

Lecture 20



Line

Integrals

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1) Definition

2) Computation

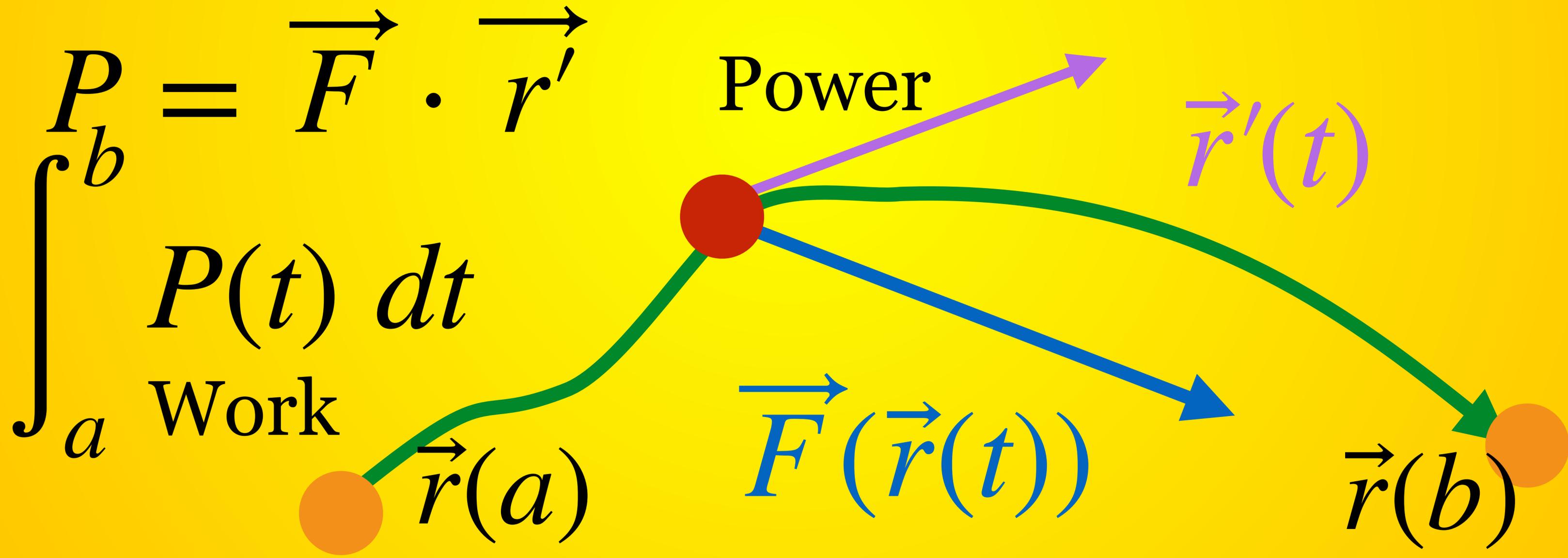
3) Where does it appear?

