Green's Theorem

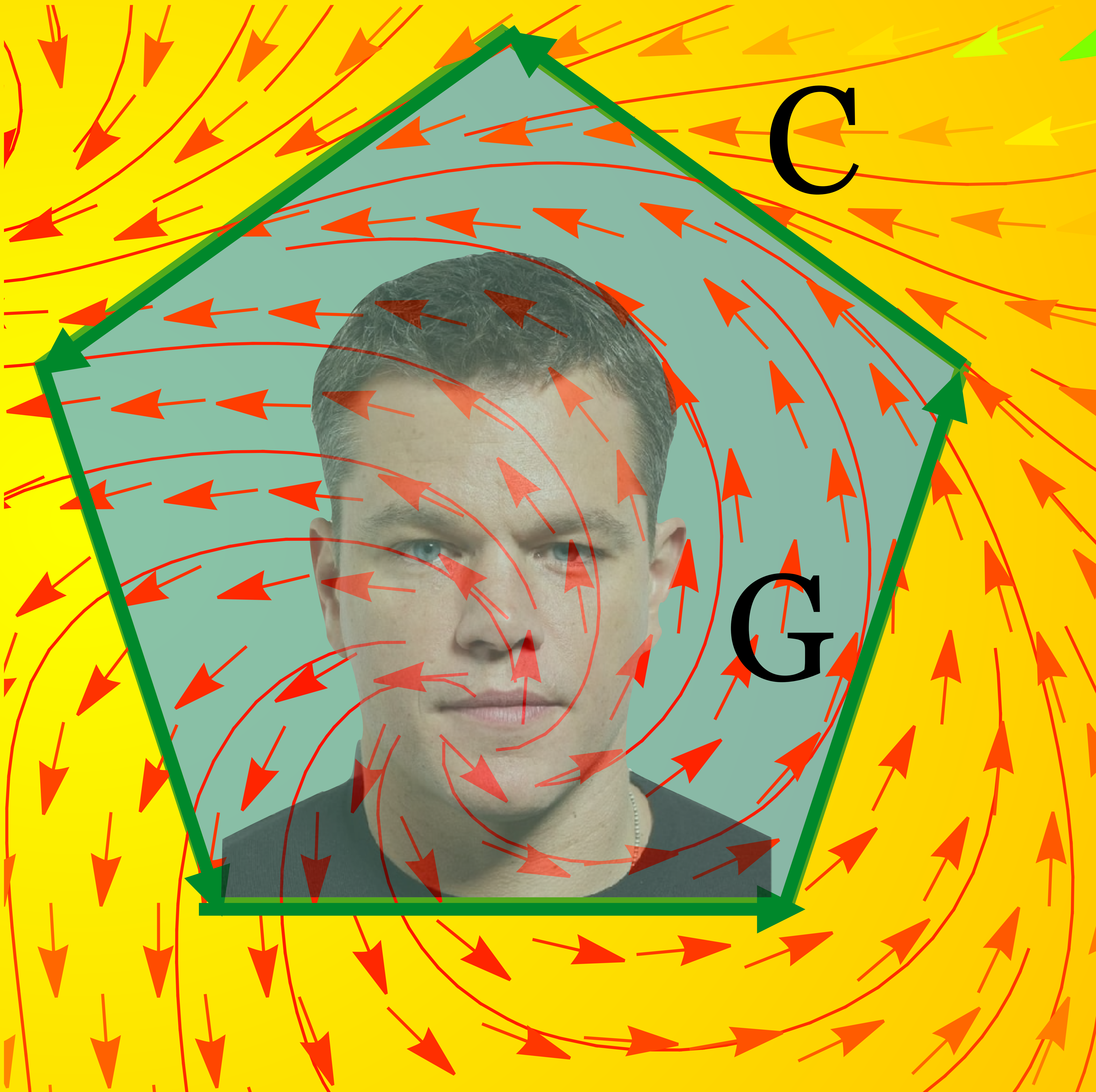


Table of Contents

1) The theorem

2) Example

3) History

4) Area computation

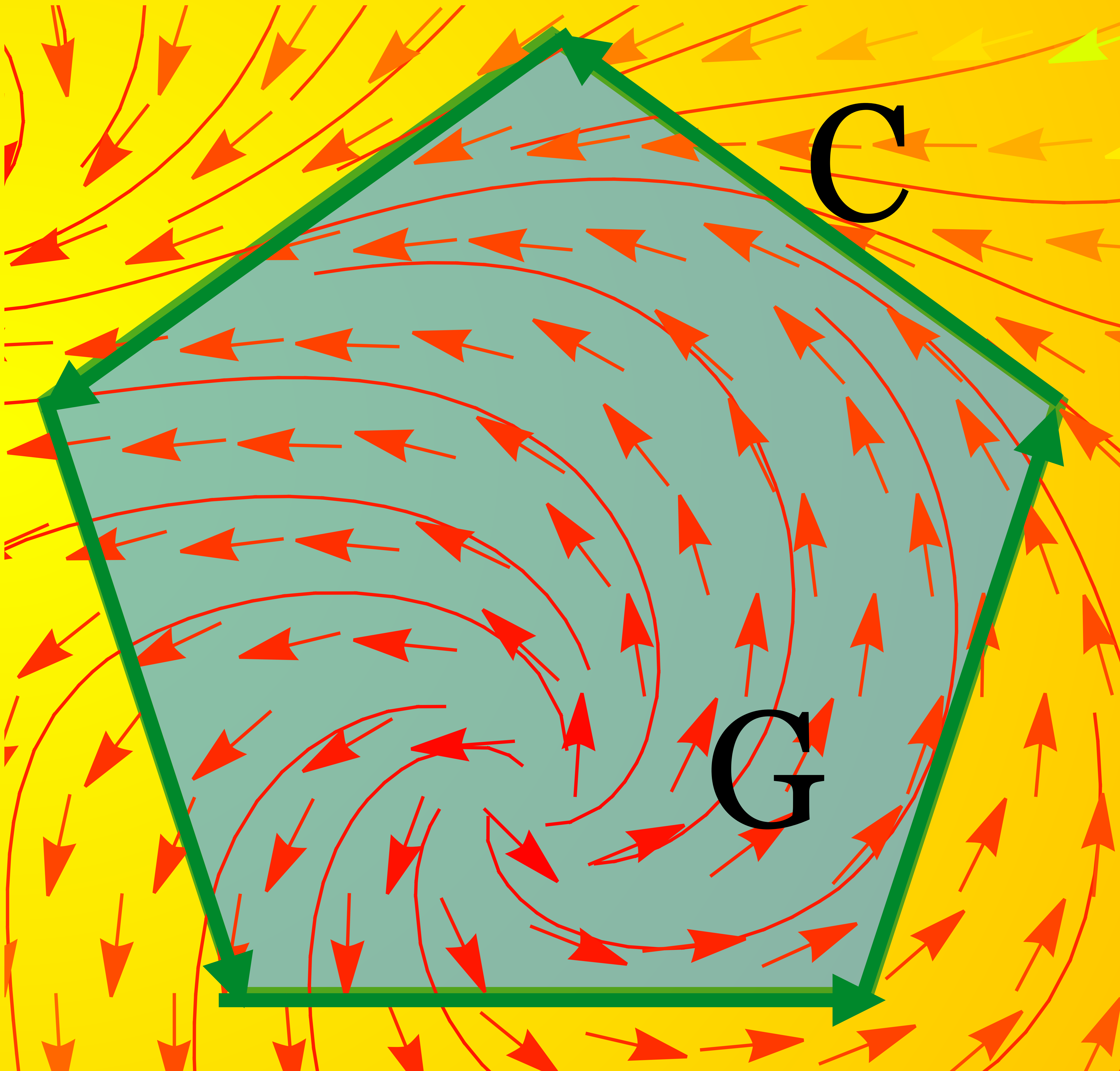
5) Work sheet problems

The Theorem

$$\int_C \vec{F} \cdot d\vec{r} = \iint_G \text{curl}(\vec{F})\, dA$$

