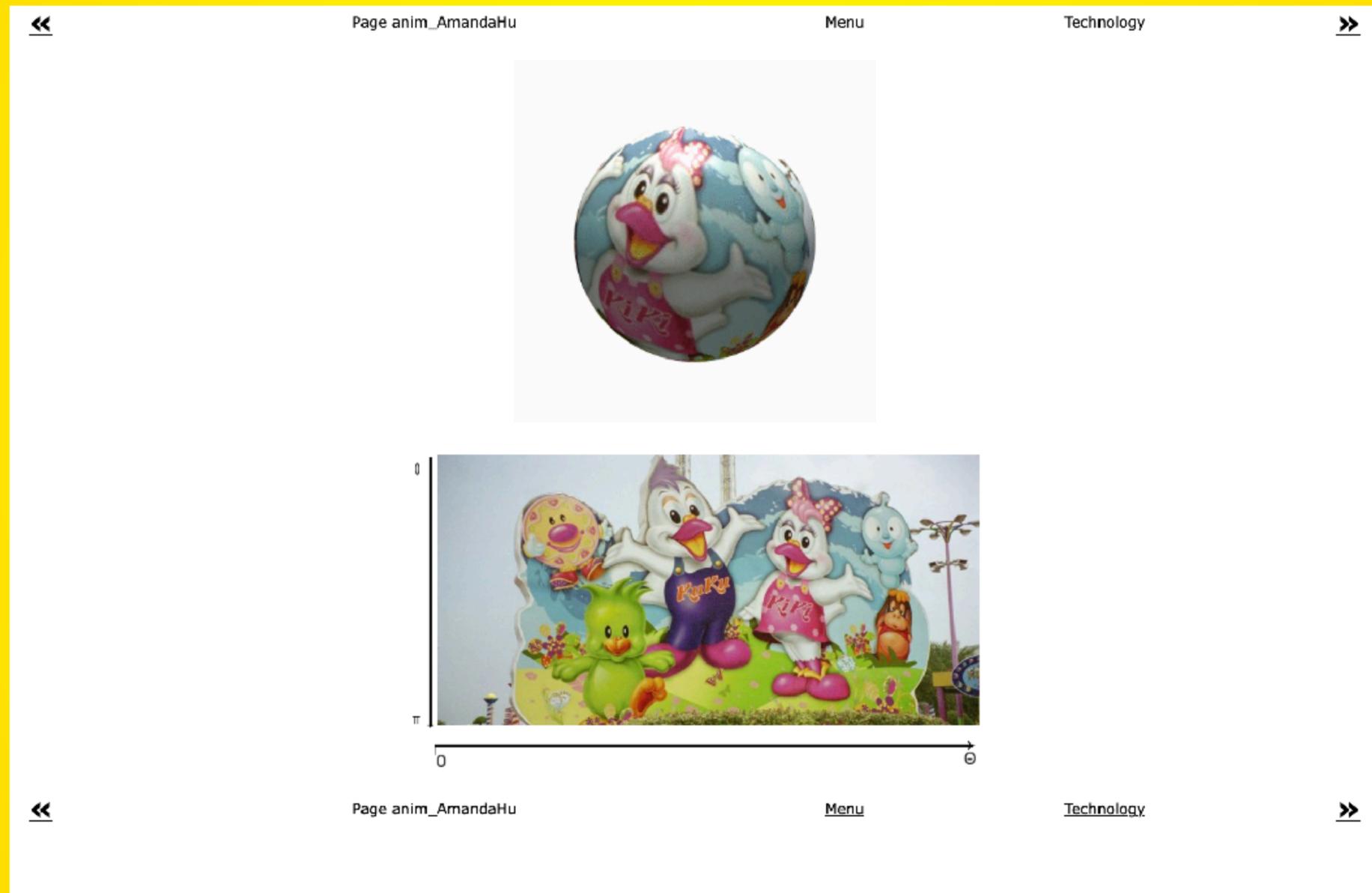
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## [Asteroid gallery](#)



[The little Prince](#) by  
Antoine de Saint-  
Exupéry, ([online book](#)).