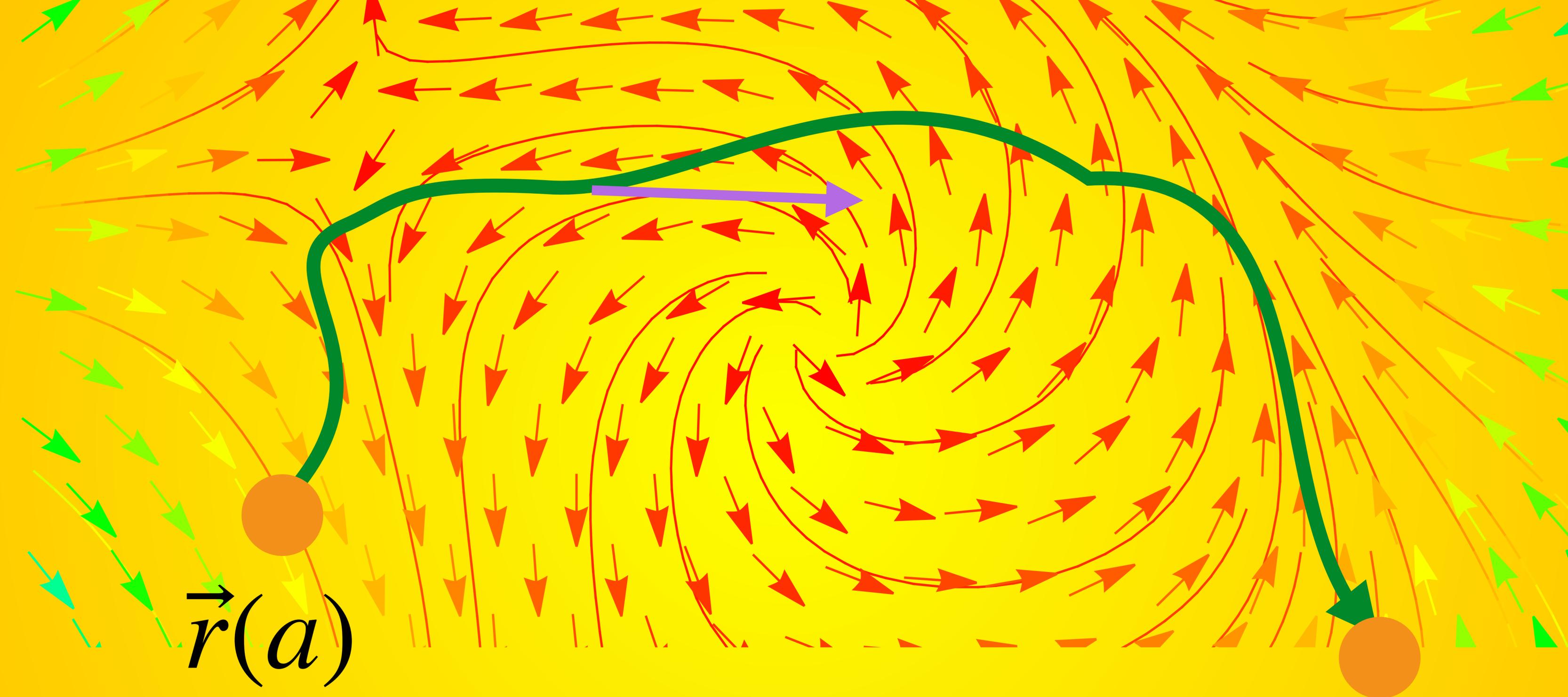
4) Work sheet problems

Line integral

$$\vec{F}(x, y) = \langle P(x, y), Q(x, y) \rangle \quad \text{Field}$$

$$\vec{r}(t) = \langle x(t), y(t) \rangle \quad \text{Curve}$$



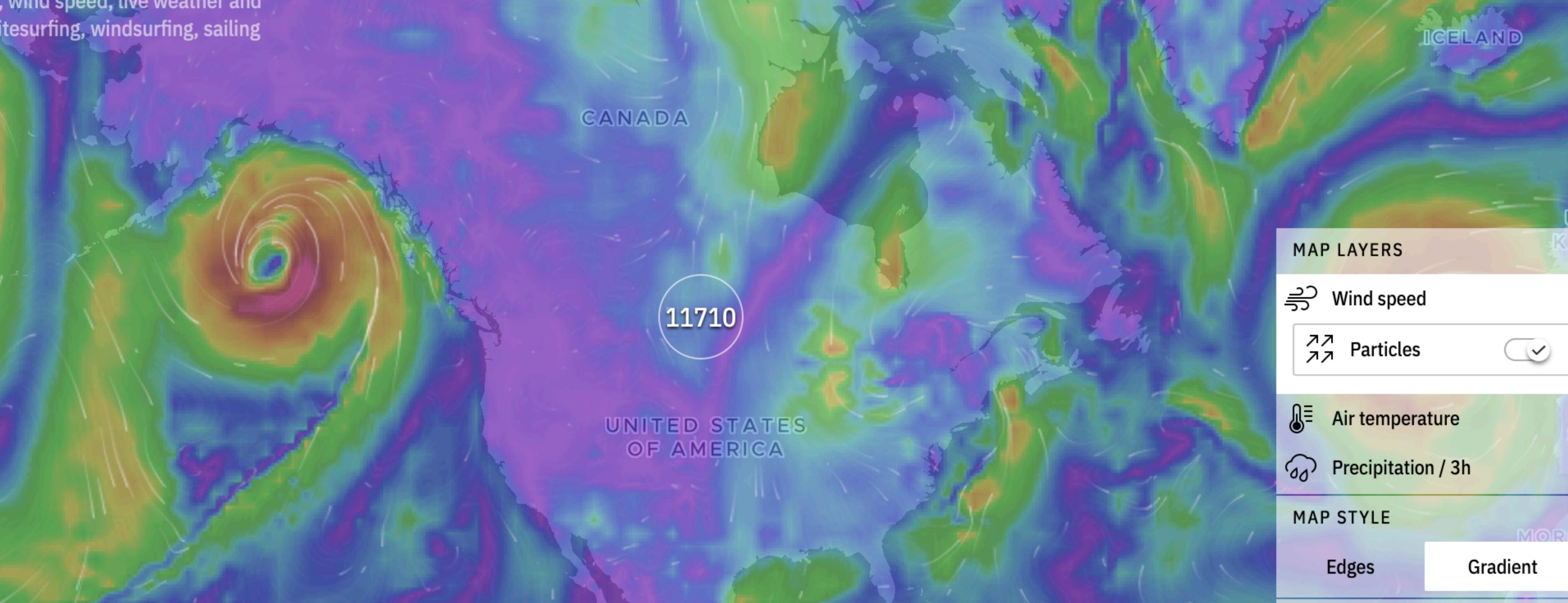


$\vec{r}(a)$

$\vec{r}(b)$

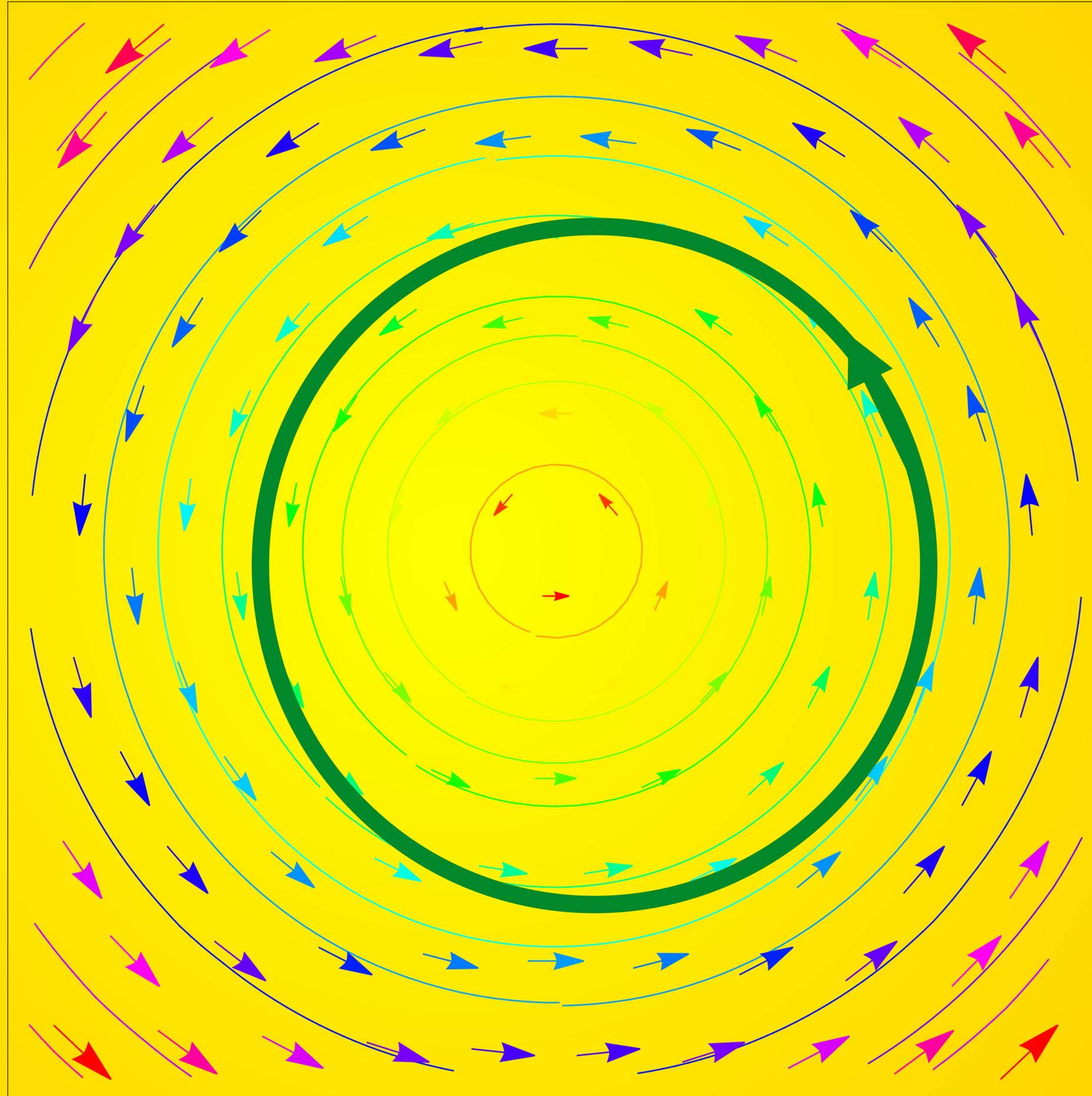
Positive or negative?