Green's Theorem

C is oriented
so that G is
to the left



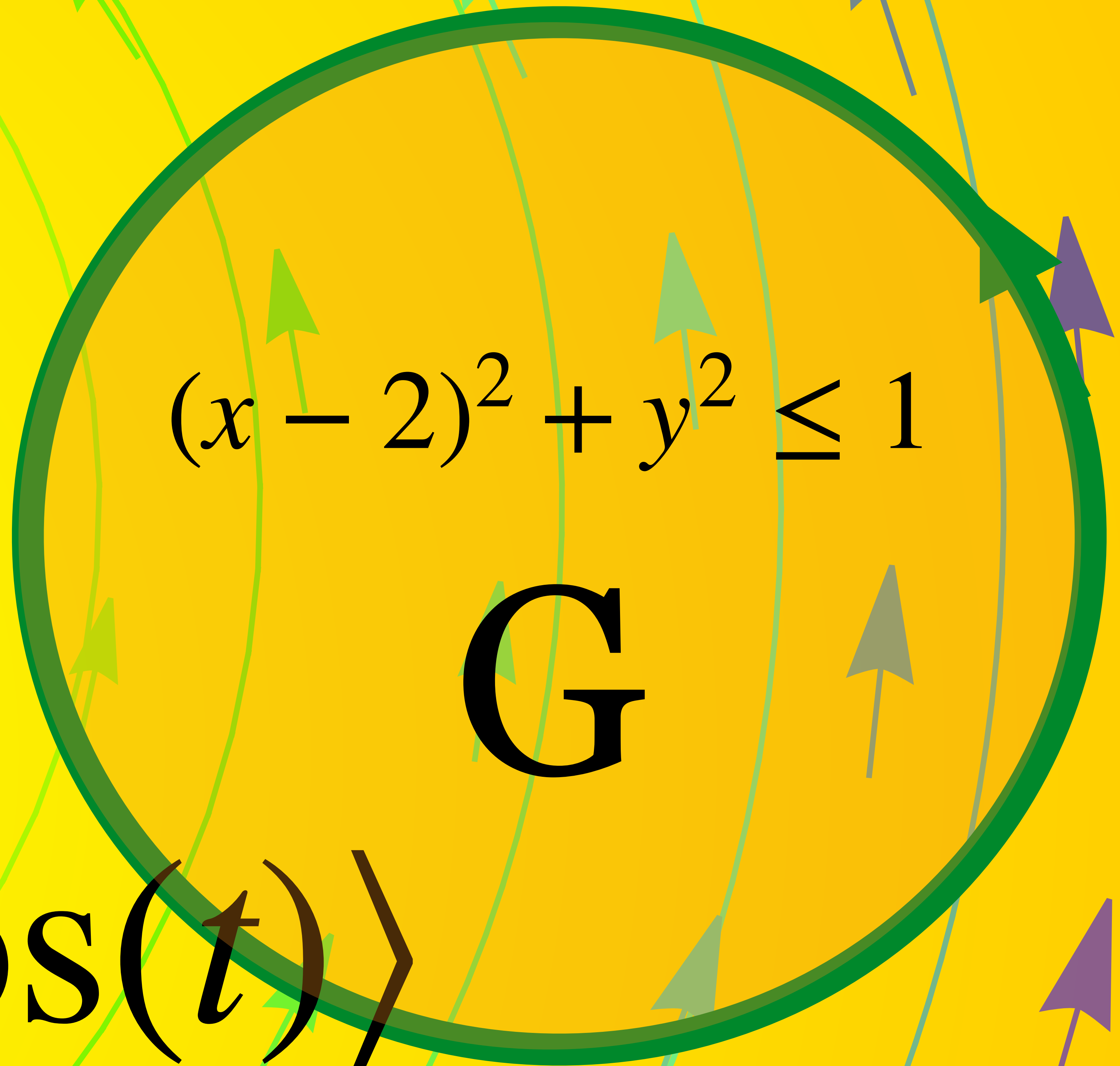
Example

$$\vec{F} = \langle -y, x \rangle$$

$$\vec{r}(t) = \langle 2 + \cos(t), \sin(t) \rangle$$

$$\vec{r}'(t) = \langle -\sin(t), \cos(t) \rangle$$

$$\vec{F}(\vec{r}(t)) = \langle -\sin(t), 2 + \cos(t) \rangle$$