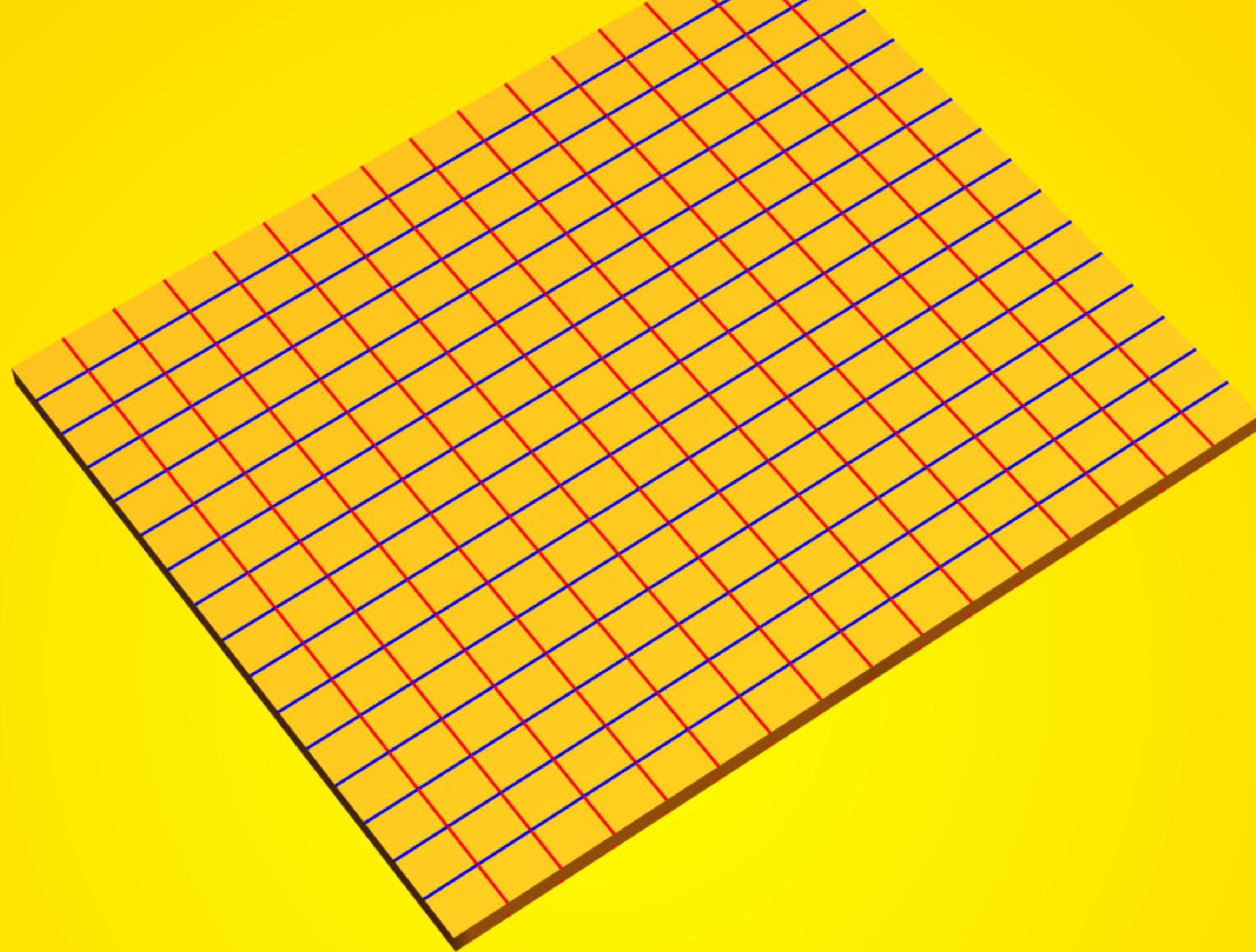
HOMEWORK PROBLEM A: (week 4, second HW set): to get full credit for this 5'th homework problem, submit a picture to [asteroid@ulam.fas.harvard.edu](mailto:asteroid@ulam.fas.harvard.edu). You have to submit the picture from an email account which we have on file. Send the picture (preferably 600x300 pixels in JPG or PNG form) as an attachment and write into the subject header: **ASTEROID YOURFIRSTNAME YOURLASTNAME**. The email can be empty but you could add into the email body a sentence explaining the context of the picture. The picture will be scaled to 600x300 pixels. So make it twice as long as wide. Otherwise it will be rescaled and look distorted.