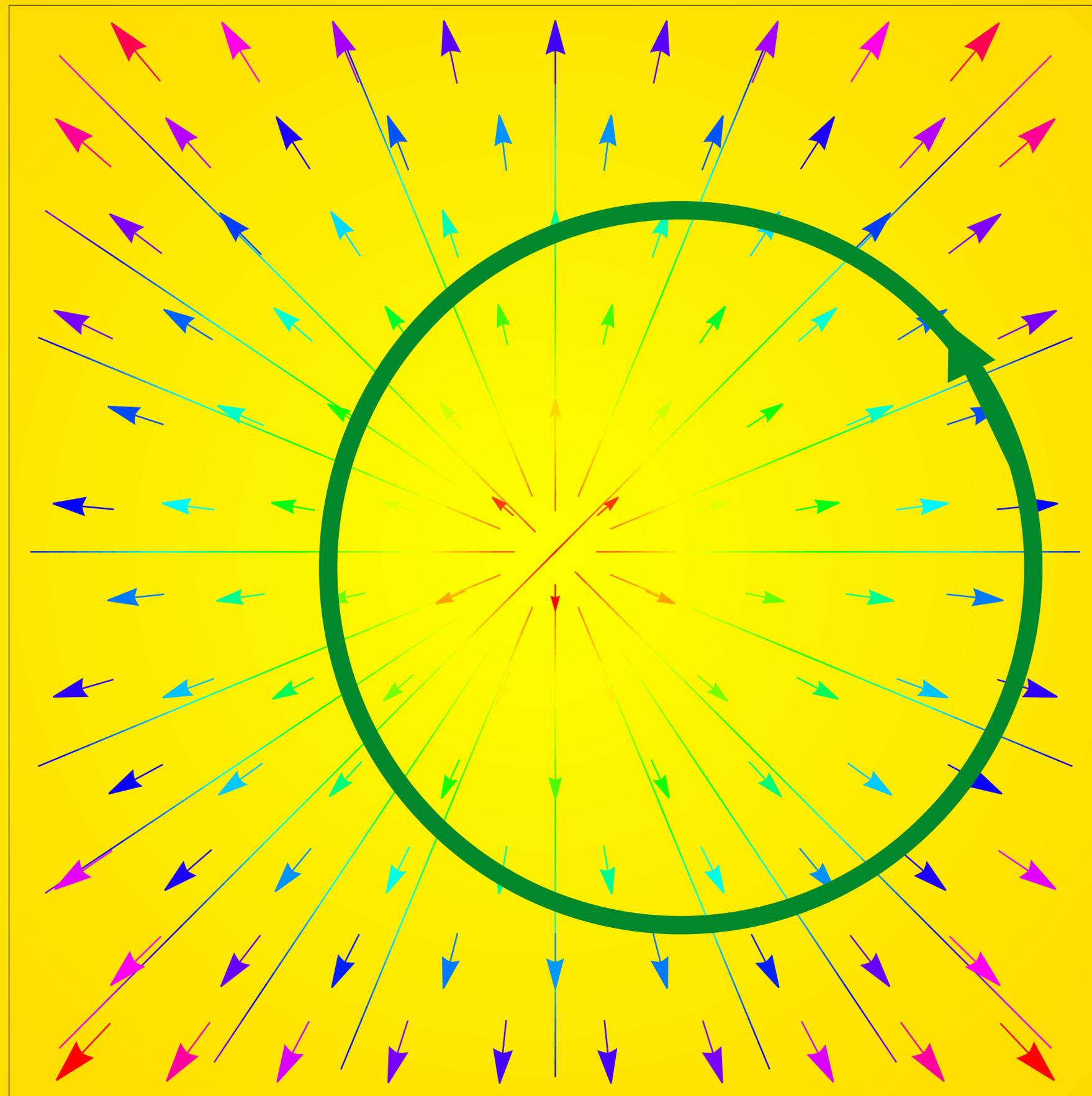
Wind speed, live weather and
kitesurfing, windsurfing, sailing