$$\int_0^{2\pi} \langle -\sin(t), 2 + \cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle \, dt = 2\pi$$

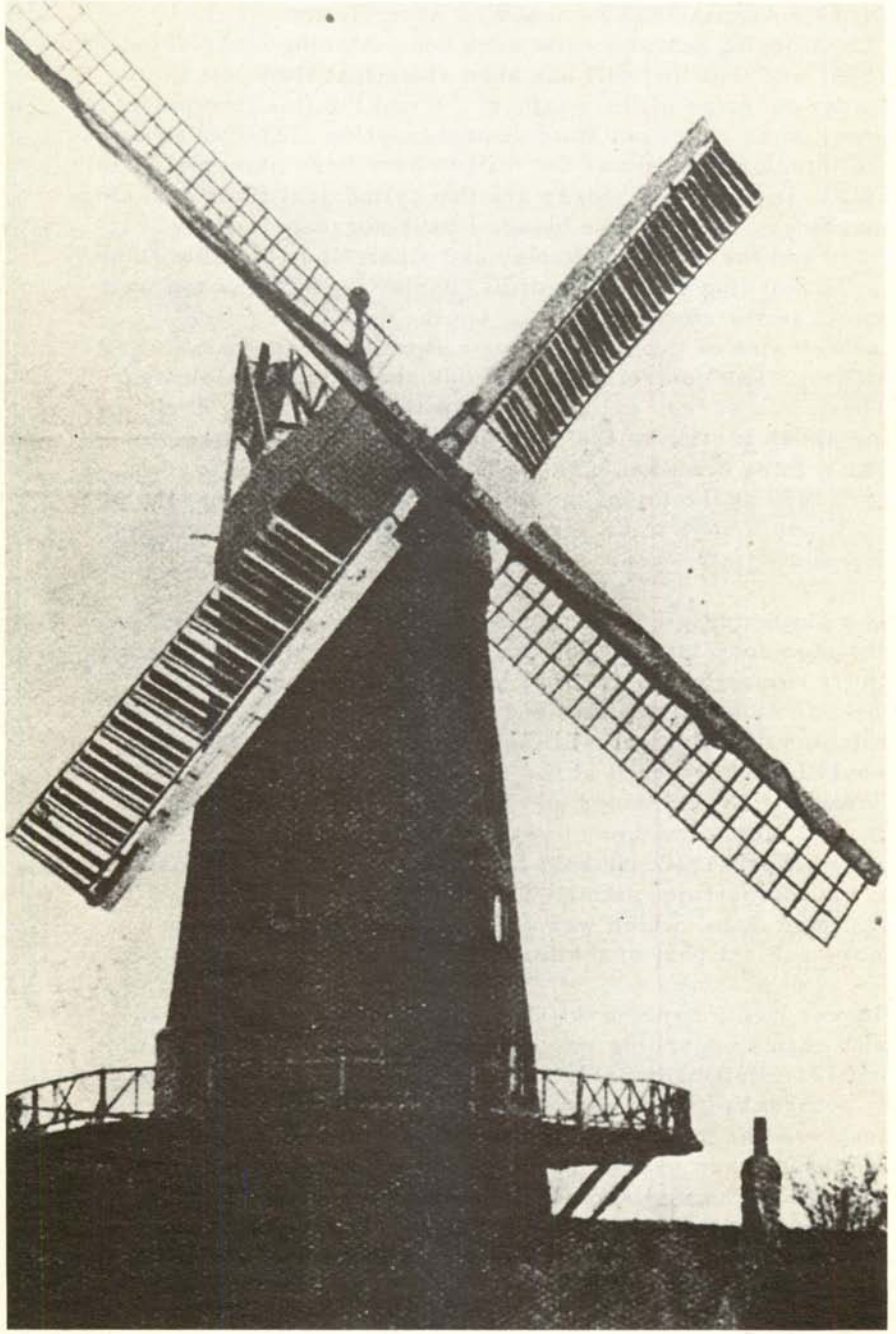
$$\operatorname{curl}(\vec{F}) = 2 \qquad \iint_G \operatorname{curl}(\vec{F}) \, dA = \iint_G 2 \, dA = 2\pi$$