[Here is an example](#).

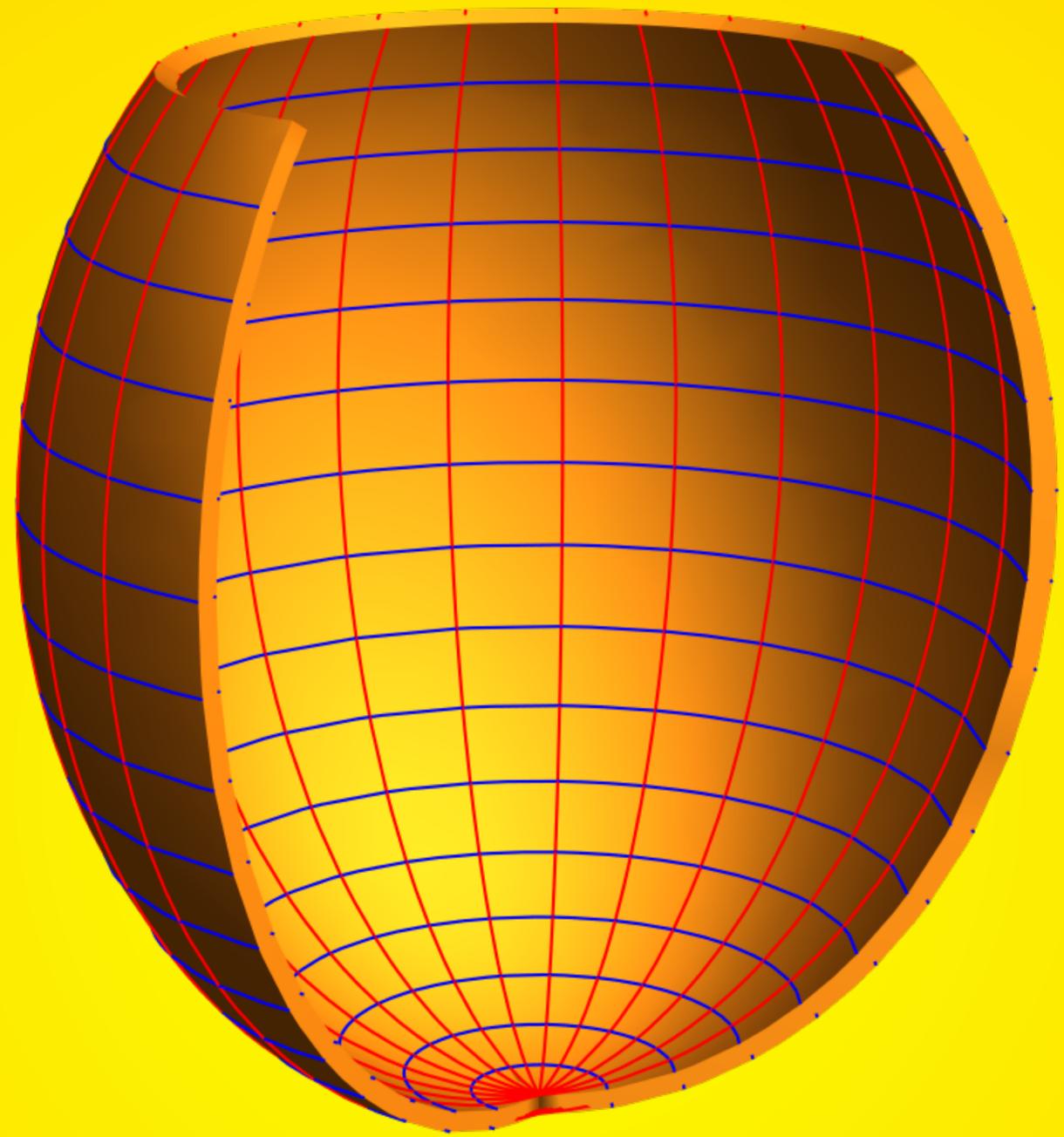
# 2007 Math 