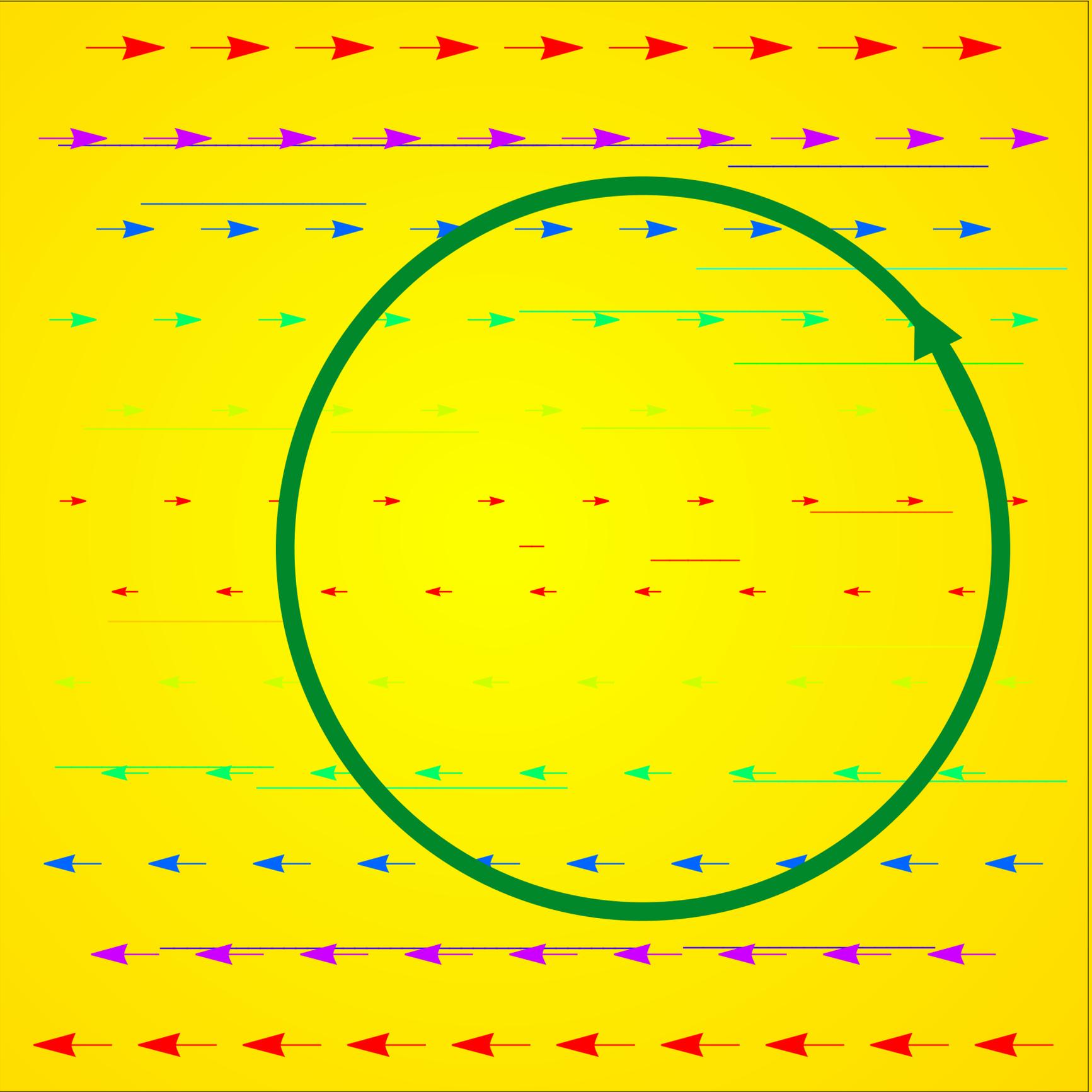


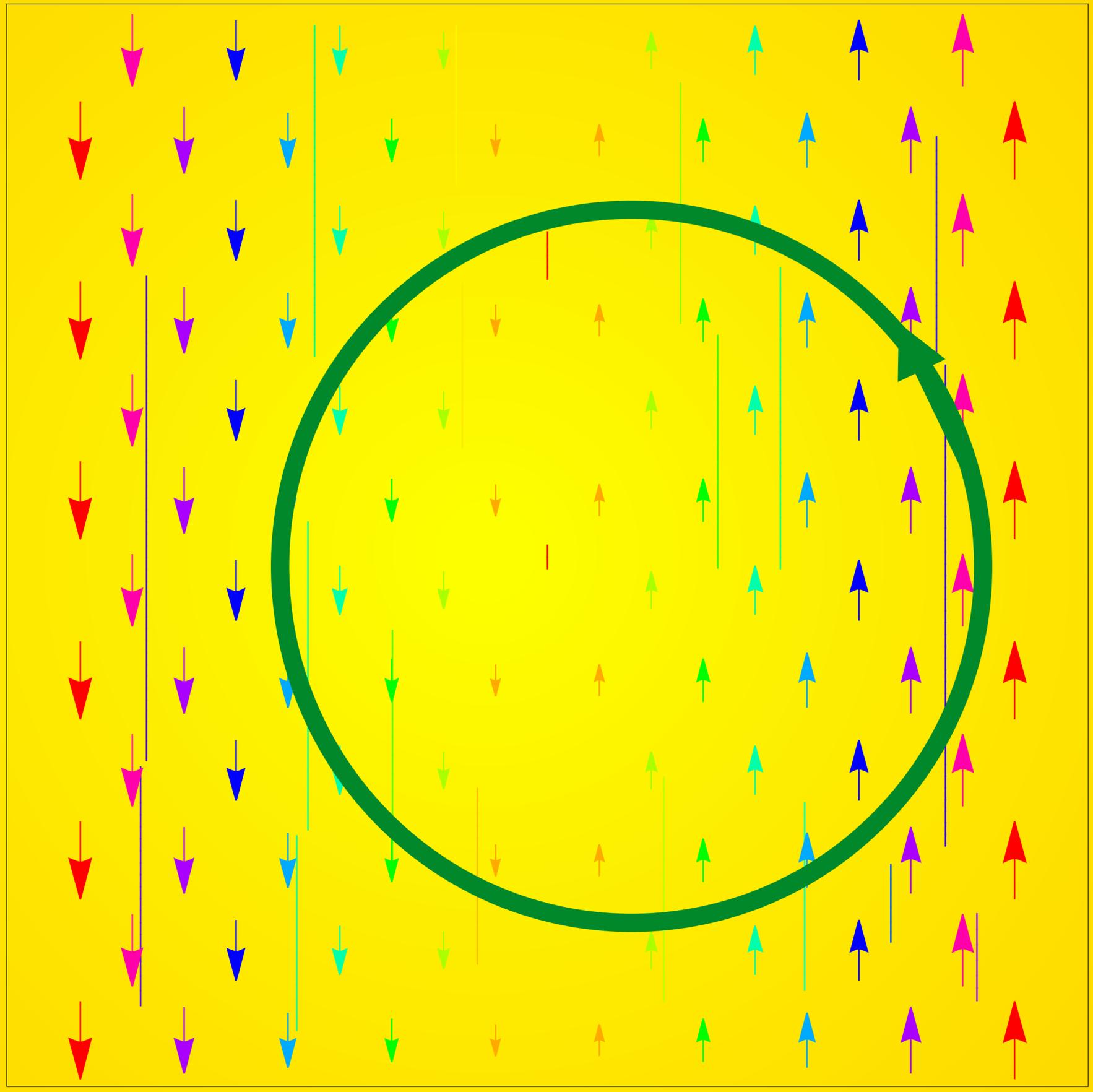
Fly from LA to Boston

Examples



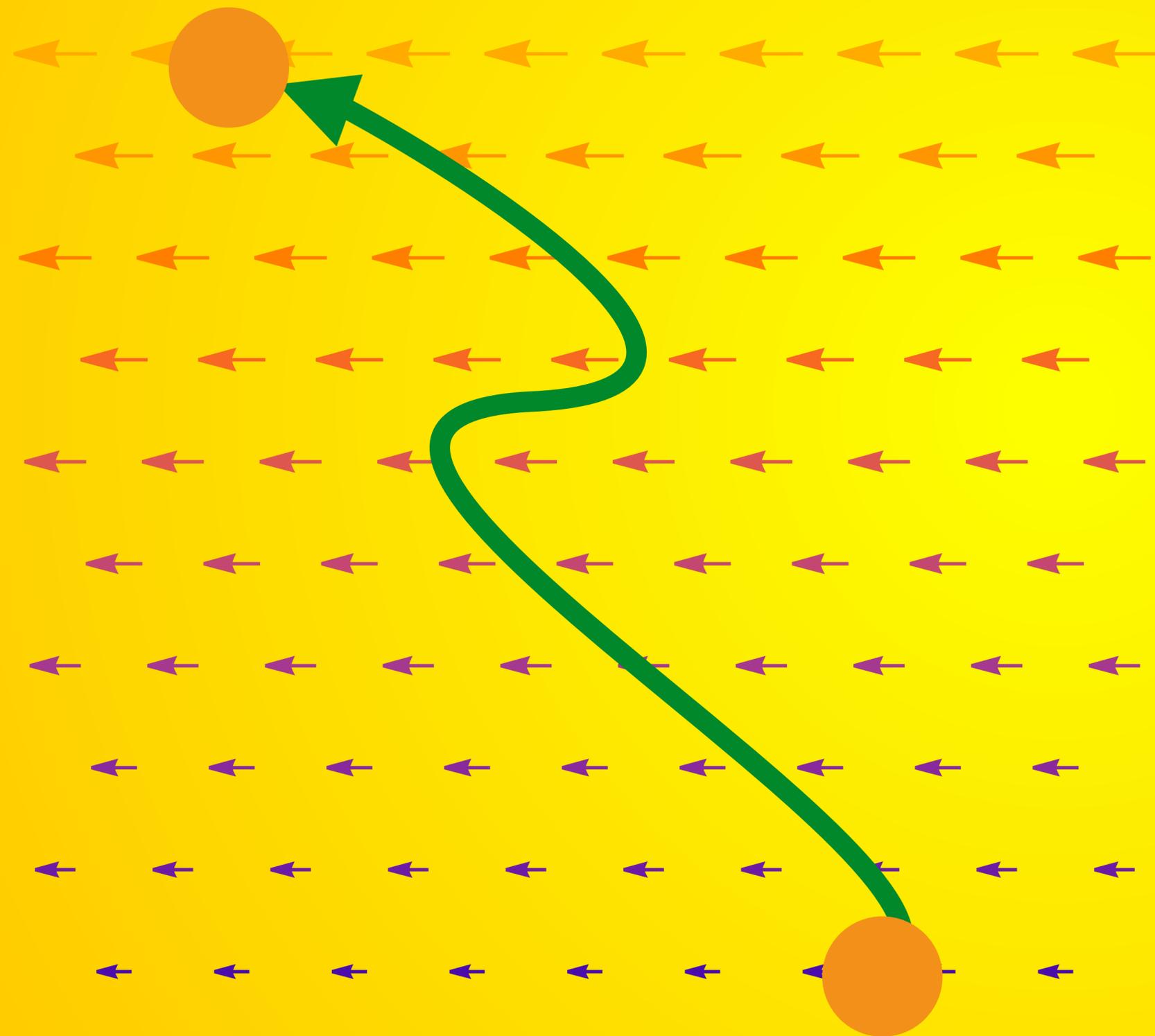






