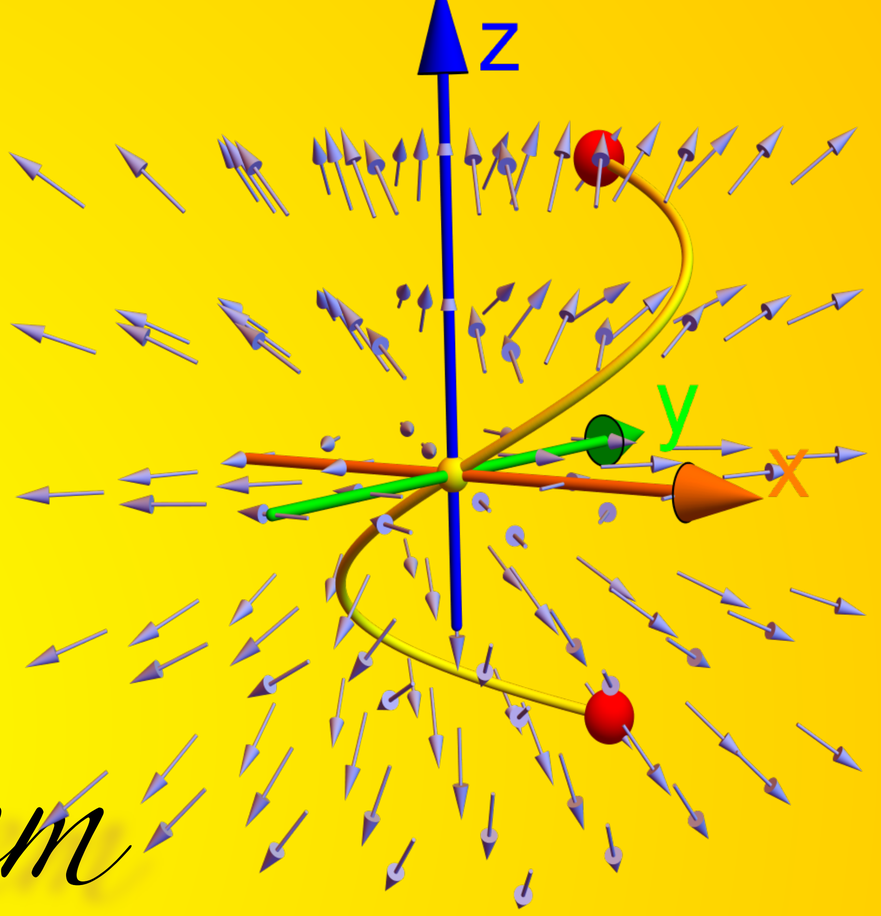
Lecture 25

Fundamental Theorem

Line Integrals



Reminders

2. Midterm 11/10/2022: 5:30-7:30
Focus on 15-25 Psets. So, today's lecture is the last topic. So, CURVES, SURFACES, Line and surface INTEGRALS, Linear and Quadratic APPROXIMATION, GREEN and FTOLI are the main topics.

