Lecture 33

Stokes Theorem

Peak
Solvay Hut
Hörnli Hut

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The Theorem

$$\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$$

Discussion

$$\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$$

What happens if F is a gradient field?

Difference Stokes and Green?

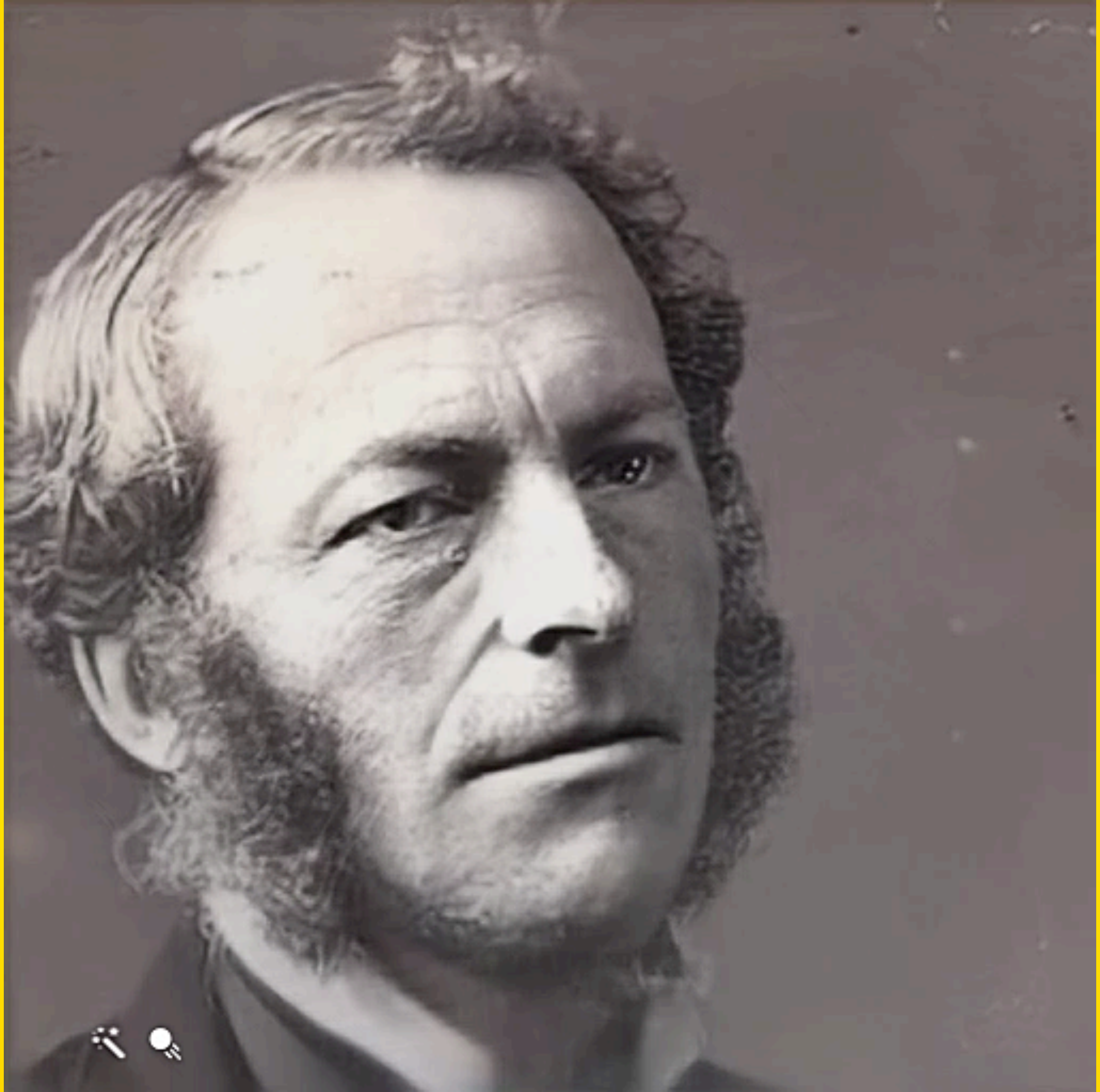
What happens if S is closed?

