Lecture 32

Grad-Curl-Div

Table of Contents

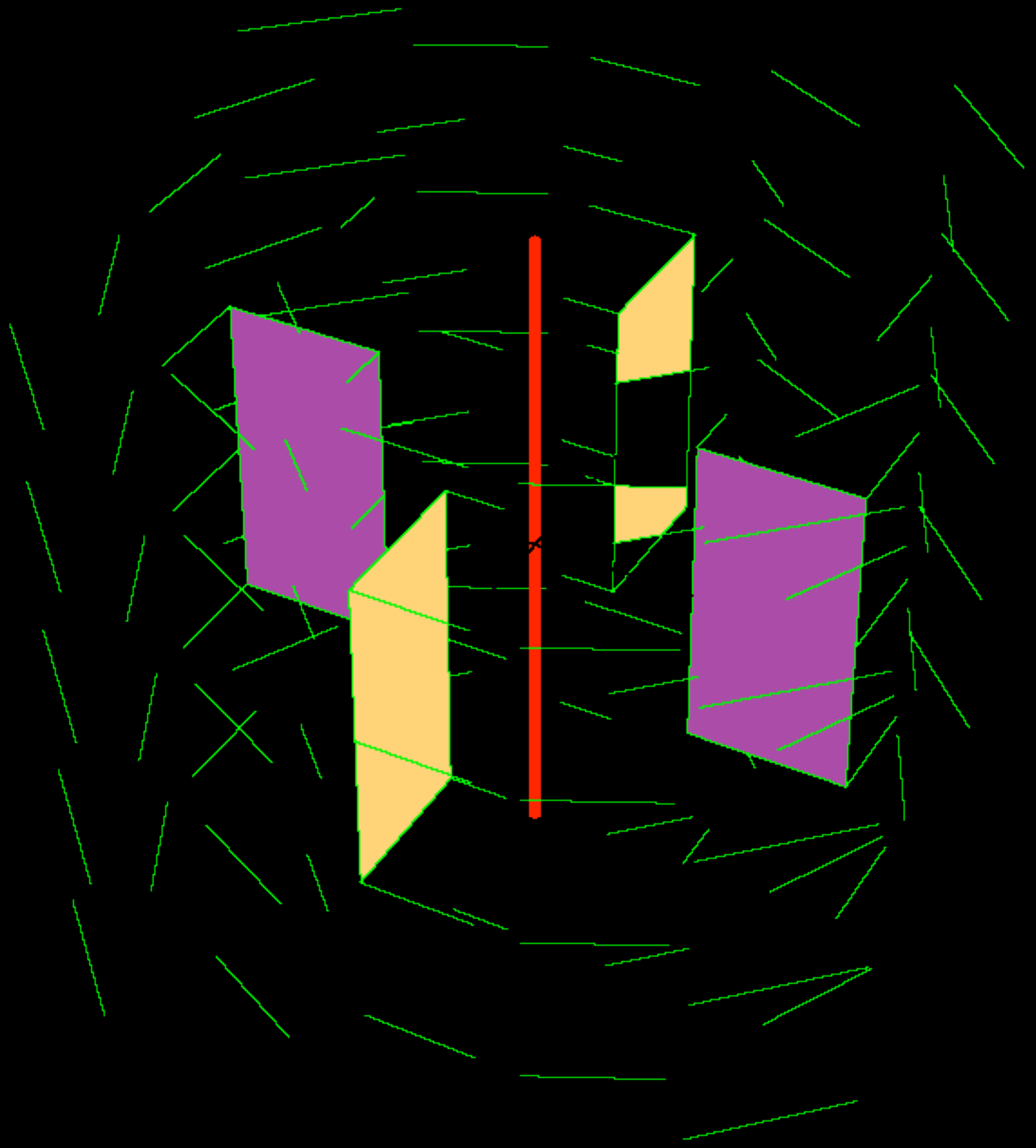
Curl in 3D

Curl

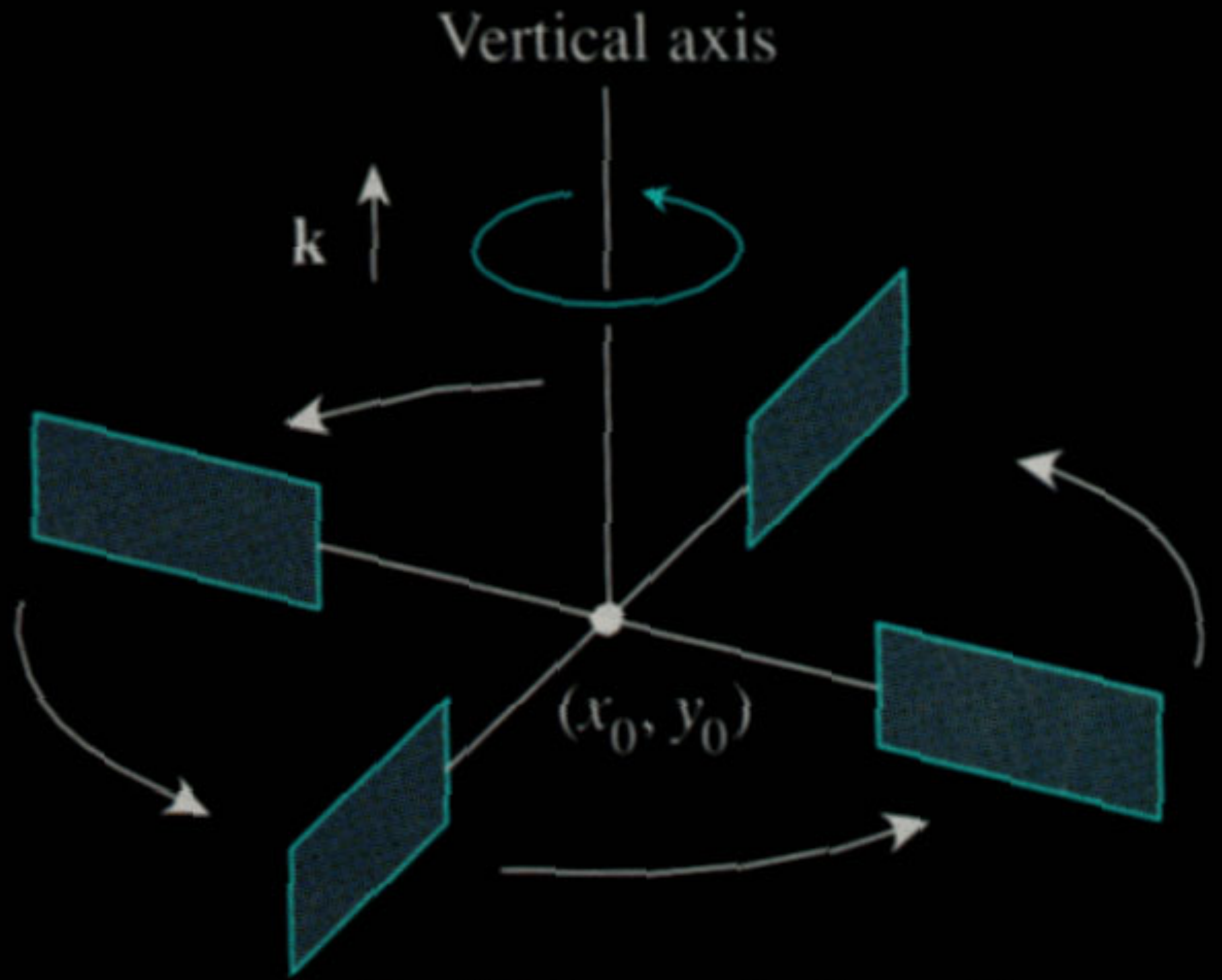
$$\vec{F} = \langle P, Q, R \rangle$$

$$\text{curl}(\vec{F}) = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$$

$$\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ \text{P} & \text{Q} & \text{R} \end{bmatrix}$$