21a



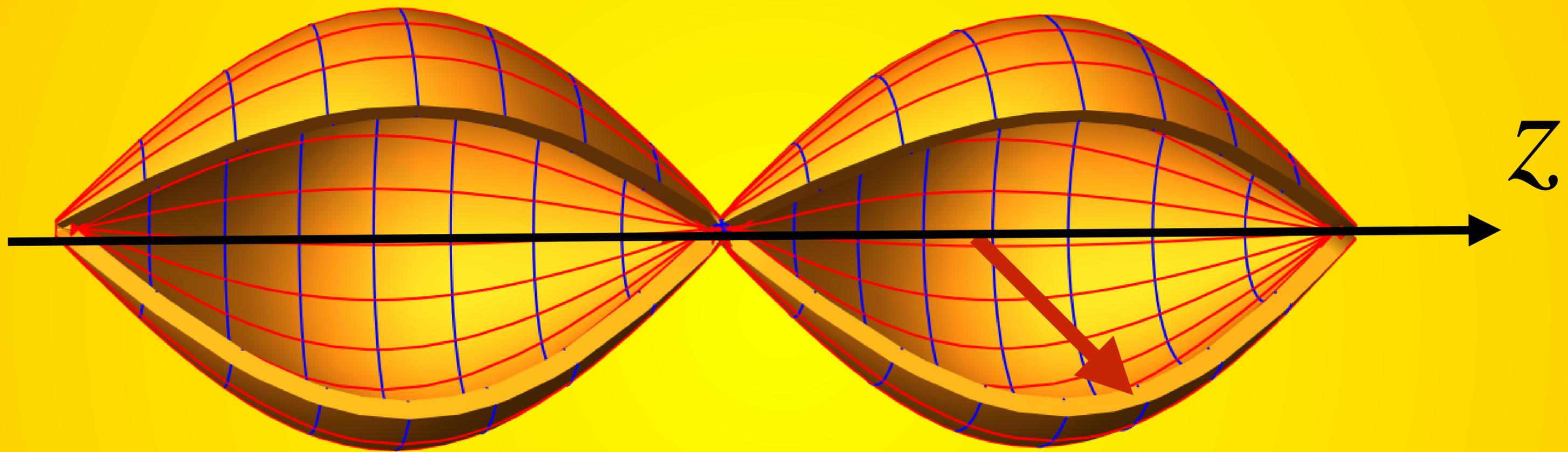
*Basic Examples*