Applications

Electric Field



Path of an
electron
in an
electric
field \vec{E}

Charge q

Voltage U

$$qU = q \int_a^b \vec{E} \cdot d\vec{r}$$

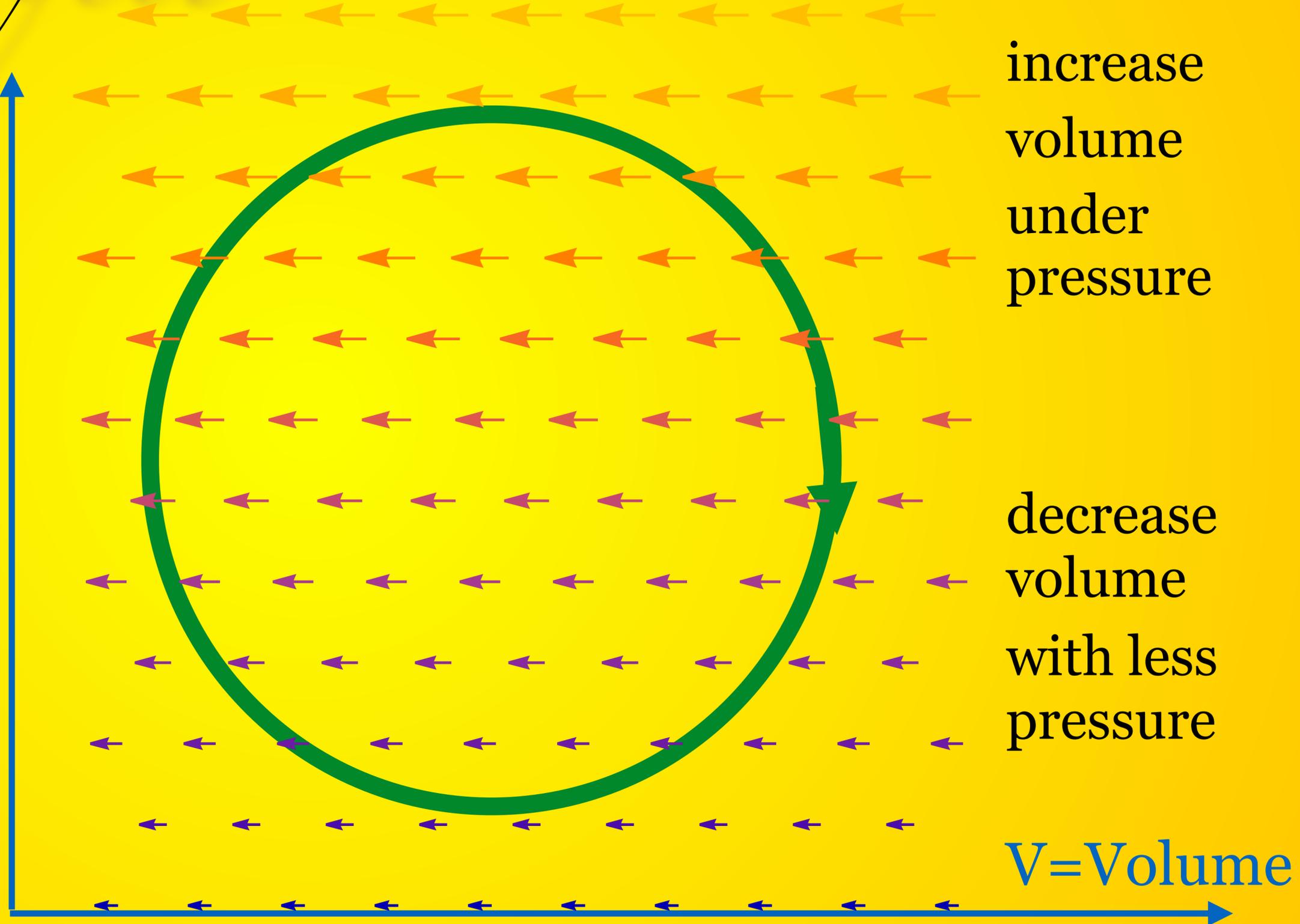