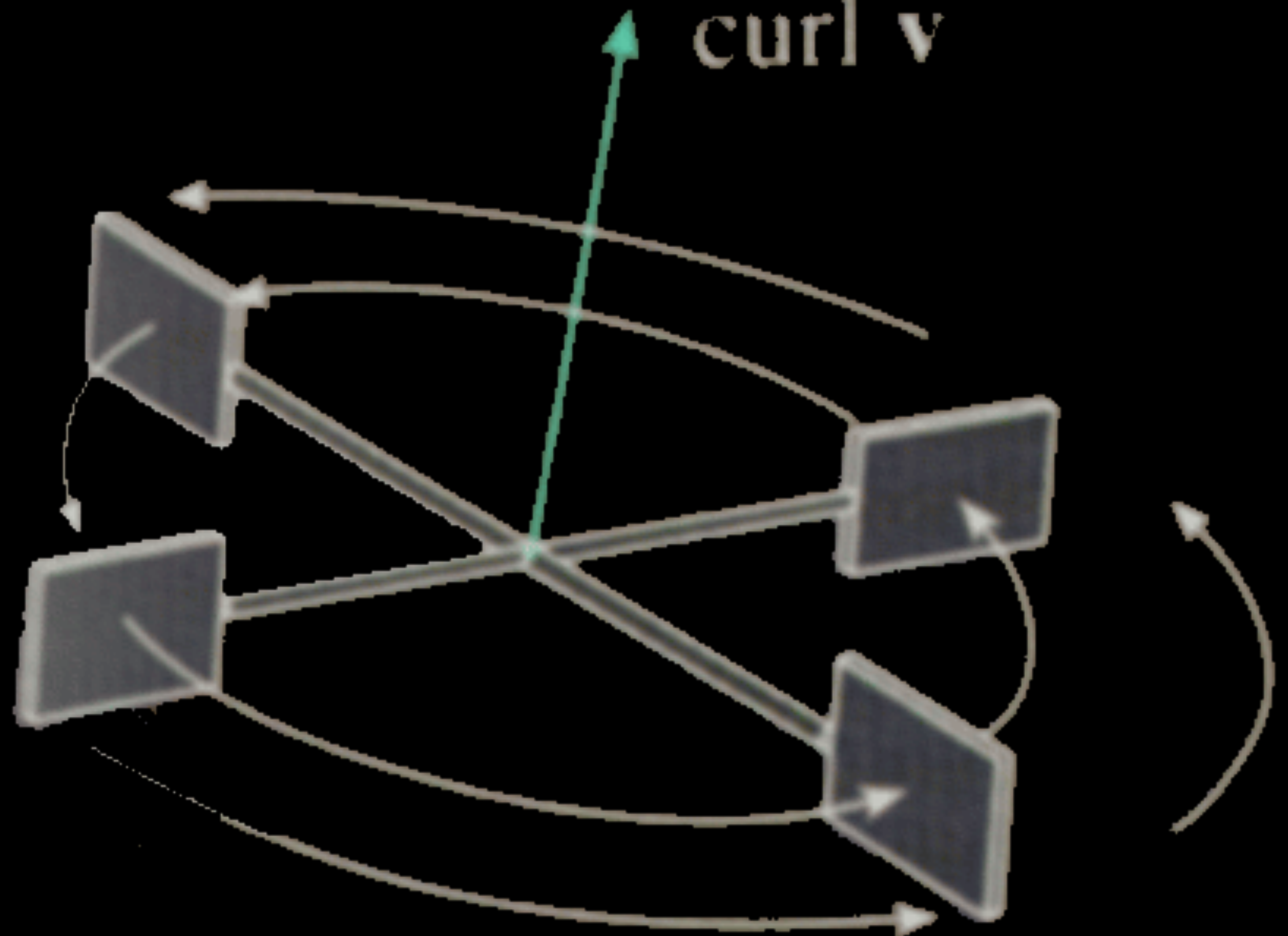


In essentially all multivariable textbooks, the paddlewheel appears.