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Fundamental Theorem

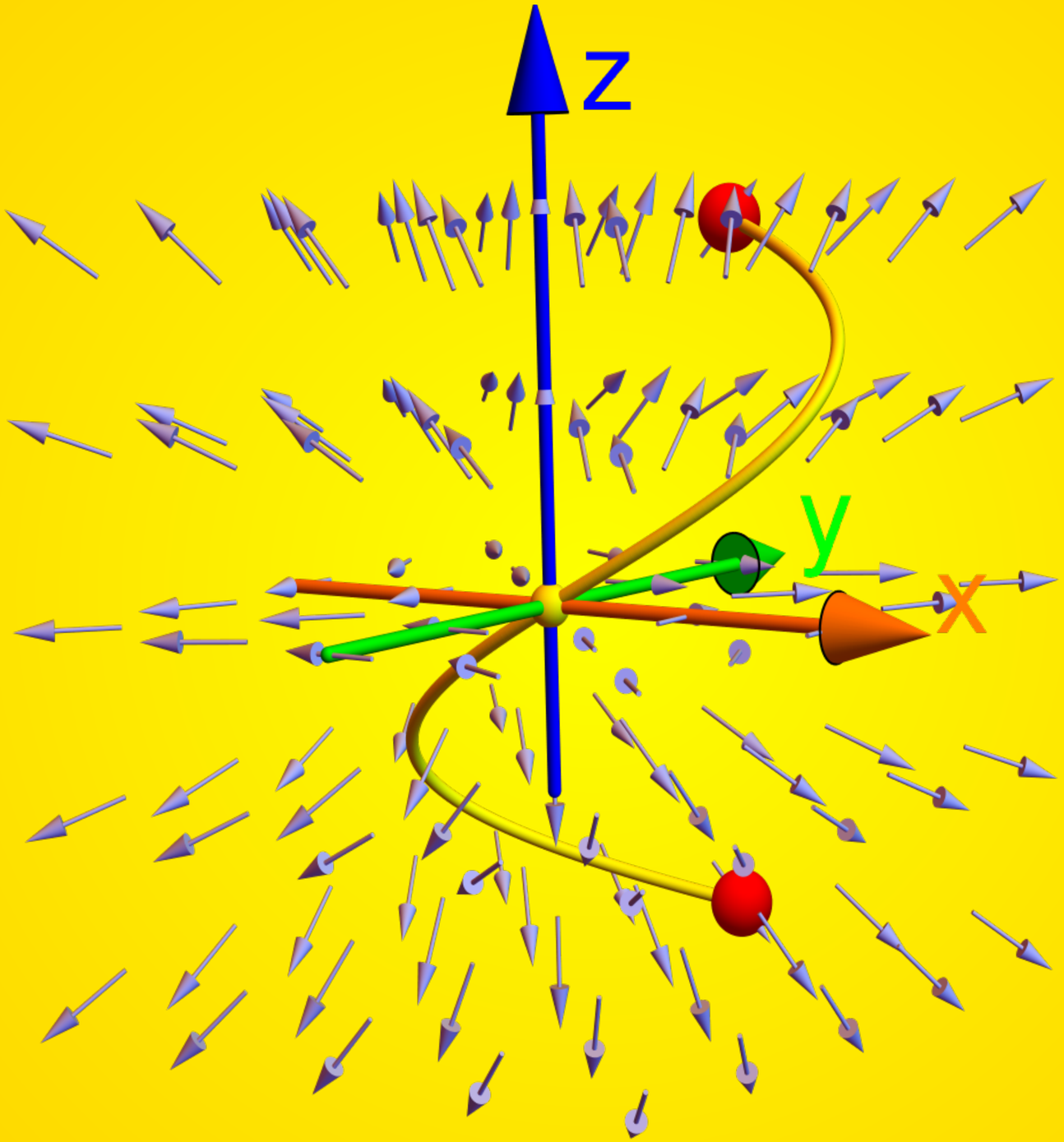
Fundamental Theorem of calculus

$$\int_a^b f'(t)\,dt = f(b) - f(a)$$

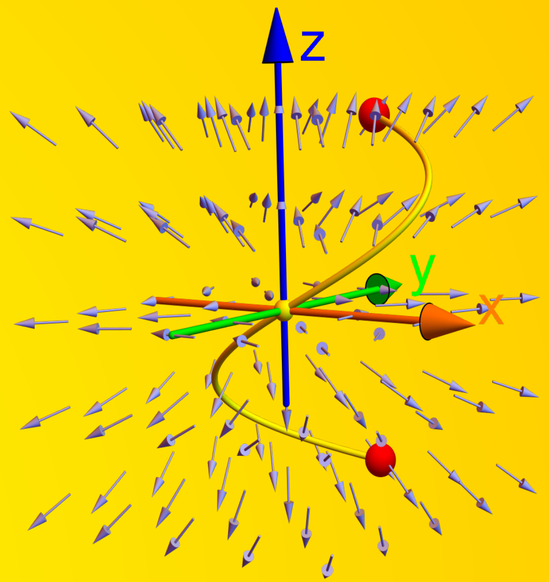
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Look at the sour

faces of the inventors!


z
y
x

Fundamental Theorem



$$\vec{F} = \nabla f$$

$$\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$$

Proof

Integrate $\dfrac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$

$$f(B) - F(A) = \int_a^b \frac{d}{dt} f(\vec{r}(t)) = \int \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \int_C \vec{F} \cdot d\vec{r}$$