Carnot Cycle

p=Pressure

$$\vec{F} = \langle -p, 0 \rangle$$

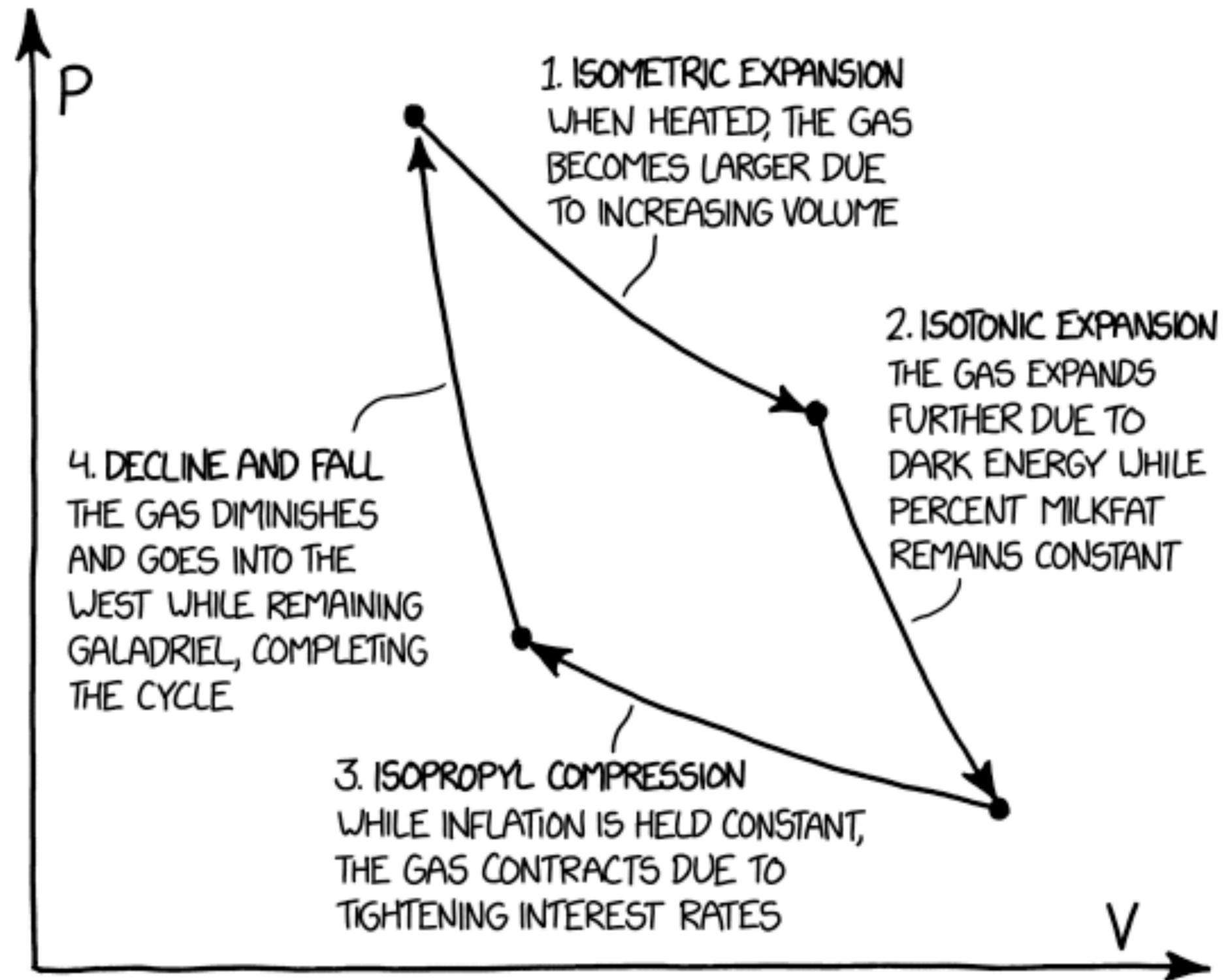
$$dU = -pdV$$

$$= -\langle p, 0 \rangle \cdot \langle V', p' \rangle dt$$