Curl $\mathbf{F}(x_0, y_0) \cdot \mathbf{k} > 0$

Counterclockwise circulation


curl v

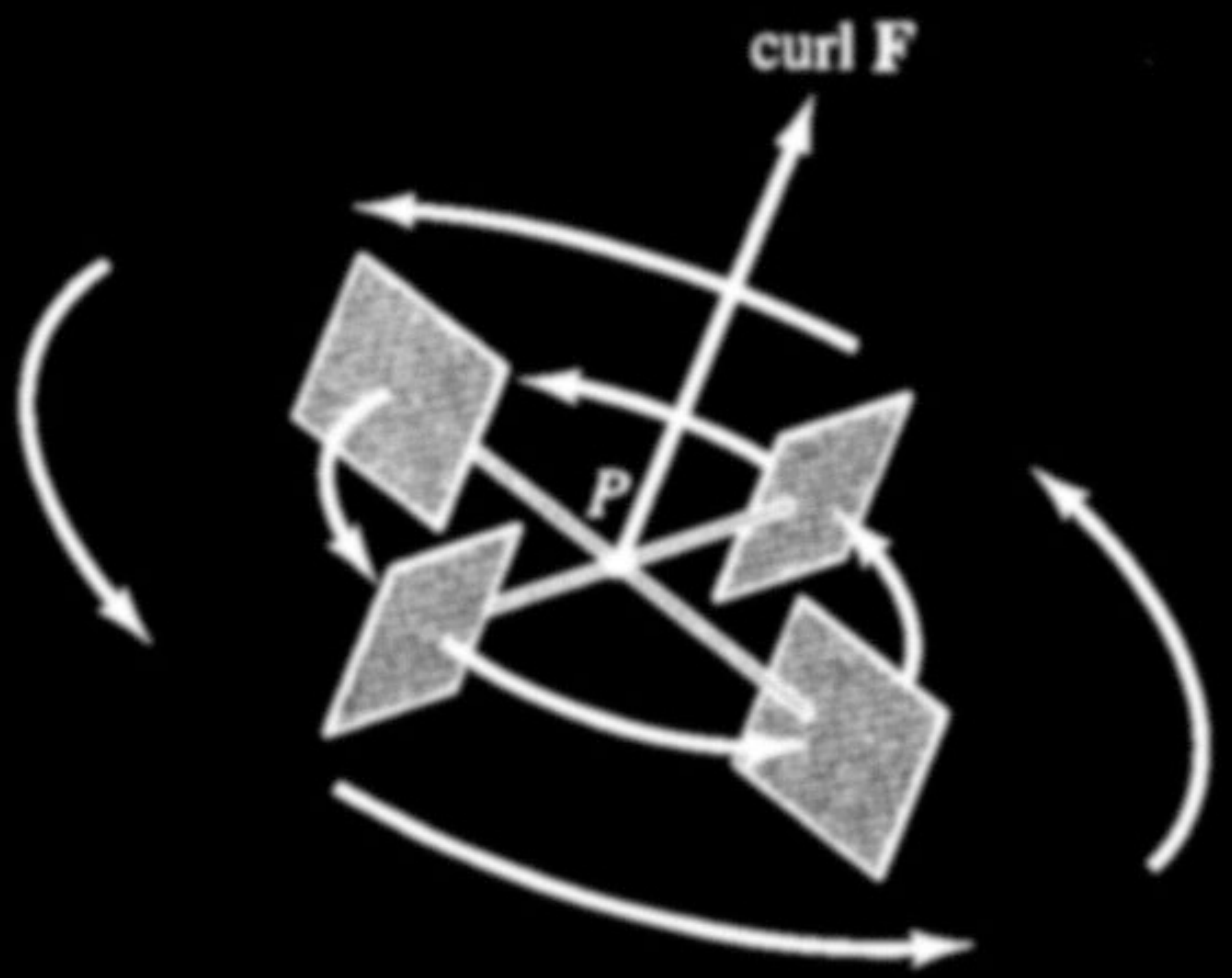


FIGURE 15.7.6 The paddle-wheel interpretation of curl **F**.