Example

Find the line integral of

$$\vec{F}(x,y) = \langle yx + x^2, y^2 \rangle$$

along the path

$$\vec{r}(t) = \langle t + \sin(\pi t), \cos(\pi \cos(\pi t)) \rangle$$

from t=0 to t=1

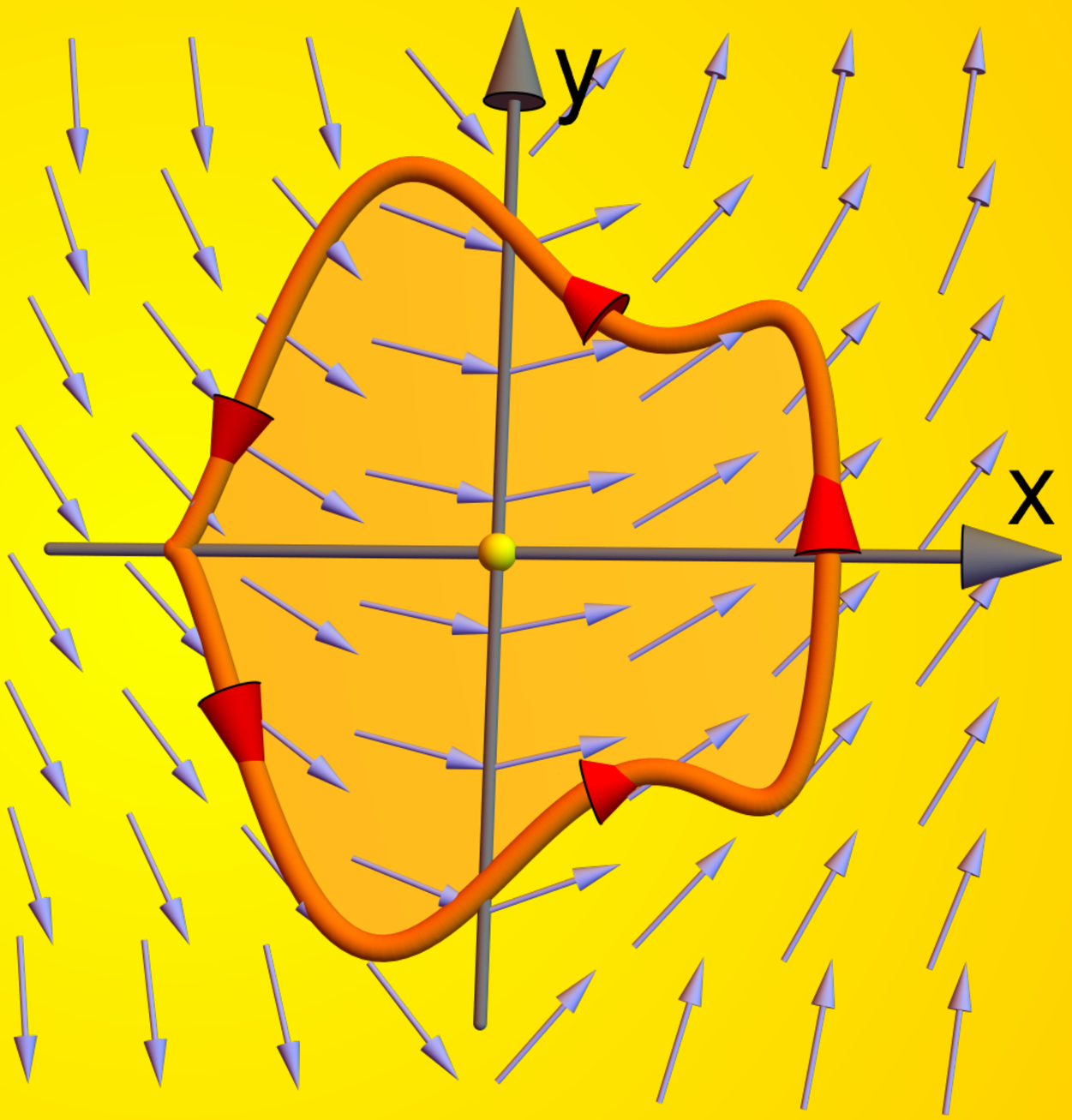
How to get the potential

$$\overrightarrow{F}(x,y) =$$

$$\langle 4x^3y + 3x^2 + e^y\cos(x), x^4 + e^y\sin(x)\rangle$$

Relation with Green

What happens if F is a gradient field?