NKCD

THE FOUR STAGES OF THE CARNOT CYCLE

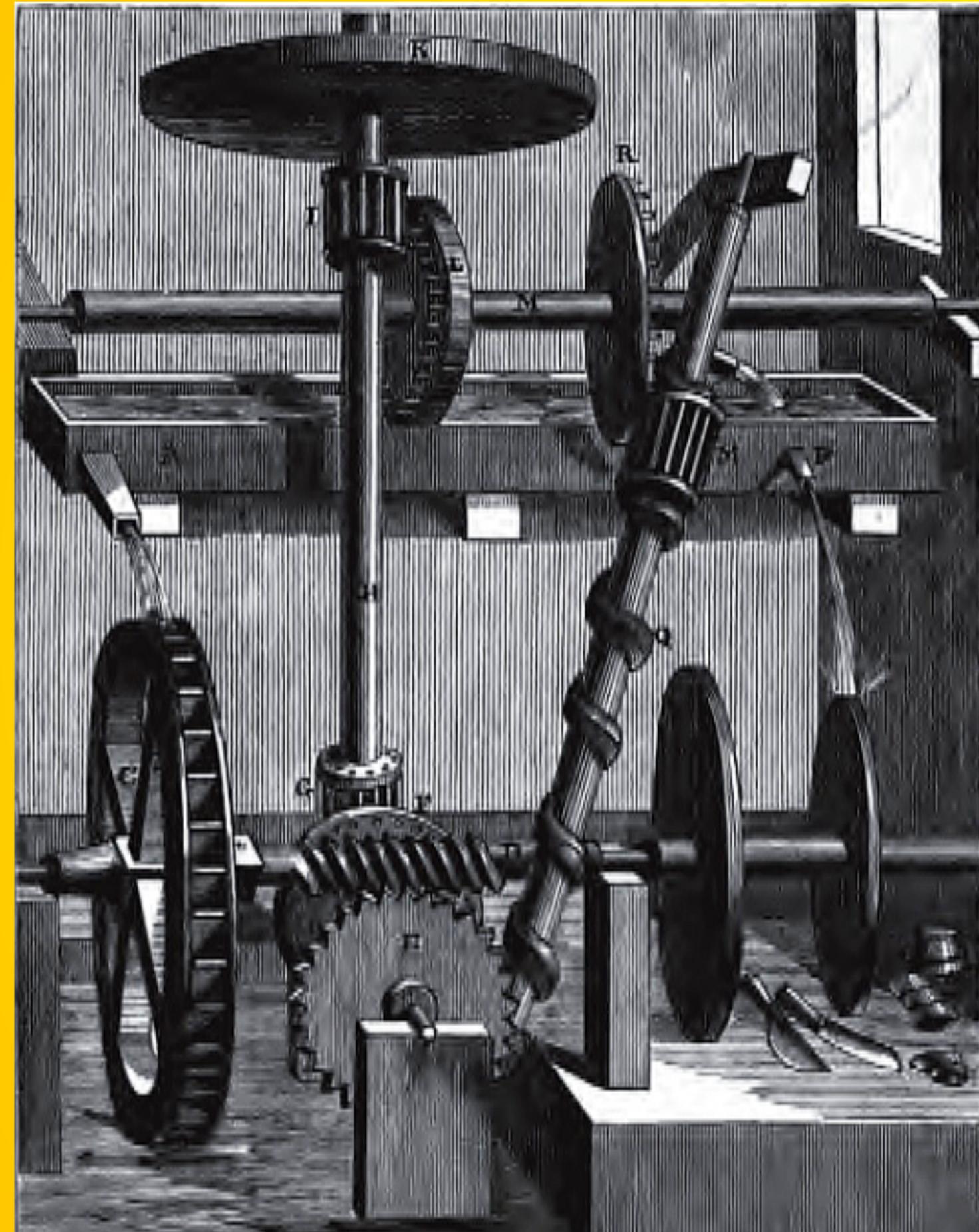


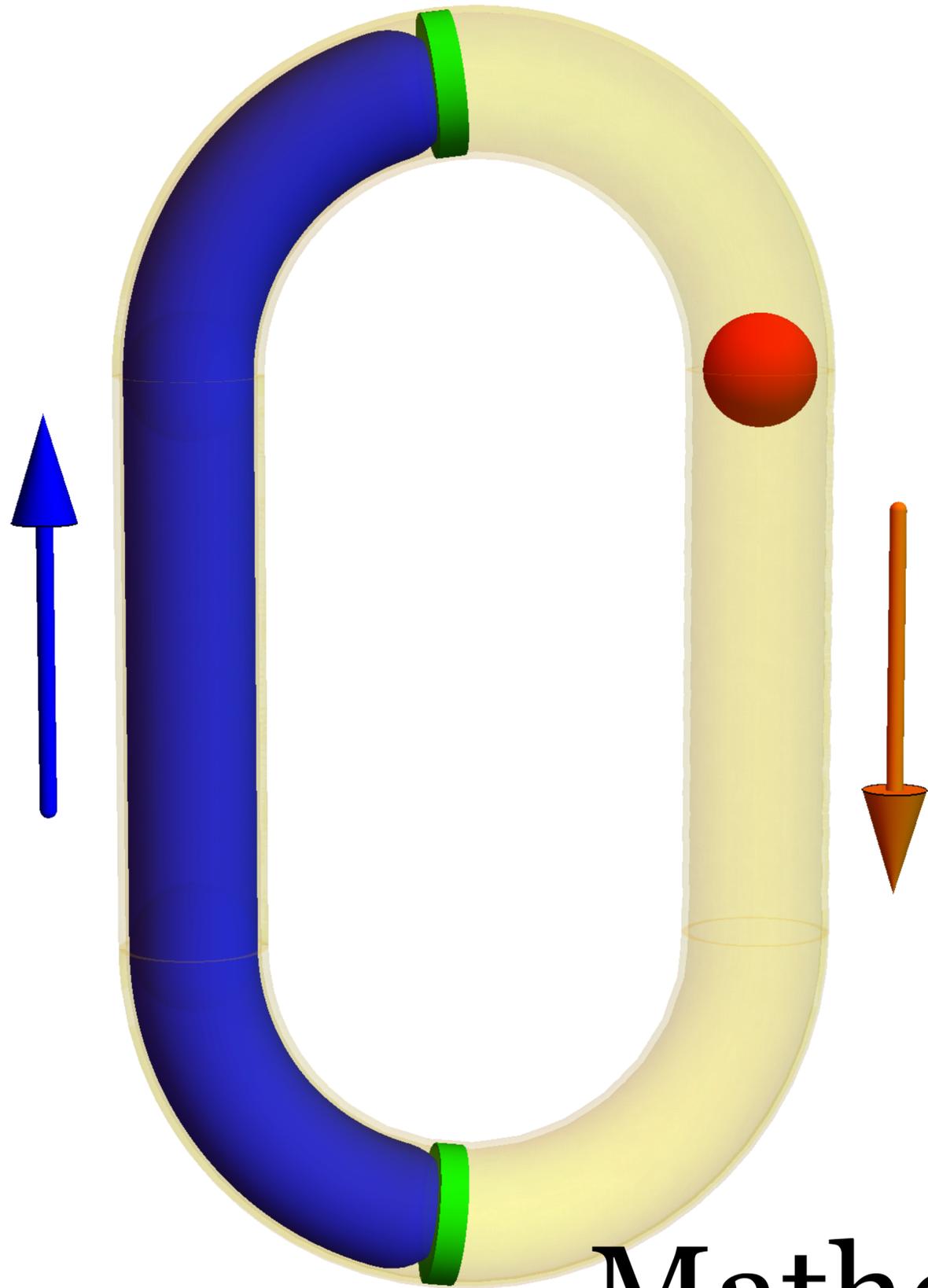
Perpetual motion