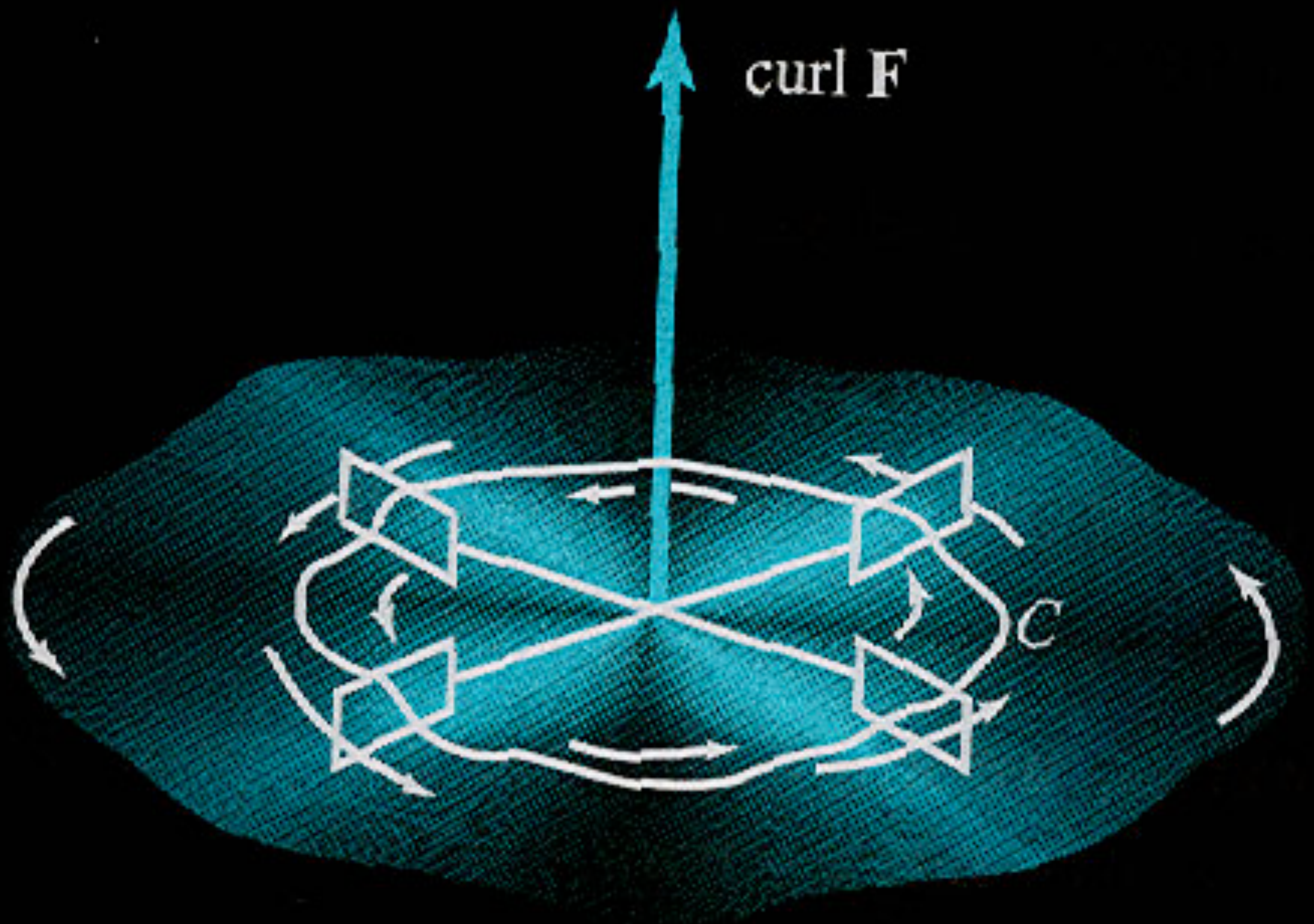
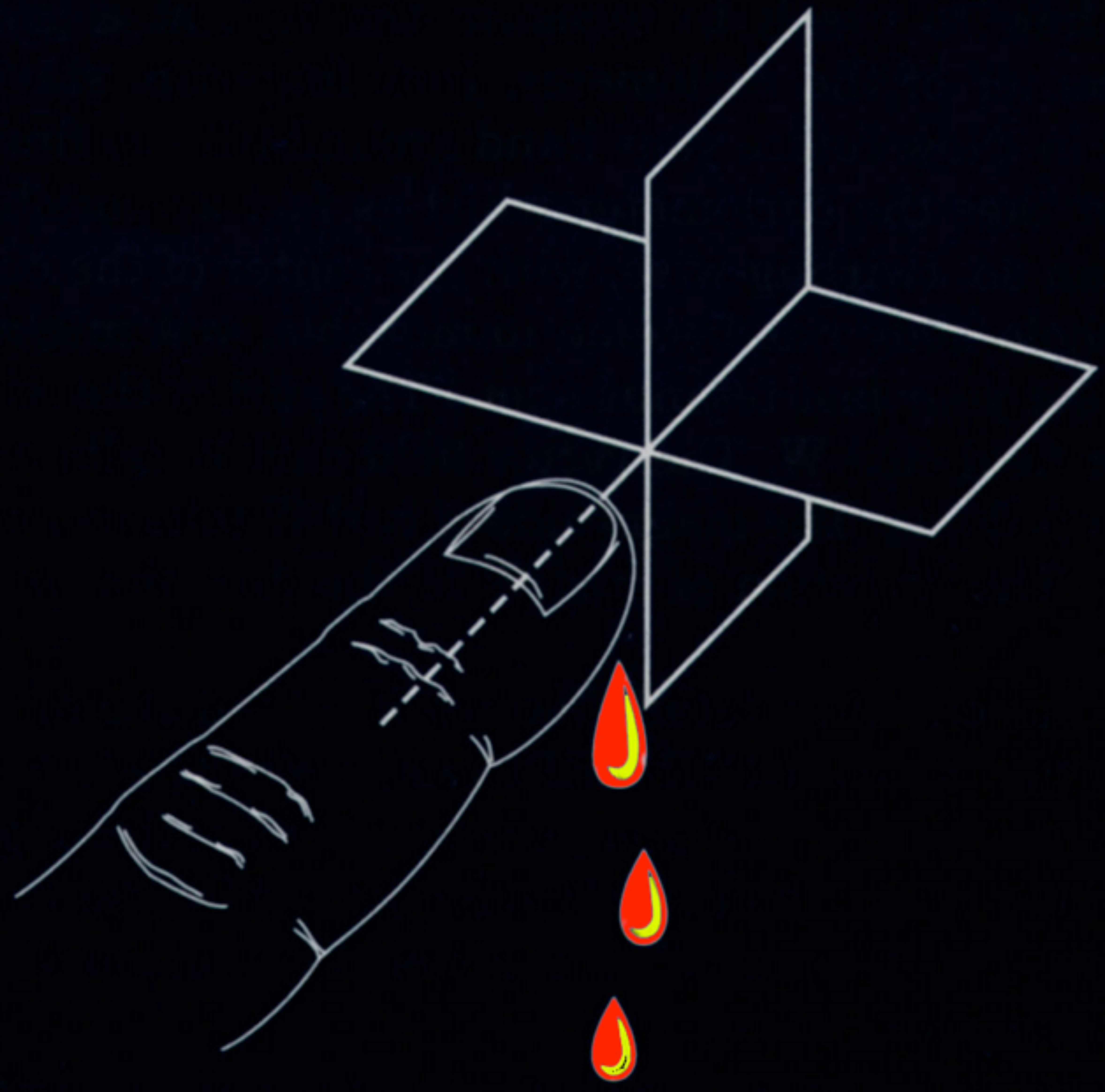