Proof
prove first
for squares
notice
cancellations

History

George Green

1793-1841



Green's Mill in working order in about 1870. From The Miller, May 5th 1924

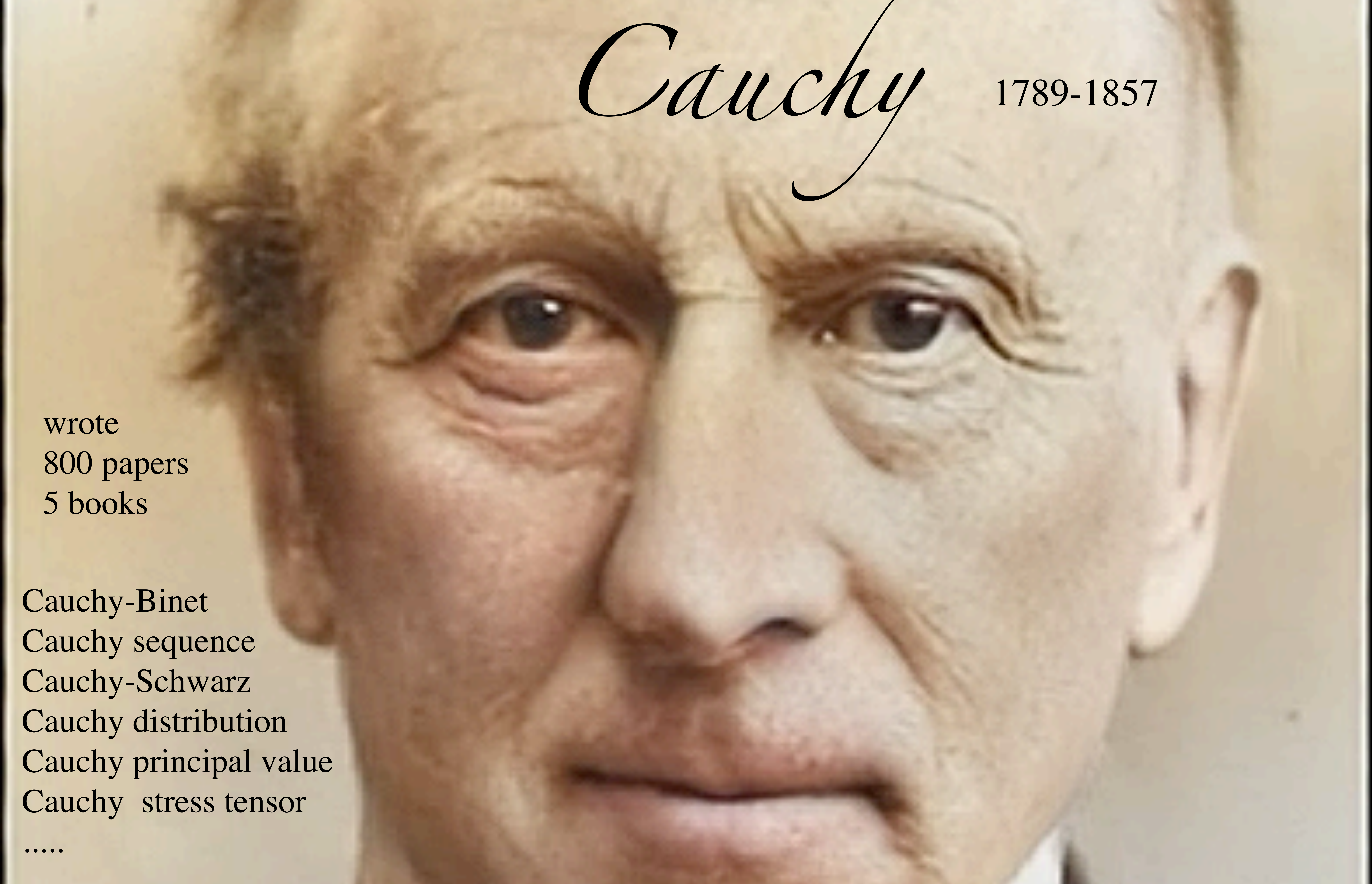

No picture of Green exists! Just imagine Matt Damon!
Good will hunting 1997


From the movie
Good will hunting 1997


From October 22, 2022

An other true Good Will Hunting story

The man who knew infinity, 2015



Cauchy 1789-1857

wrote