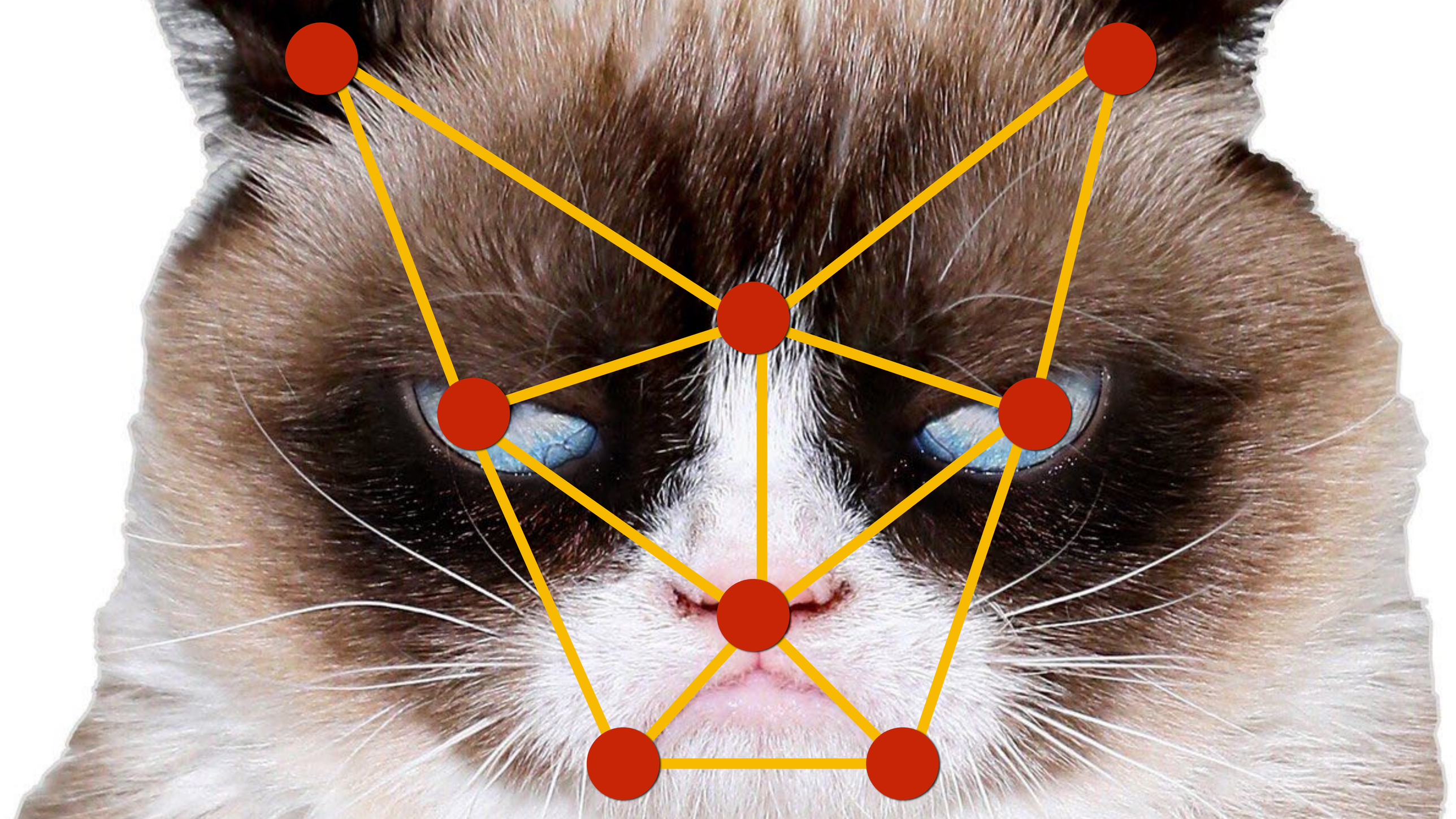


What will your kids learn in college? What about your grand kids?

Quantum Calculus!