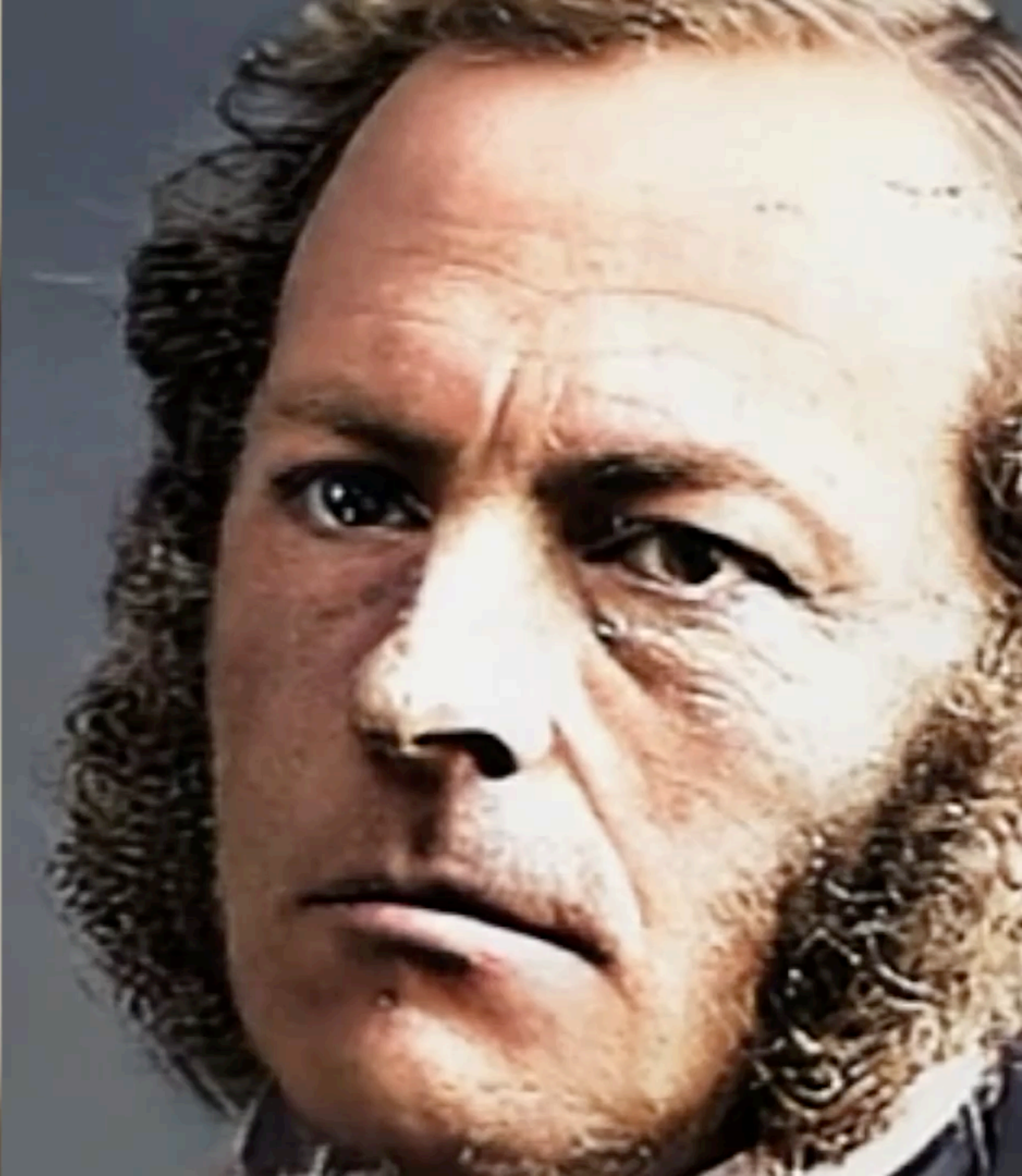

THE MORE COMPLICATED THE MATH, THE DUMBER YOU SOUND EXPLAINING IT.
STOKES' THEOREM? YEAH, THAT'S HOW IF YOU DRAW A LOOP AROUND SOMETHING, YOU CAN TELL HOW MUCH SWIRLY IS IN IT.

Orientation


x

History



[The following Smith's Prize Exam was taken by James Clerk Maxwell at Cambridge. Question 8 is Stokes' Theorem. (Stokes was a personal friend of Maxwell.) Maxwell completed the exam tied for first.]

February, 1854.

BY GEORGE GABRIEL STOKES, ESQ. M. A.

Lucasian Professor.

1. STRAIGHT lines *AP*, *BP* pass through the fixed points *A*, *B*, and are always equally inclined to a fixed line; shew that the locus of P is a hyperbola, and find its asymptotes.

2. A number of equal vessels communicate successively with each other by small pipes, the last vessel opening into the air. The vessels being at first filled with air, a gas is gently forced at a uniform rate into the first; find the quantity of air remaining in the n^{th} vessel at the end of a given time, supposing the gas and air in each vessel at a given instant to be uniformly mixed.

3. Separate the roots of the equation

$$2x^3 - 9x^2 + 12x - 4.4 = 0,$$

begin to move along a generating line of an elliptic cone having Q for vertex in order that consecutive tangents may ultimately intersect, but that the conditions of the problem may be impossible.

8. If X, Y, Z be functions of the rectangular co-ordinates x, y, z, dS an element of any limited surface, l, m, n the cosines of the inclinations of the normal at dS to the axes, ds an element of the bounding line, shew that

$$\iint \left\{ l\left(\frac{dZ}{dy} - \frac{dY}{dx}\right) + m\left(\frac{dX}{dz} - \frac{dZ}{dx}\right) + n\left(\frac{dY}{dx} - \frac{dX}{dy}\right) \right\} dS$$
$$= \int \left(X\frac{dx}{ds} + Y\frac{dy}{ds} + Z\frac{dz}{ds} \right) ds,$$

the differential coefficients of X, Y, Z being partial, and the single integral being taken all round the perimeter of the surface.

9. Explain the geometrical relation between the curves, referred to the rectangular co-ordinates x, y, z, whose differential equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R},$$

and the family of surfaces represented by the partial differential equation

In the movies


Math 21a
Harvard
Fall 2019
Stokes theorem
a Stairway
to Heaven

THE END