800 papers
5 books

Cauchy-Binet
Cauchy sequence
Cauchy-Schwarz
Cauchy distribution
Cauchy principal value
Cauchy stress tensor
.....

List of things named after Augustin-Louis Cauchy

From Wikipedia, the free encyclopedia

More concepts and theorems have been named for Cauchy than for any other mathematician

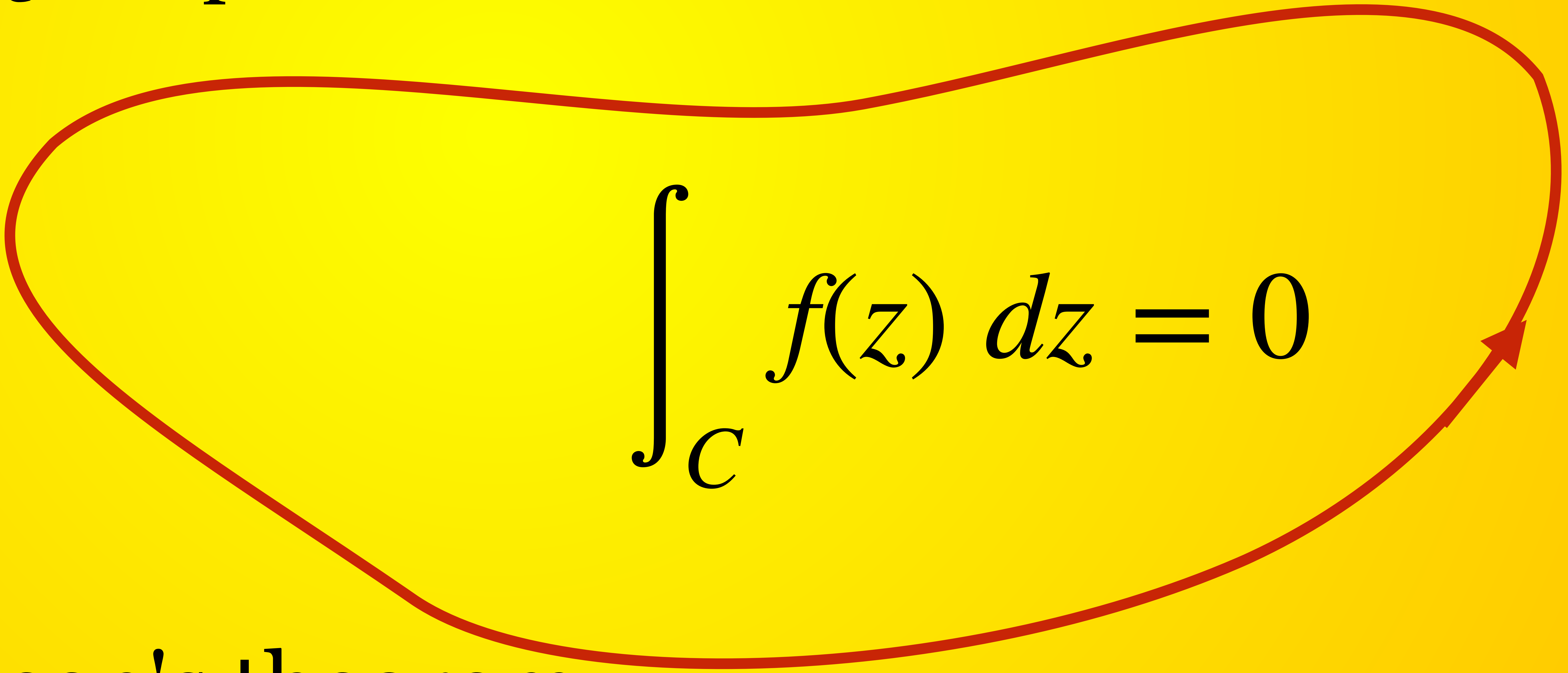
Things named after the 19th-century French mathematician Augustin-Louis Cauchy include:

This list is incomplete; you can help by adding missing items. *(March 2022)*

- Binet–Cauchy identity
- Bolzano–Cauchy theorem
- Cauchy's argument principle
- Cauchy–Binet formula
- Cauchy–Born rule
- Cauchy boundary condition
- Cauchy bounds
- Cauchy completeness
- Cauchy completion
- Cauchy condensation test
- Cauchy-continuous function
- Cauchy's convergence test
- Cauchy (crater)
- Cauchy–Davenport theorem
- Cauchy determinant
- Cauchy distribution
  - Log-Cauchy distribution
  - Wrapped Cauchy distribution
- Cauchy elastic material
- Cauchy's equation
- Cauchy–Euler equation
- Cauchy's functional equation
- Cauchy filter
- Cauchy formula for repeated integration
- Cauchy–Frobenius lemma
- Cauchy–Green deformation tensor
- Cauchy–Hadamard theorem
- Cauchy horizon
- Cauchy identity
- Cauchy index
- Cauchy inequality
- Cauchy's integral formula
- Cauchy's integral theorem
- Cauchy interlacing theorem
- Cauchy–Kovalevskaya theorem
- Cauchy–Kowalevski theorem
- Cauchy–Lipschitz theorem
- Cauchy matrix
- Cauchy momentum equation
- Cauchy net
- Cauchy number
- Cauchy–Peano theorem
- Cauchy point
- Cauchy principal value
- Cauchy problem
  - Abstract Cauchy problem
- Cauchy process
- Cauchy product
- Cauchy's radical test
- Cauchy–Rassias stability
- Cauchy ratio test
- Cauchy–Riemann equations
- Cauchy–Riemann manifold
- Cauchy's Residue Theorem
- Cauchy–Schlömilch transformation
- Cauchy–Schwarz inequality
- Cauchy sequence
  - Uniformly Cauchy sequence
- Cauchy space
- Cauchy surface
- Cauchy's mean value theorem
- Cauchy stress tensor
- Cauchy's theorem (geometry)
- Cauchy's theorem (group theory)
- Cauchy's two-line notation
- Euler–Cauchy stress principle
- Maclaurin–Cauchy test

Cauchy's integral theorem

f(x+iy) = P+i Q complex valued



$$\int_C f(z)\, dz = 0$$

reduces to Green's theorem

Gauss


measuring the world

Area computation

Area of Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\vec{r}(t) = \langle a\cos(t), b\sin(t)\rangle$$

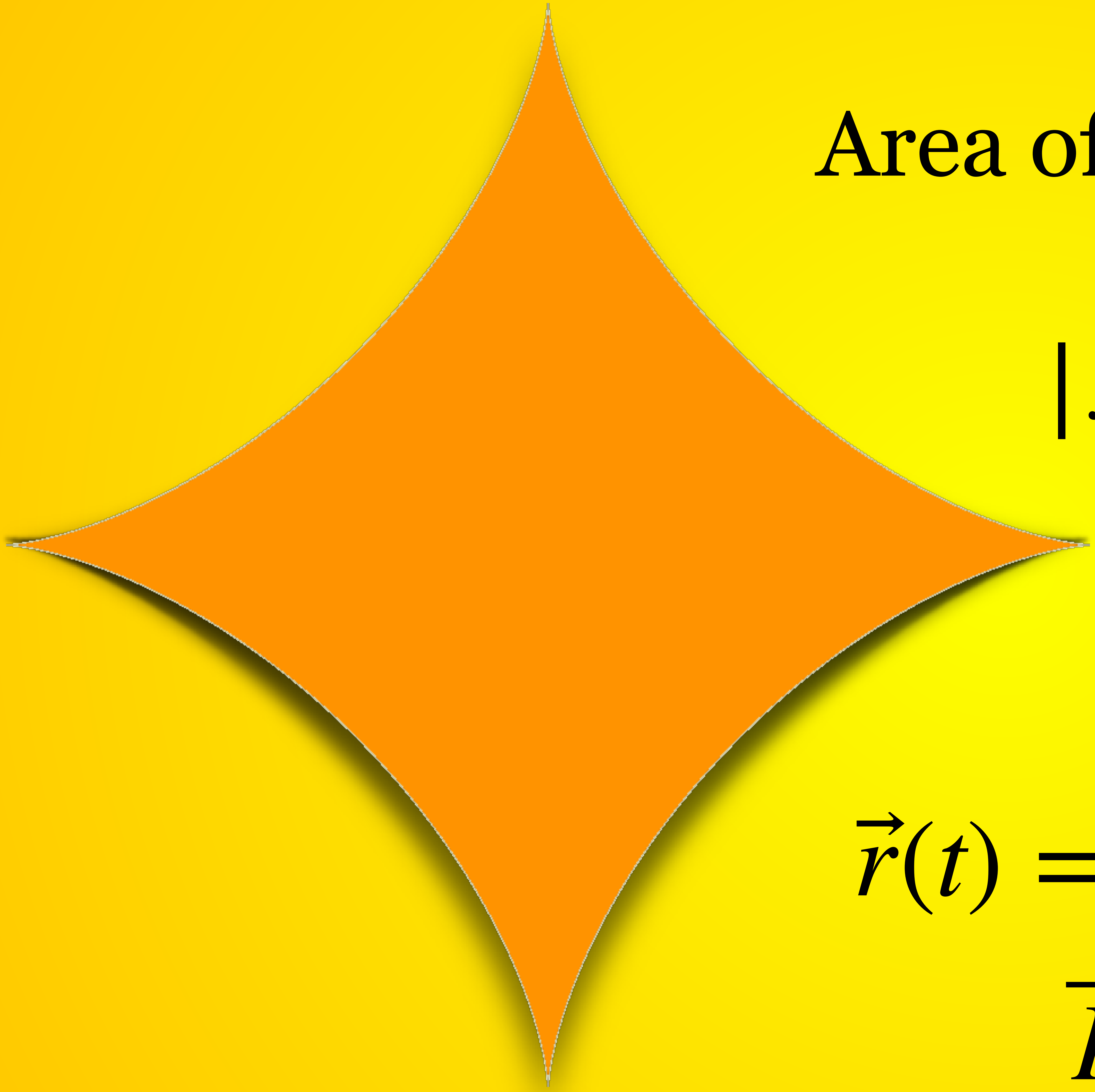
$$\vec{r}\,'(t) = \langle -a\sin(t), b\cos(t)\rangle$$