Networks

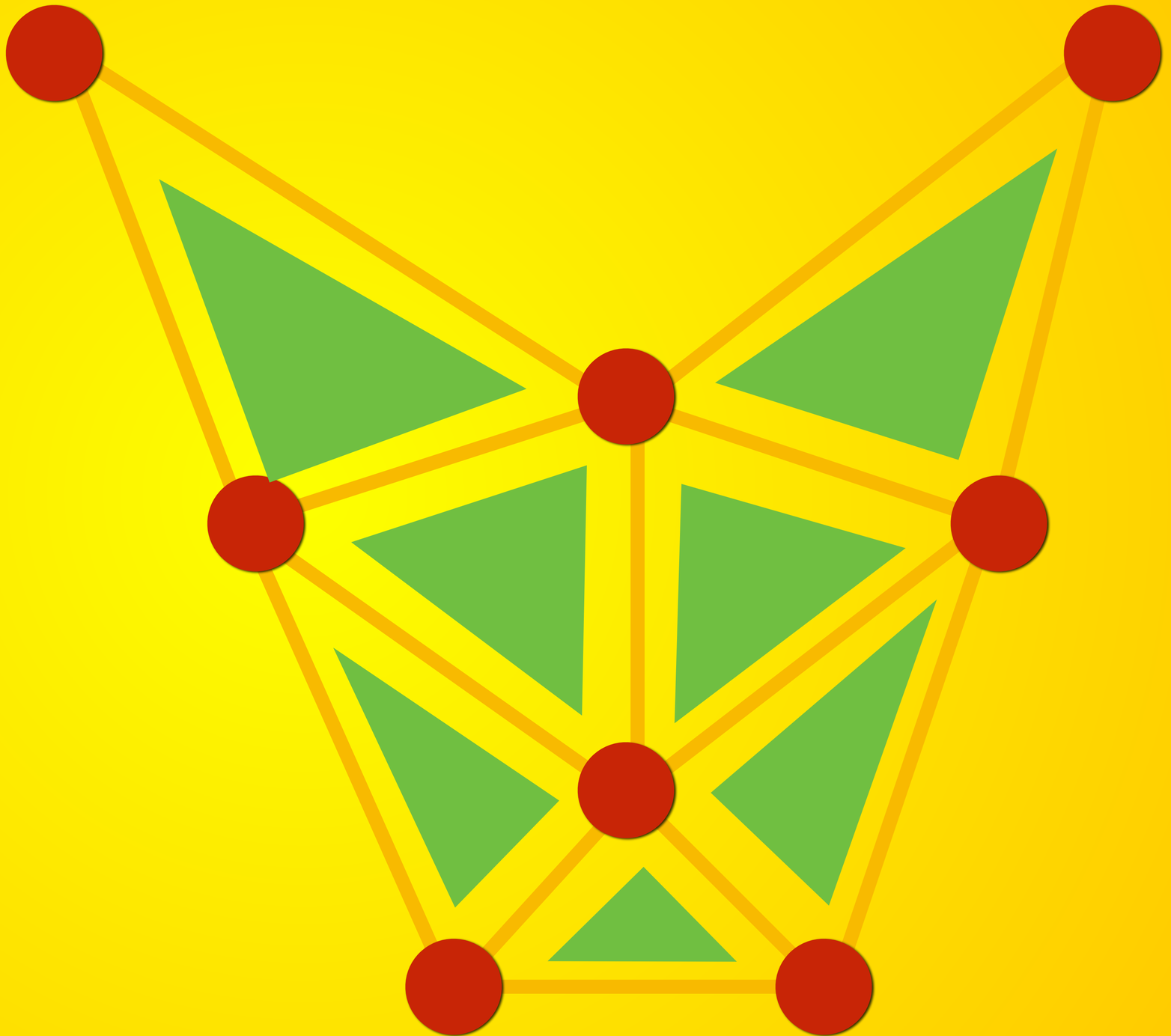
Networks



Nodes

Connections =dr

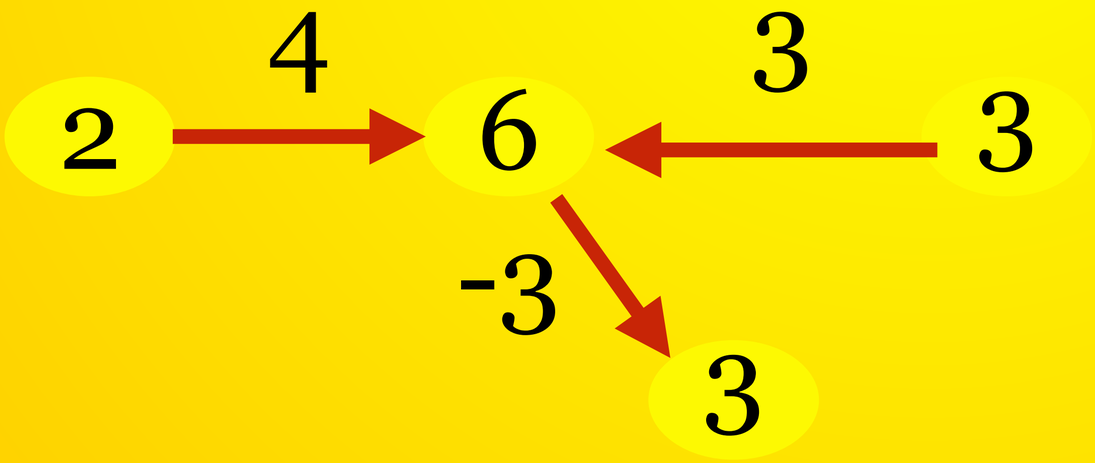
Triangles =dA