Figure 8.3.4 $\int_C \mathbf{F} \cdot d\mathbf{s} \neq 0$ imp
paddle wheel in a fluid with velo
rotate around its axis.



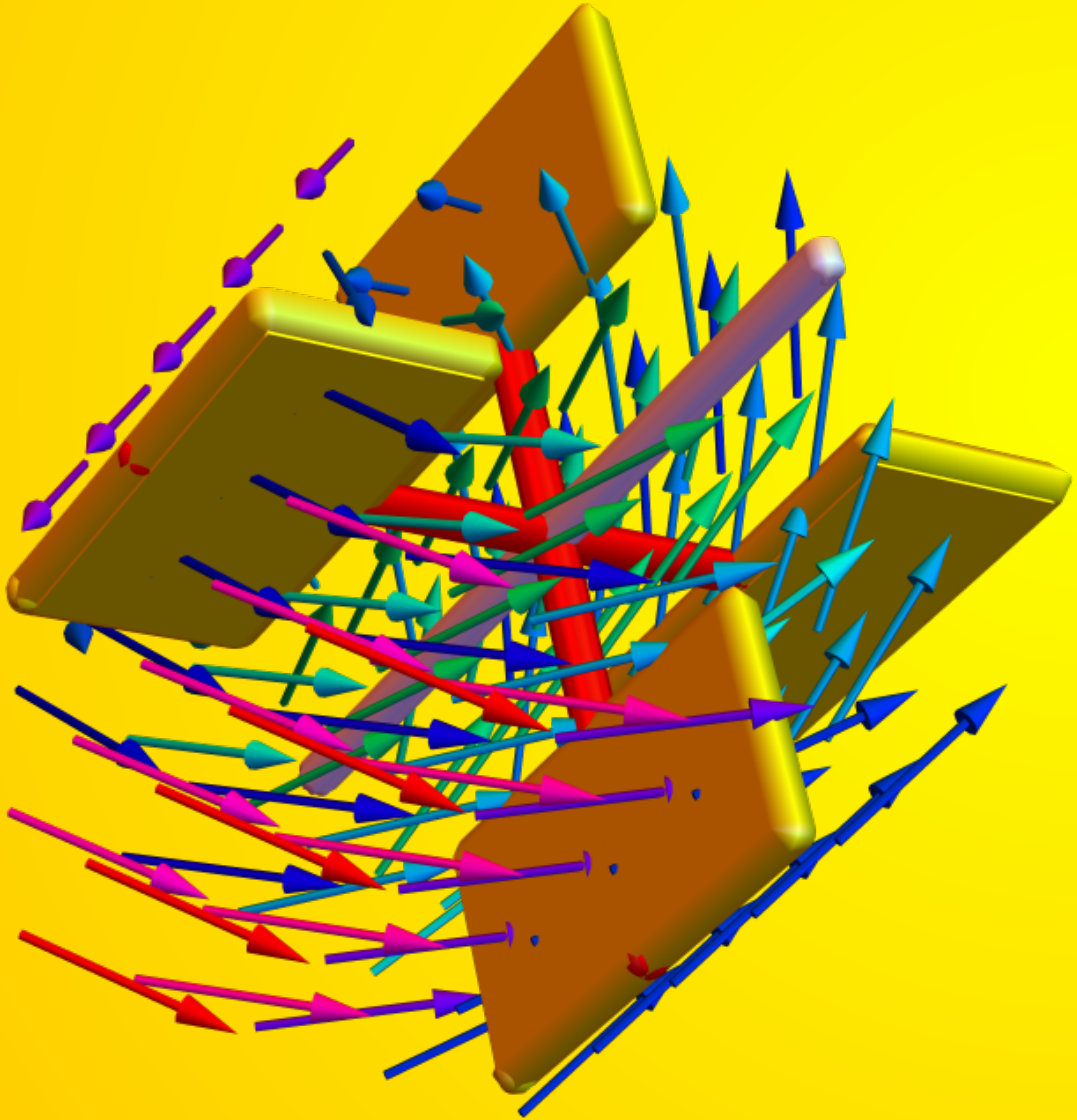
Div in 3D

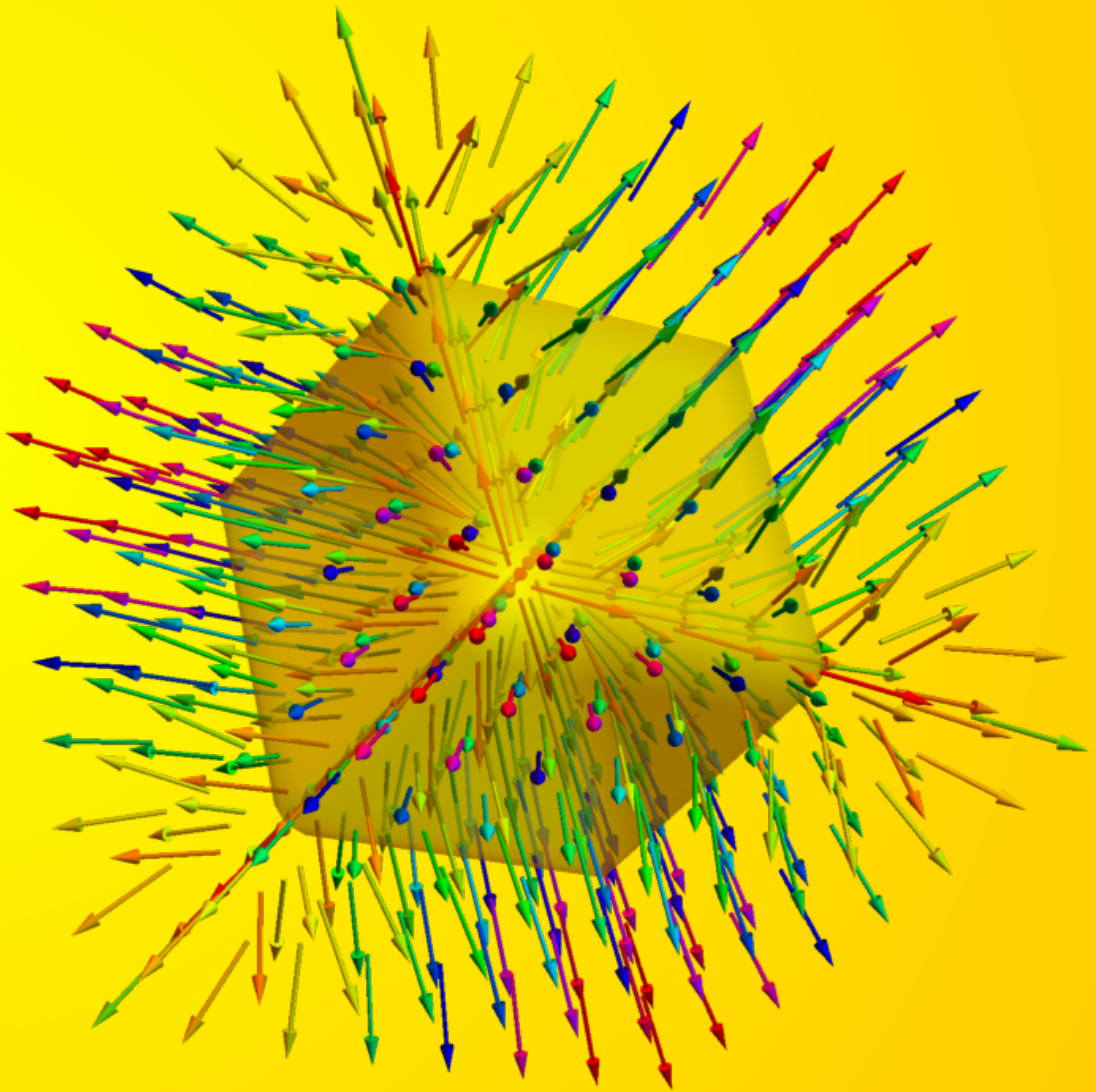
Div

$$\overrightarrow{F} = \langle P, Q, R \rangle$$

$$\operatorname{div}(\overrightarrow{F}) = P_x + Q_y + R_z$$

Pictures





Nabla Calculus

Grad - Curl - Div

$\overrightarrow{F} = \langle P, Q, R \rangle$, f scalar function

$\nabla = \langle \partial_x, \partial_y, \partial_z \rangle$

"Nabla"

$$\text{grad}(f) = \nabla f$$
$$\text{curl}(\overrightarrow{F}) = \nabla \times \overrightarrow{F}$$
$$\text{div}(\overrightarrow{F}) = \nabla \cdot \overrightarrow{F}$$