$$\overrightarrow{F} = \langle 0,x\rangle$$

$$\overrightarrow{F}(\vec{r}(t)) = \langle 0,a\cos(t)\rangle$$

$$\int_0^{2\pi} ab\cos^2(t)\;dt = ab\pi$$


EXTRA CREDIT
Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
$e\to 0 \Rightarrow a\to b\to r$
$e\to 0 \Rightarrow c\wedge -c\to 0$
$A_{\odot}=\pi r^2$
To Prove: $A_E=\pi ab$
$\left(A_E=\int_{-a}^{a}(y_2-y_1)\,dx\right)$
y
(0,b)
(a,0)
(-a,0)
(-c,0)
(c,0)
x
(0,-b)
Rushmore



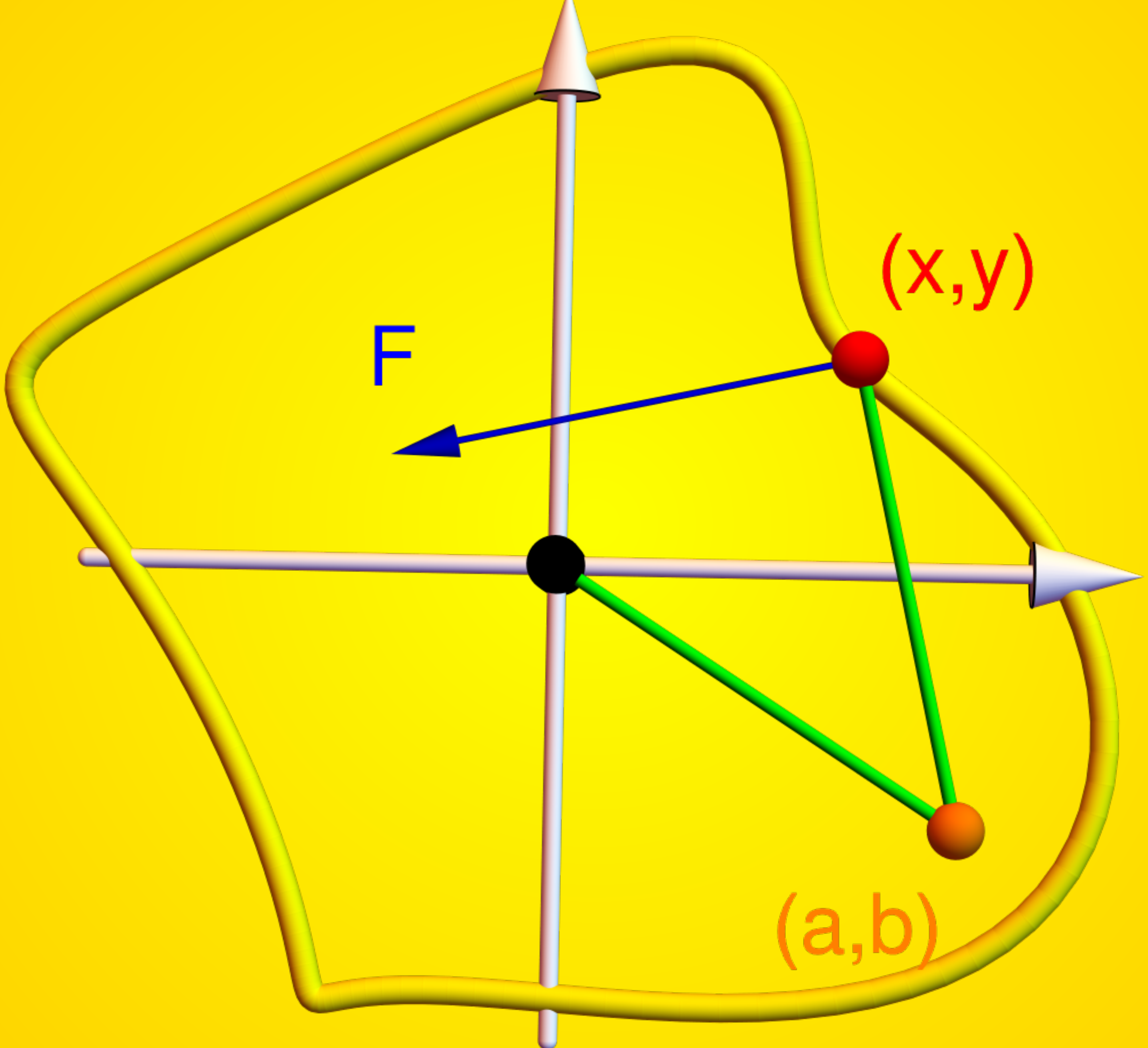
Area of Hypercycloid

$$|x|^{2/3} + |y|^{2/3} = 1$$

$$\vec{r}(t) = \langle \cos(t)^3, \sin(t)^3 \rangle >$$

$$\overrightarrow{F} = \langle 0, x \rangle$$

Planimeter


(x,y)
F
(a,b)

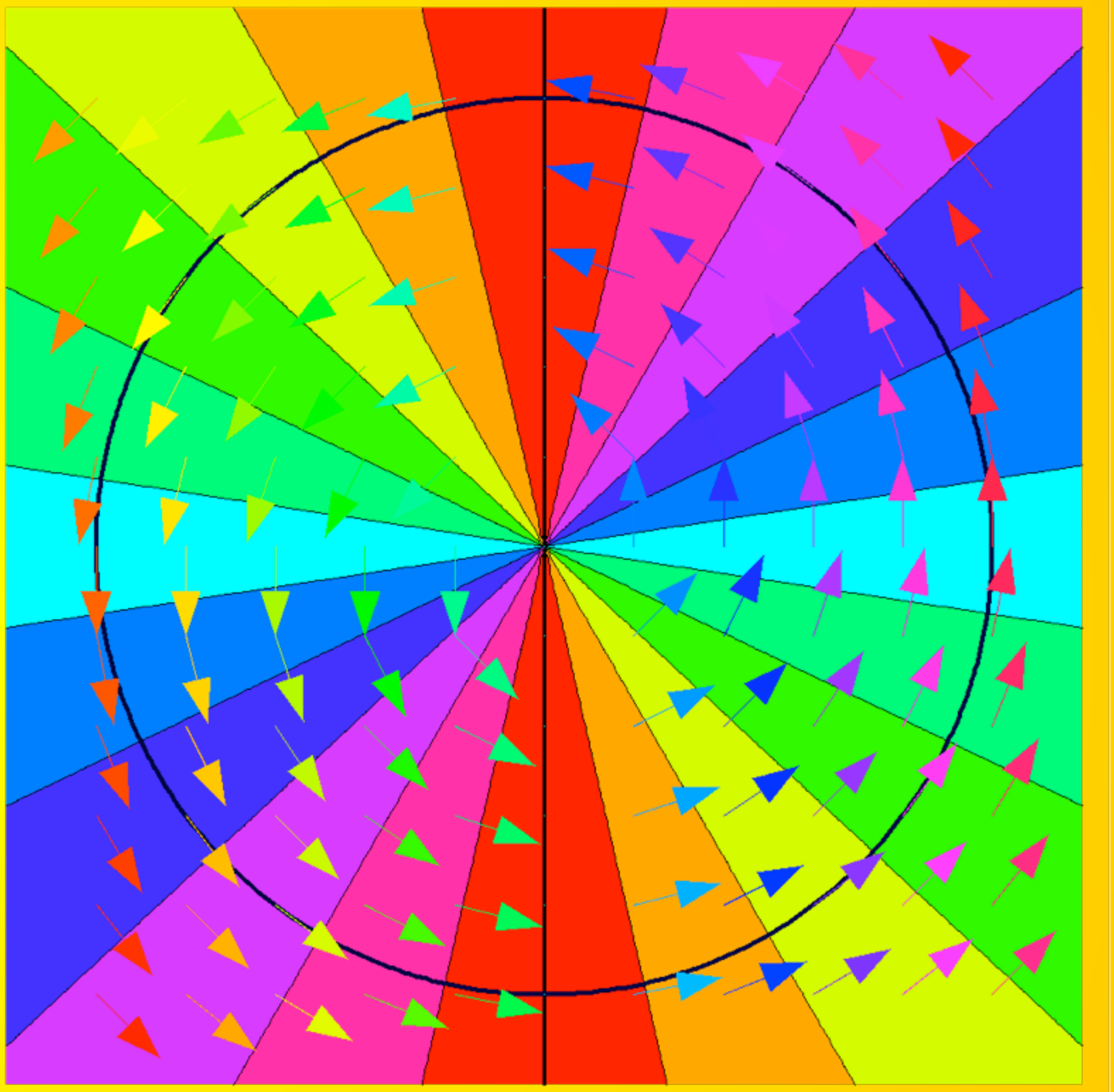

90□CM

Limitations

Vortex

$$\vec{F} = \begin{bmatrix} \dfrac{-y}{x^2+y^2} \\ \dfrac{x}{x^2+y^2} \end{bmatrix}$$

$$\text{curl}(\vec{F}) = 0$$



We need nice F's!

THE END