Robert Fludd

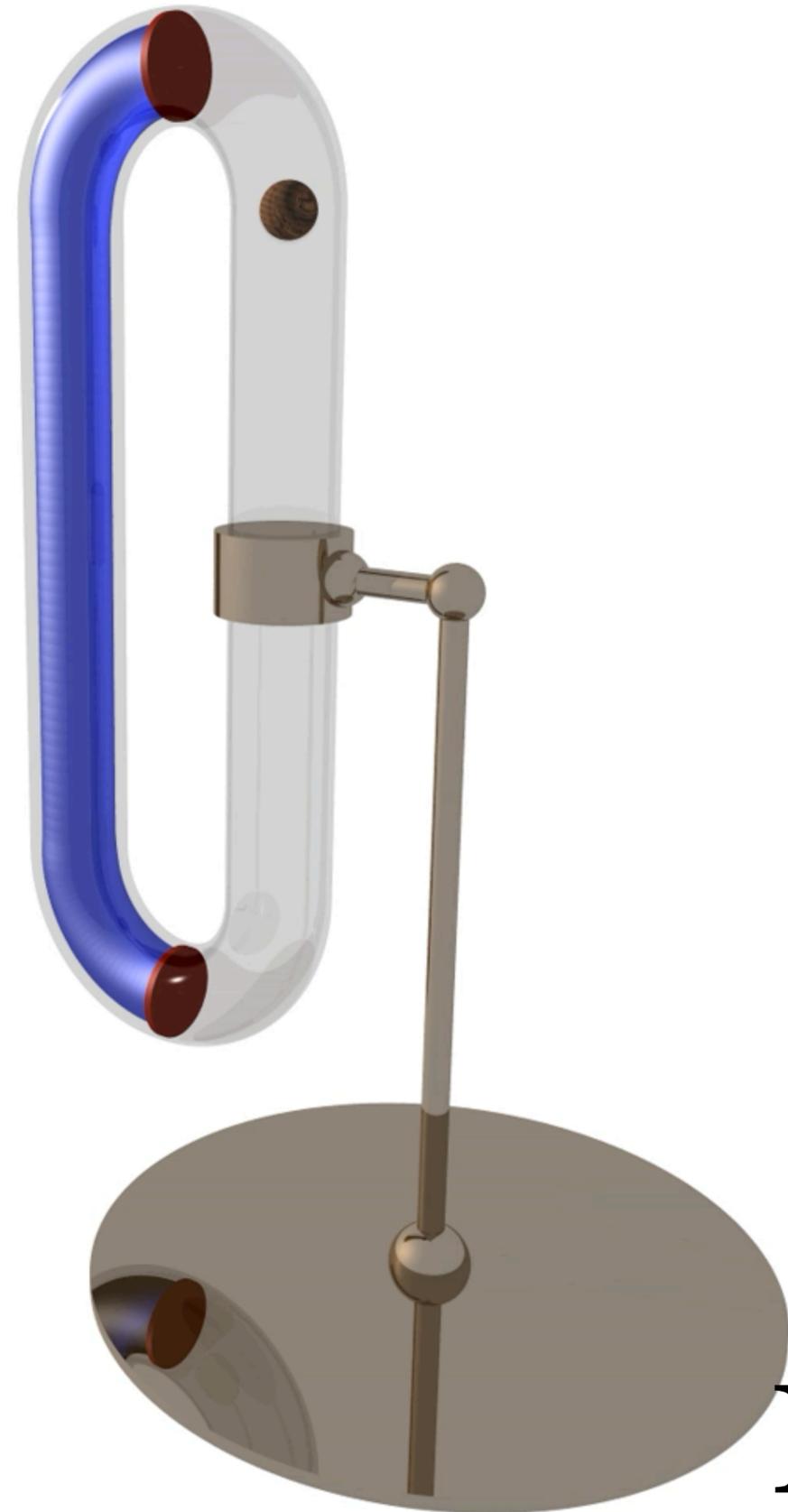


SOURCE: WIKIPEDIA

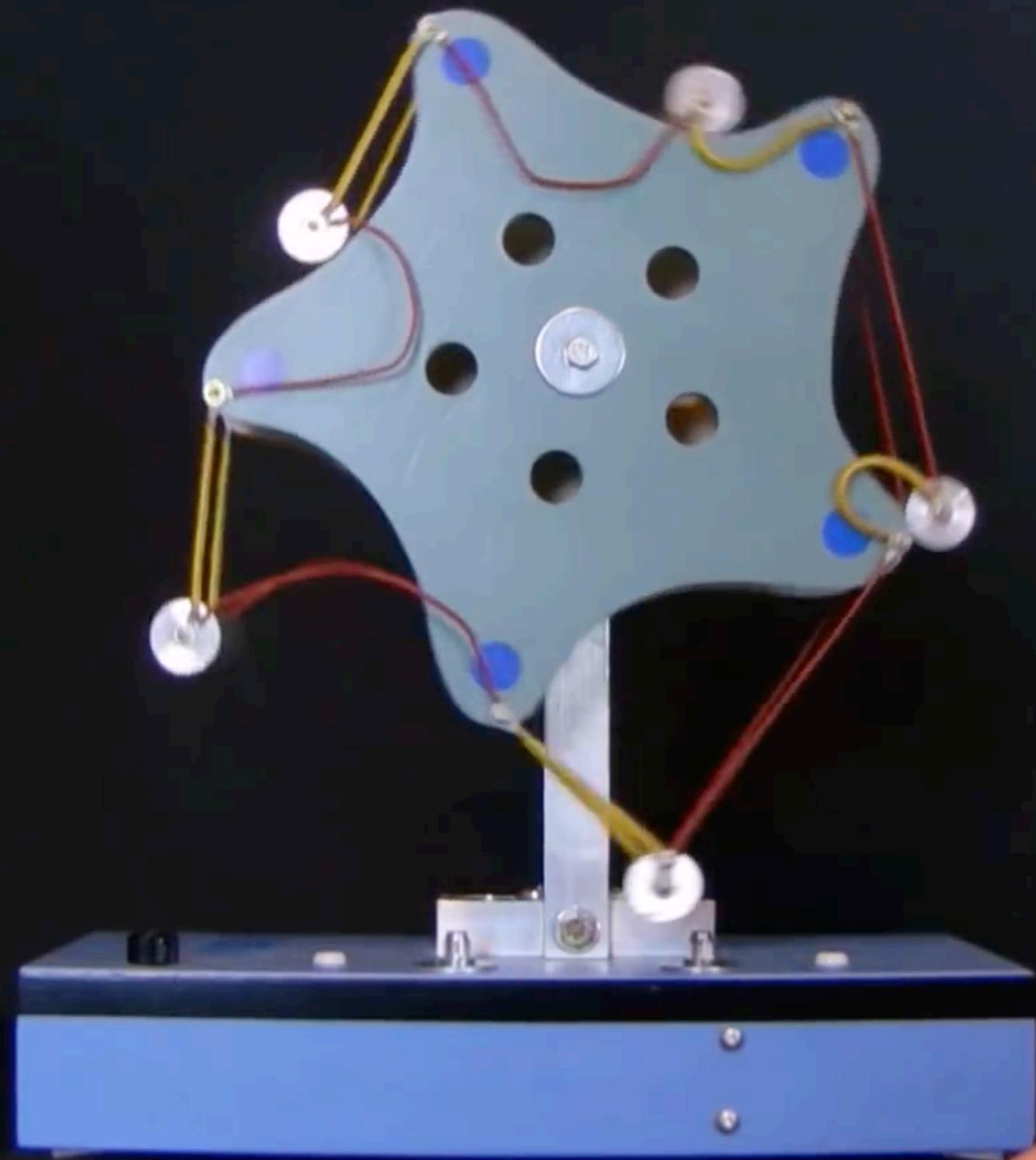




Mathematica



Povray



To be continued...

THE END