Identities

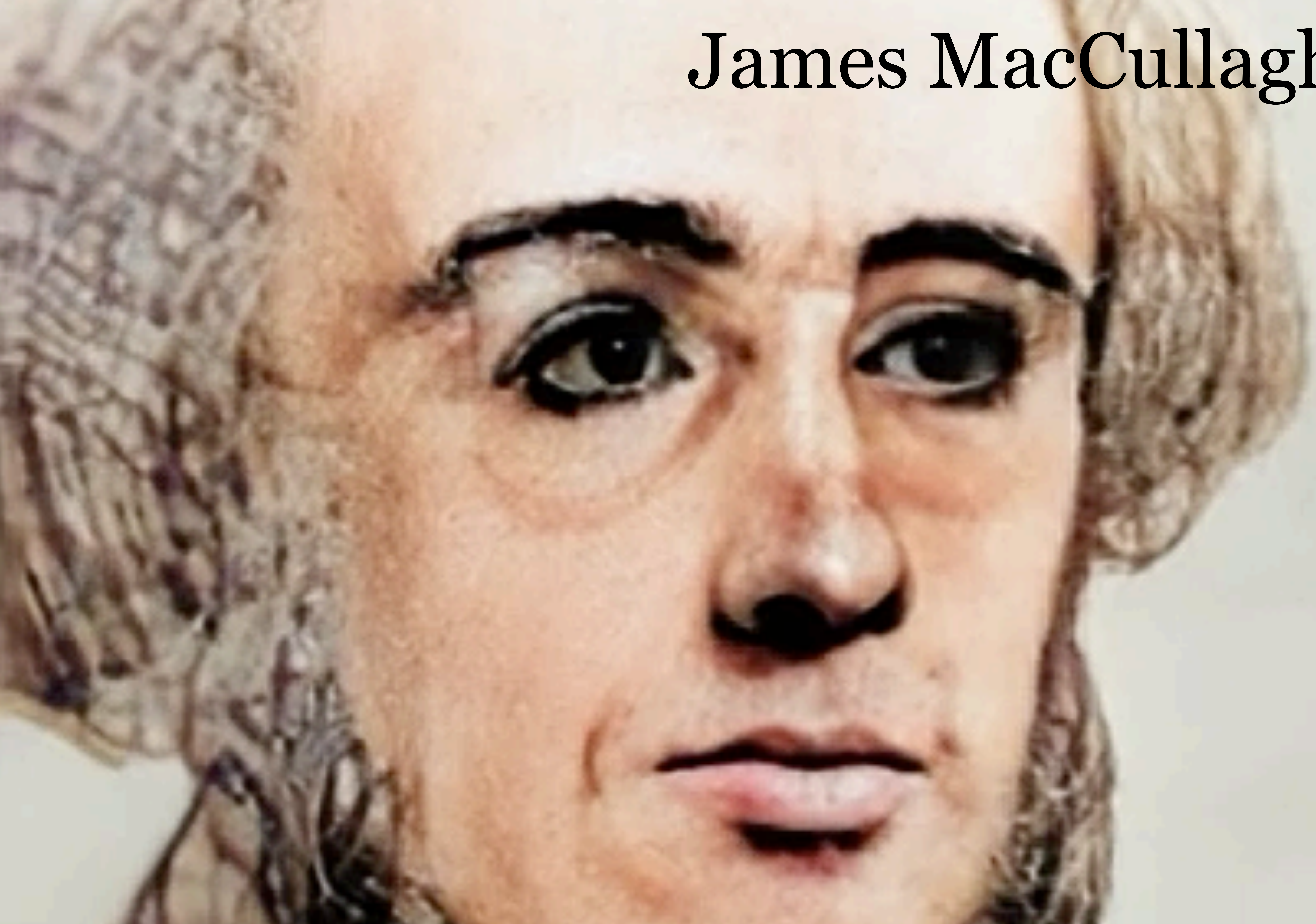
$$\mathrm{curl}(\mathrm{grad}(f)) = \langle 0,0,0 \rangle$$

$$\mathrm{div}(\mathrm{curl}(\mathrm{F})) = 0$$

Origin

James Maxwell, 1871

James MacCullagh, 1839




Test tomorrow!
covering:
$\nabla \cdot D = 4\pi\rho$ $\nabla \times H = \frac{4\pi}{c} J + \frac{1}{c}\frac{\partial D}{\partial t}$
$\nabla \cdot B = 0$ $\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$
7:30am. Show your work!

THE END