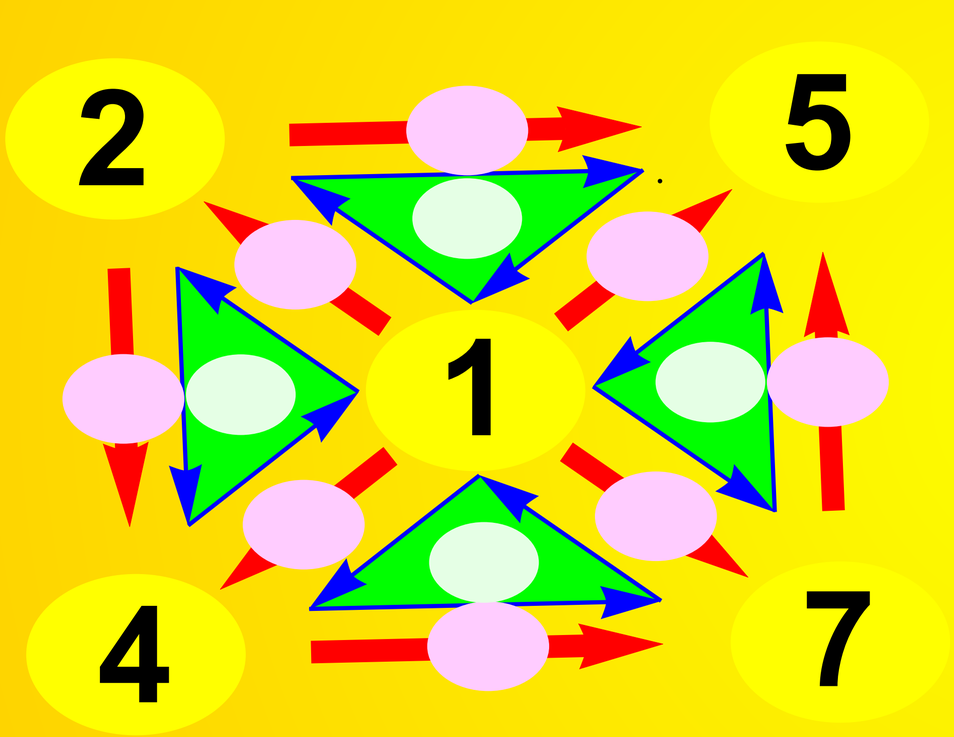
Gradient

Gradient

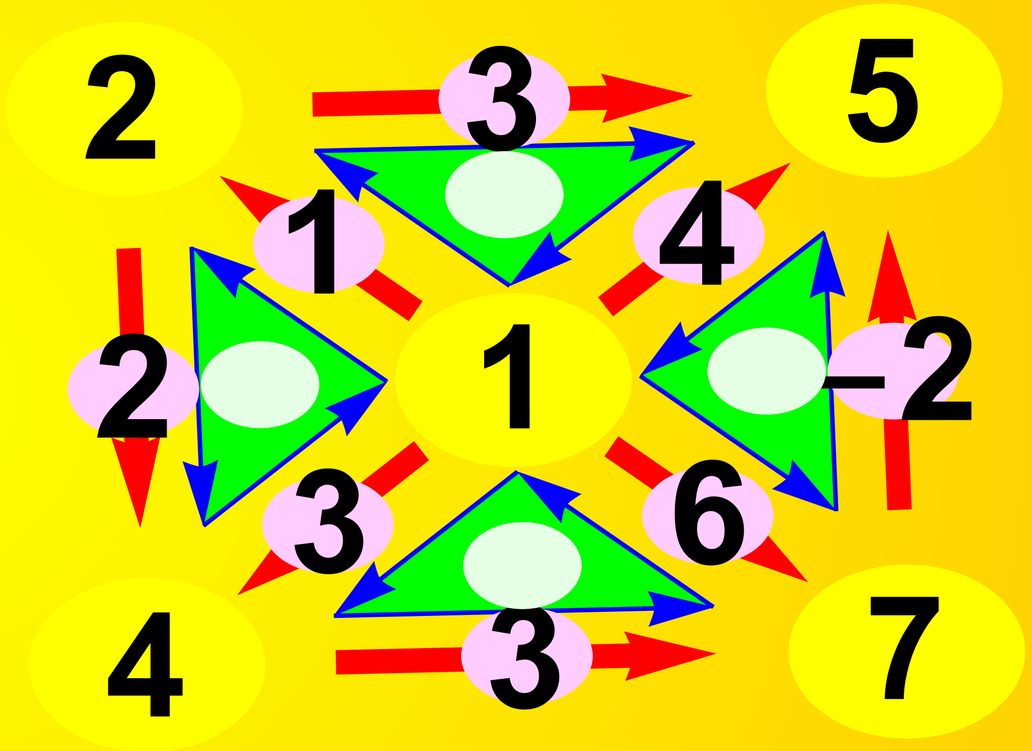
$$\nabla f(a,b) = f(b)-f(a)$$



Example:



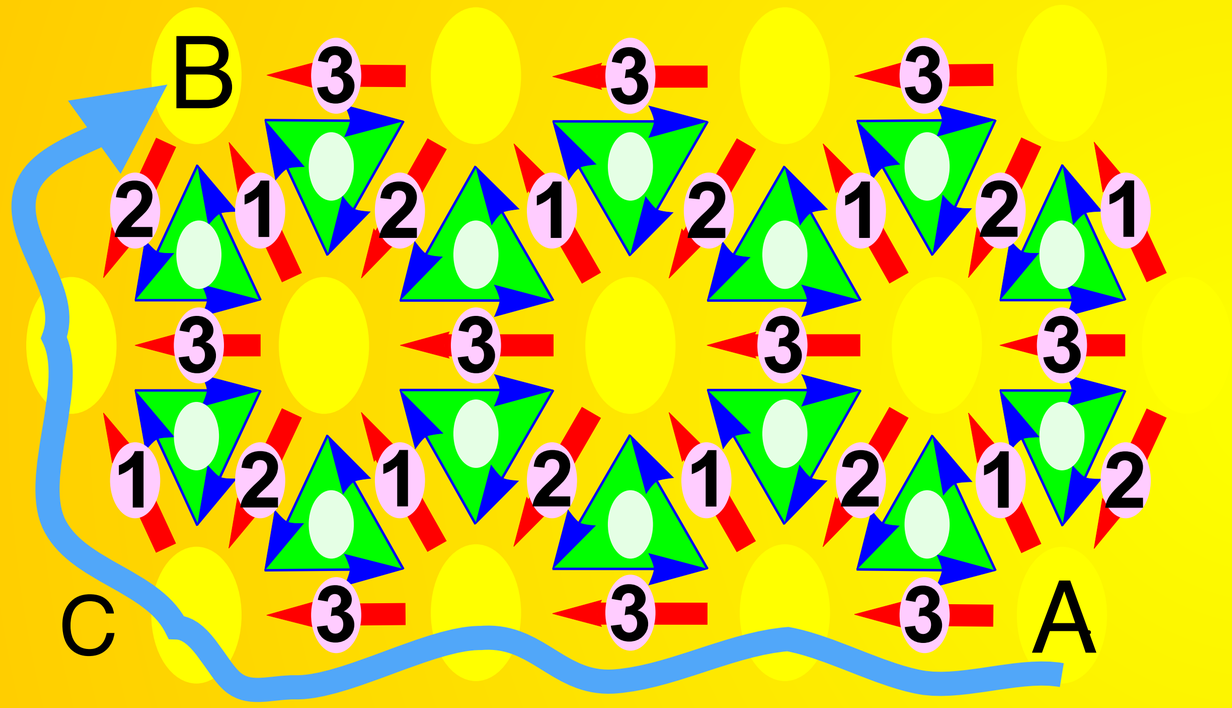
f



$F = \nabla f$

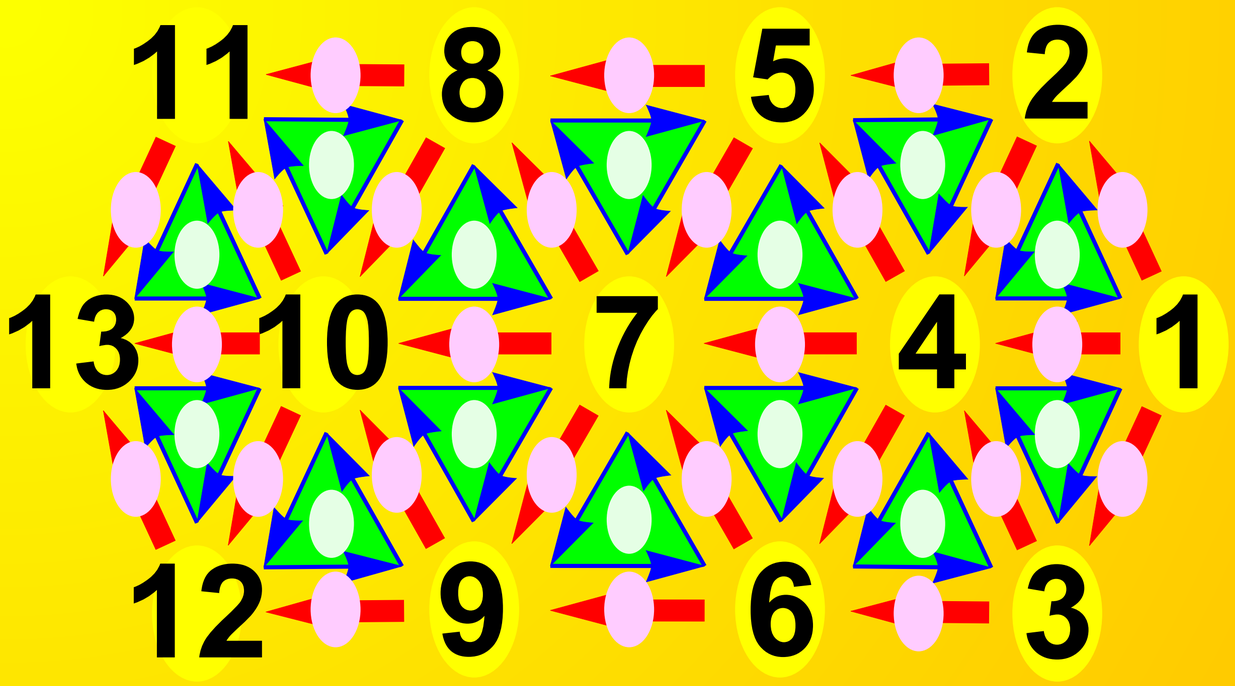
FTLI

Note that curl =0 everywhere



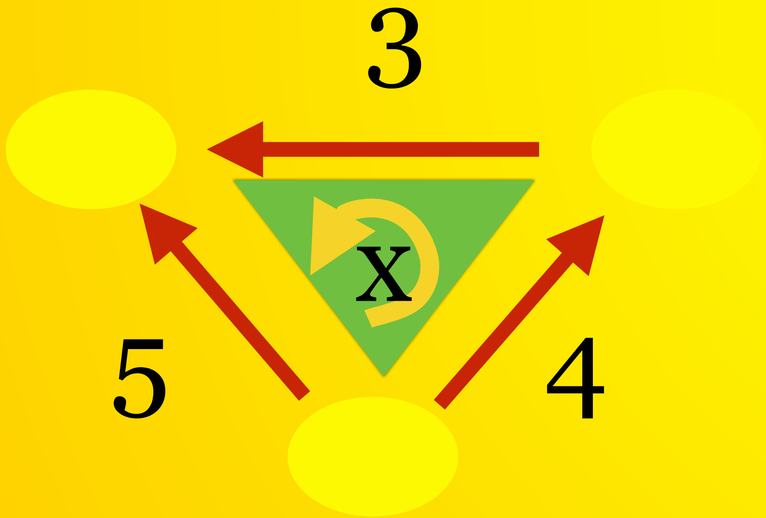
Direct sum: 3+3+3+1-2

FTLI: f(B)-f(A)=11-3

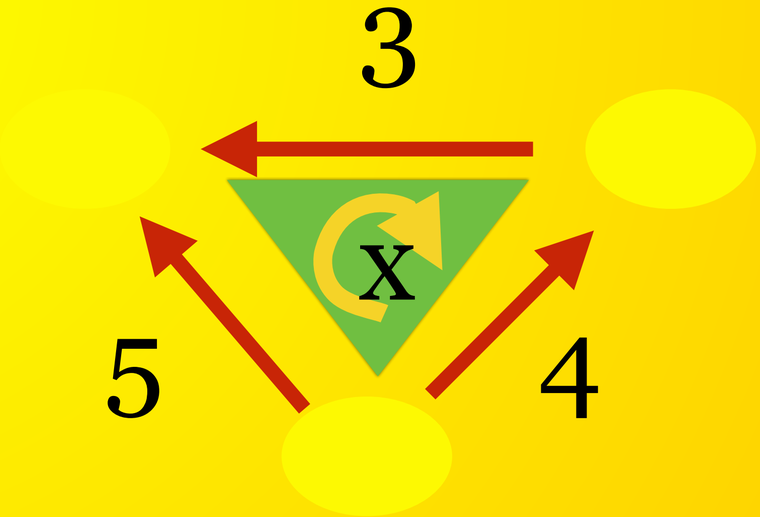


Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$

Curl



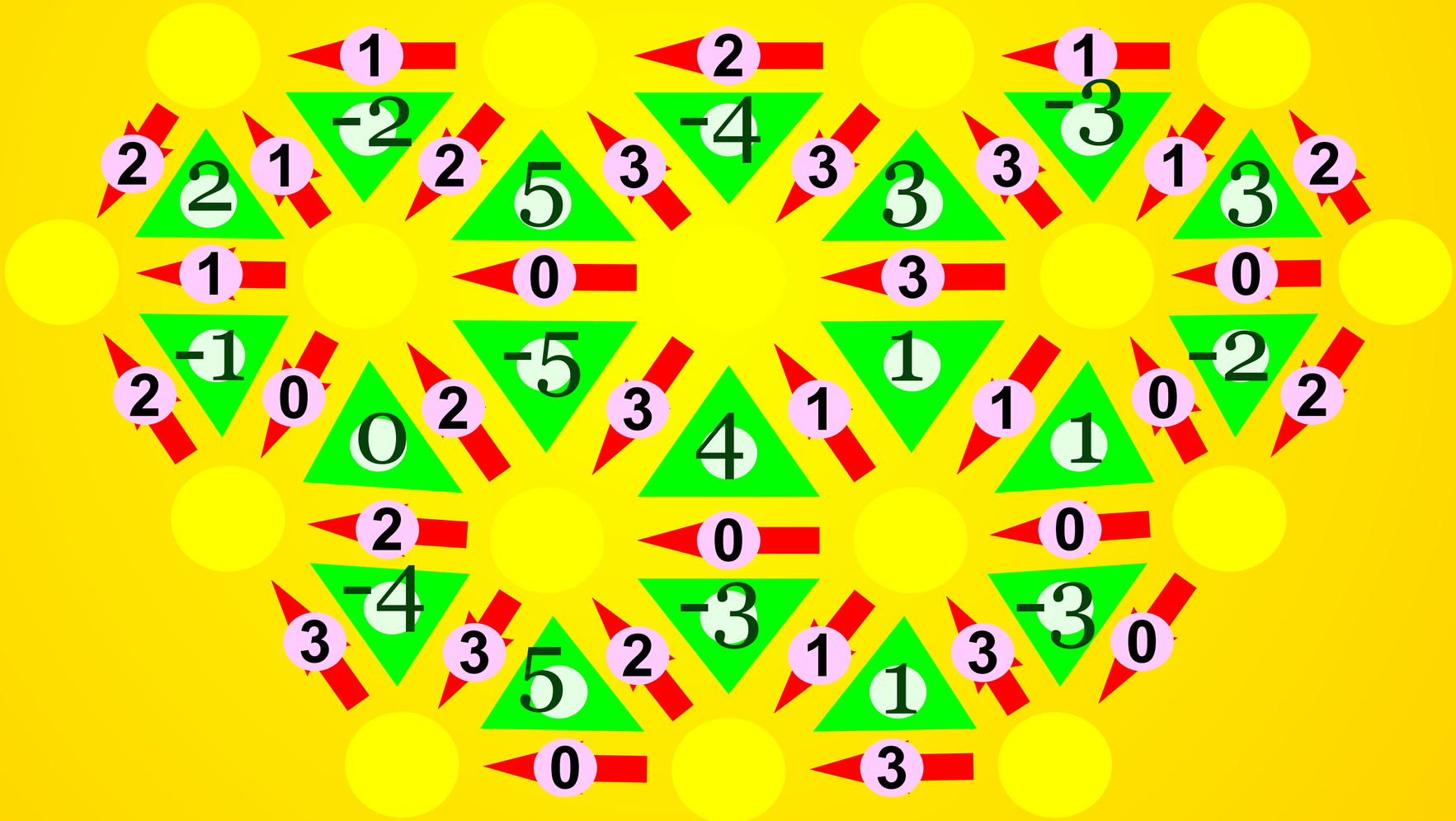
curl(F)(x)=4+3-5=2



durl(F)(x)=5-3-4=-2

Green's theorem

$$\iint_G \vec{F} \cdot dA = \int_C \vec{F} \cdot d\vec{r}$$



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THE END