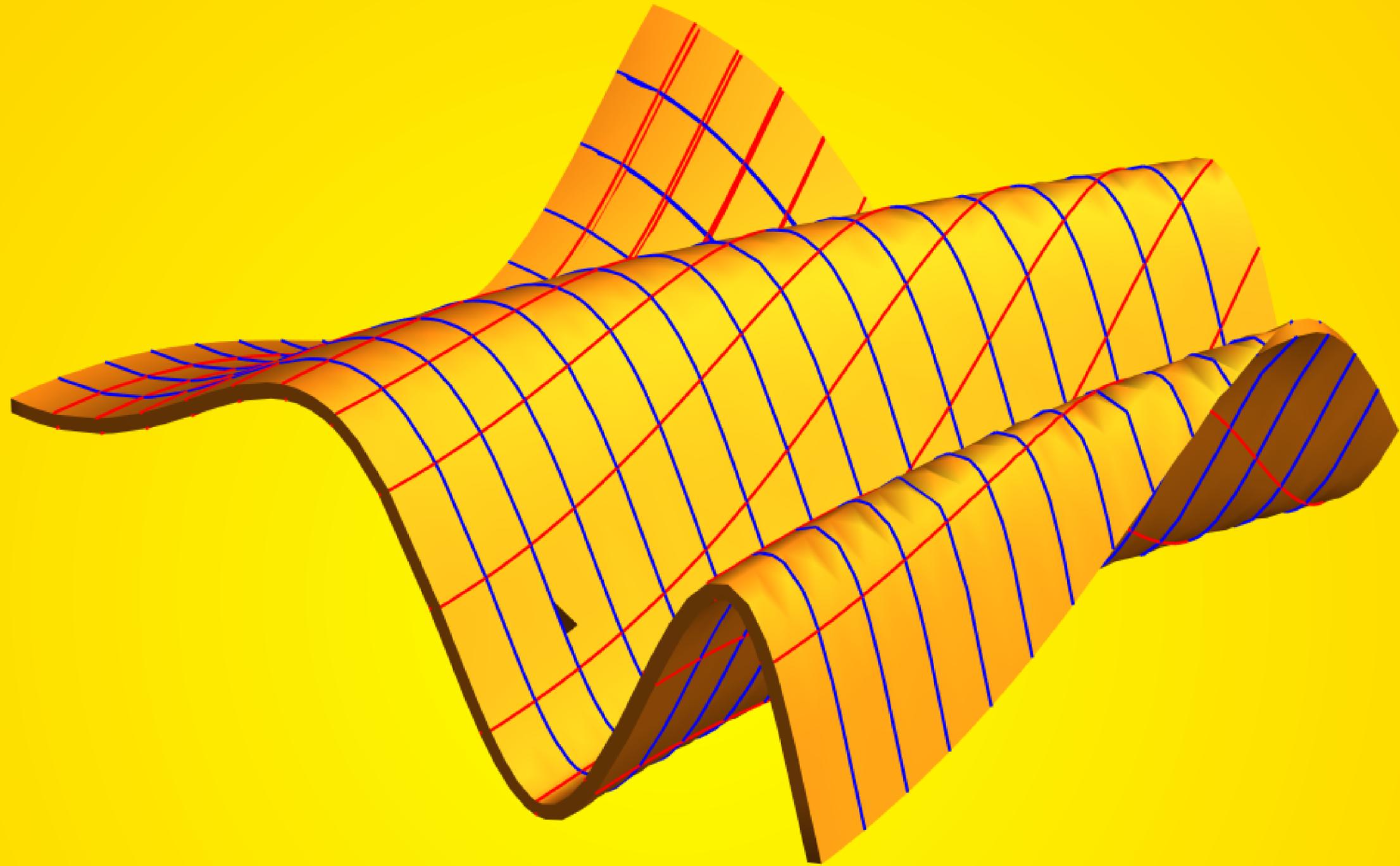
$$d\mathcal{S} = |\vec{v} \times \vec{w}| \, du \, dv$$



$$dS = \rho^2 \sin(\phi)$$



$$dS = g(z) \sqrt{1 + g'(z)^2} d\theta dz$$



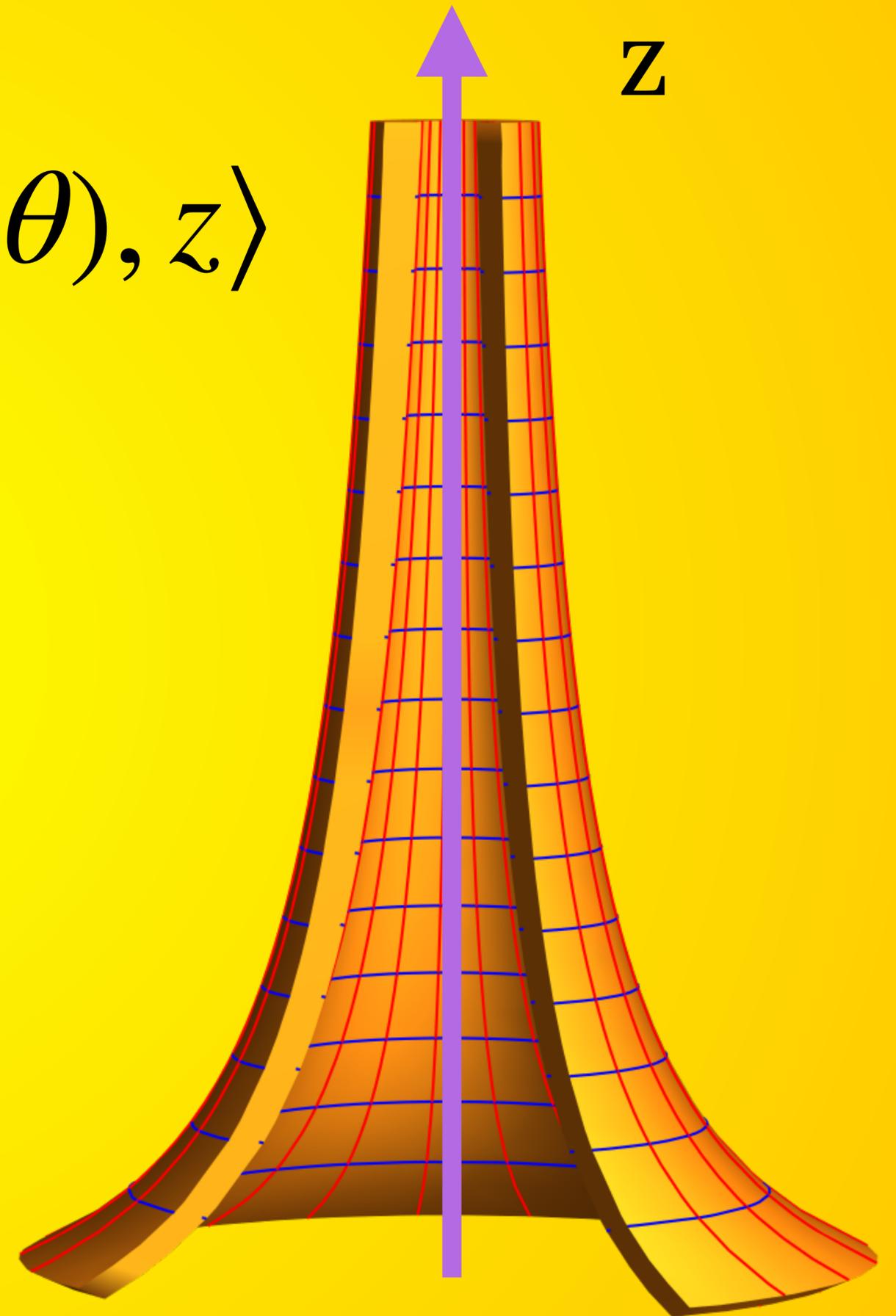
$$dS = \sqrt{1 + f_u^2 + f_v^2} \, dudv$$

*The Trumpet*

$$\vec{r}(\theta, z) = \left\langle \frac{1}{z} \cos(\theta), \frac{1}{z} \sin(\theta), z \right\rangle$$

Finite Volume but  
infinite Surface  
area

1



*Minions like it*





has  
area but  
finite volume

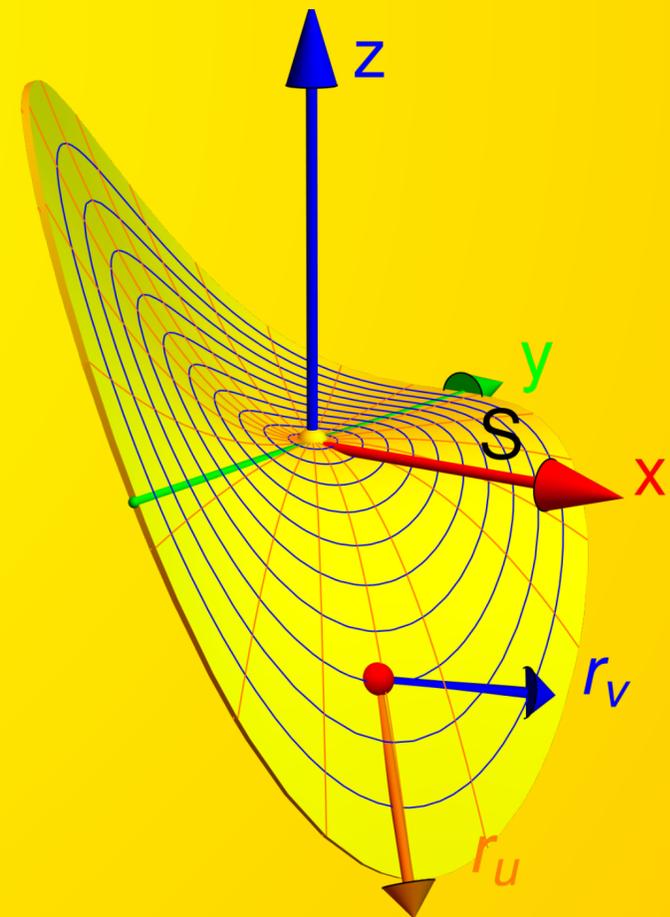
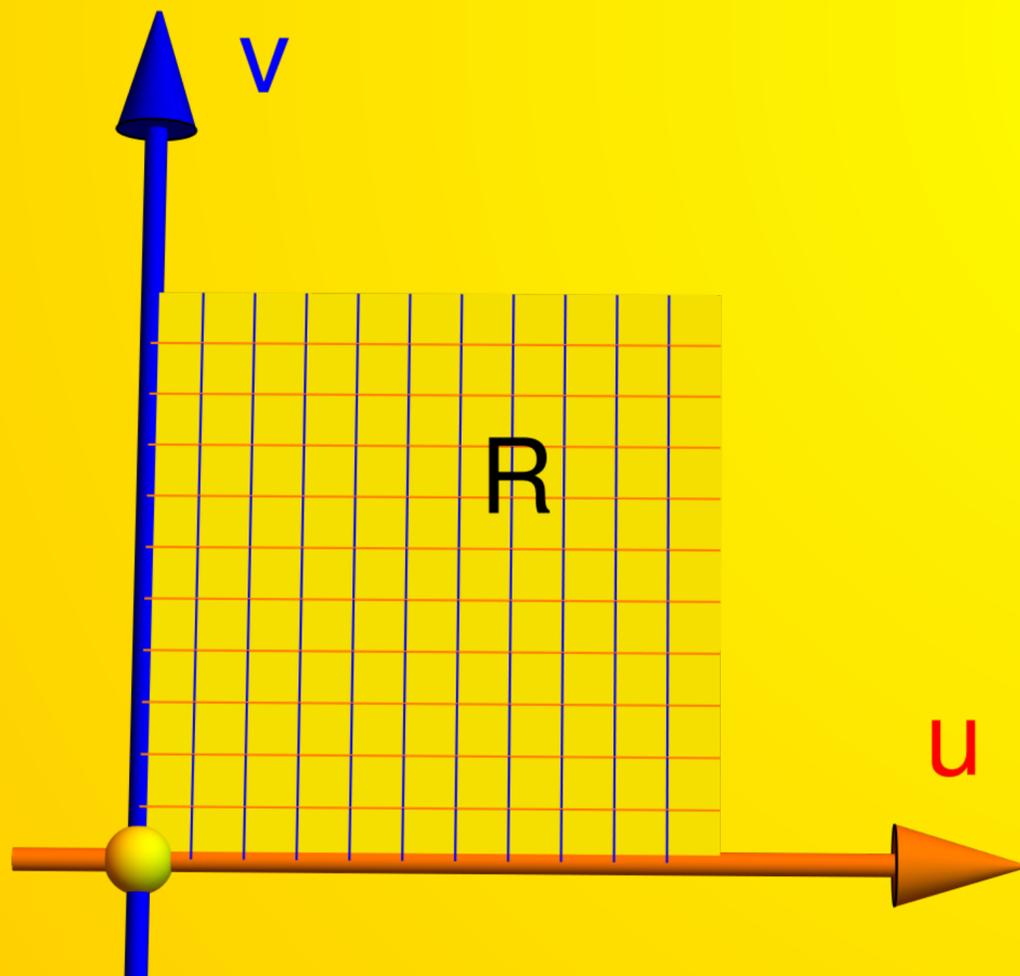
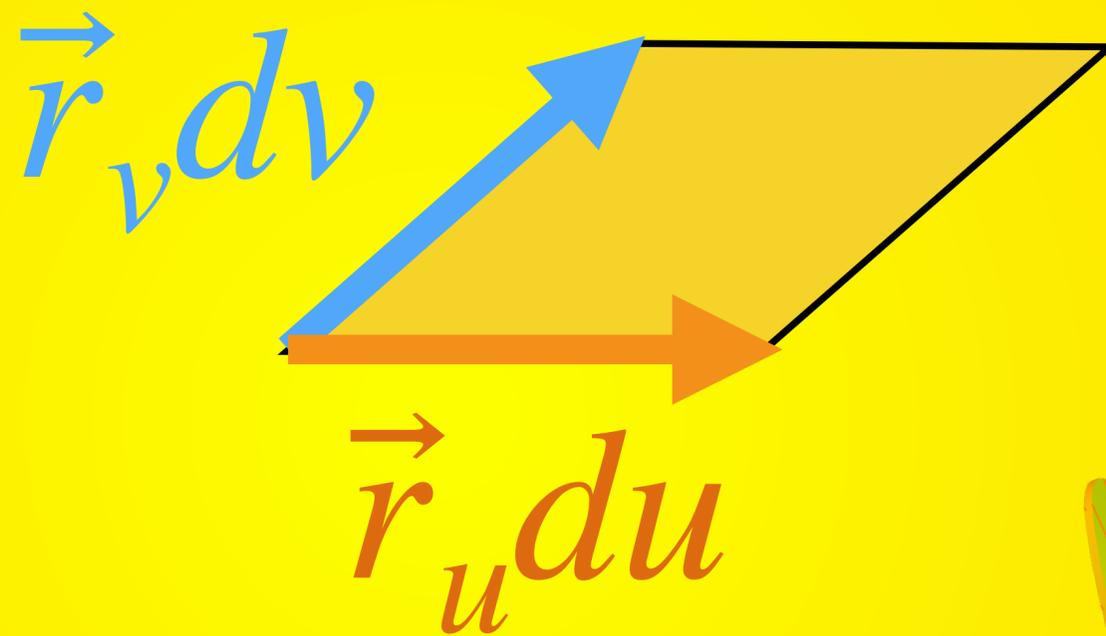
$$\int_0^{2\pi} \int_0^R \frac{1}{z} dz d\theta$$

$$= \int_0^{2\pi} \log(z)/R d\theta = 2\pi(\log R - 1)$$

$$= 2\pi \log R$$

*Surface Integral*

$$fdS = f(u, v) \left| \vec{r}_u \times \vec{r}_v \right| dudv$$



*THE END*