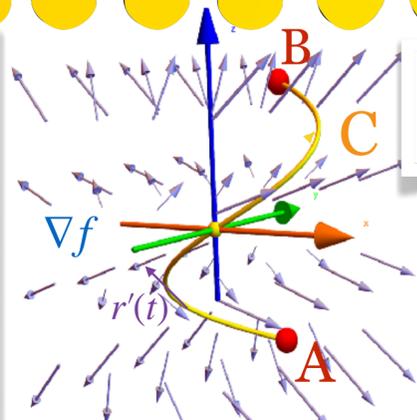


Lecture 35

Integral Theorems

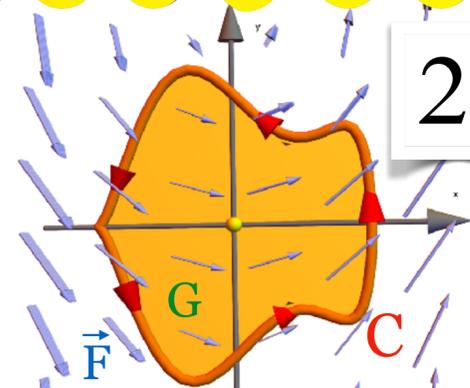
Line Integral Theorem



21

$$\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)$$

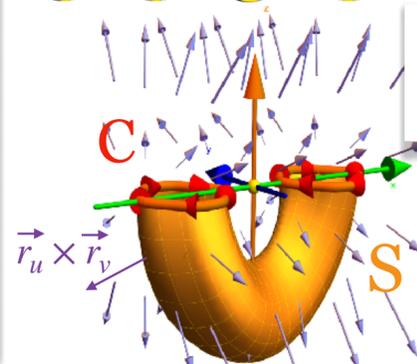
Green's Theorem



21

$$\iint_G \text{curl}(\vec{F}) \, dA = \int_C \vec{F} \cdot d\vec{r}$$

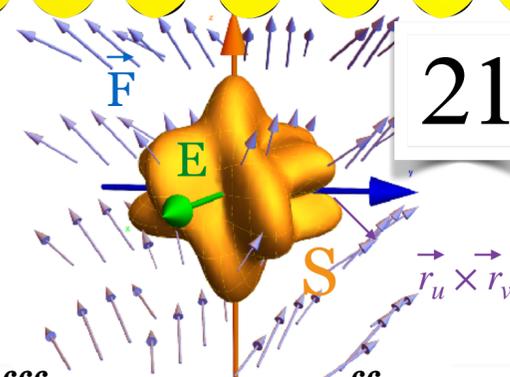
Stokes Theorem



21

$$\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$$

Divergence Theorem



21

$$\iiint_E \text{div}(\vec{F}) \, dV = \iint_S \vec{F} \cdot d\vec{S}$$

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1) The Theorems

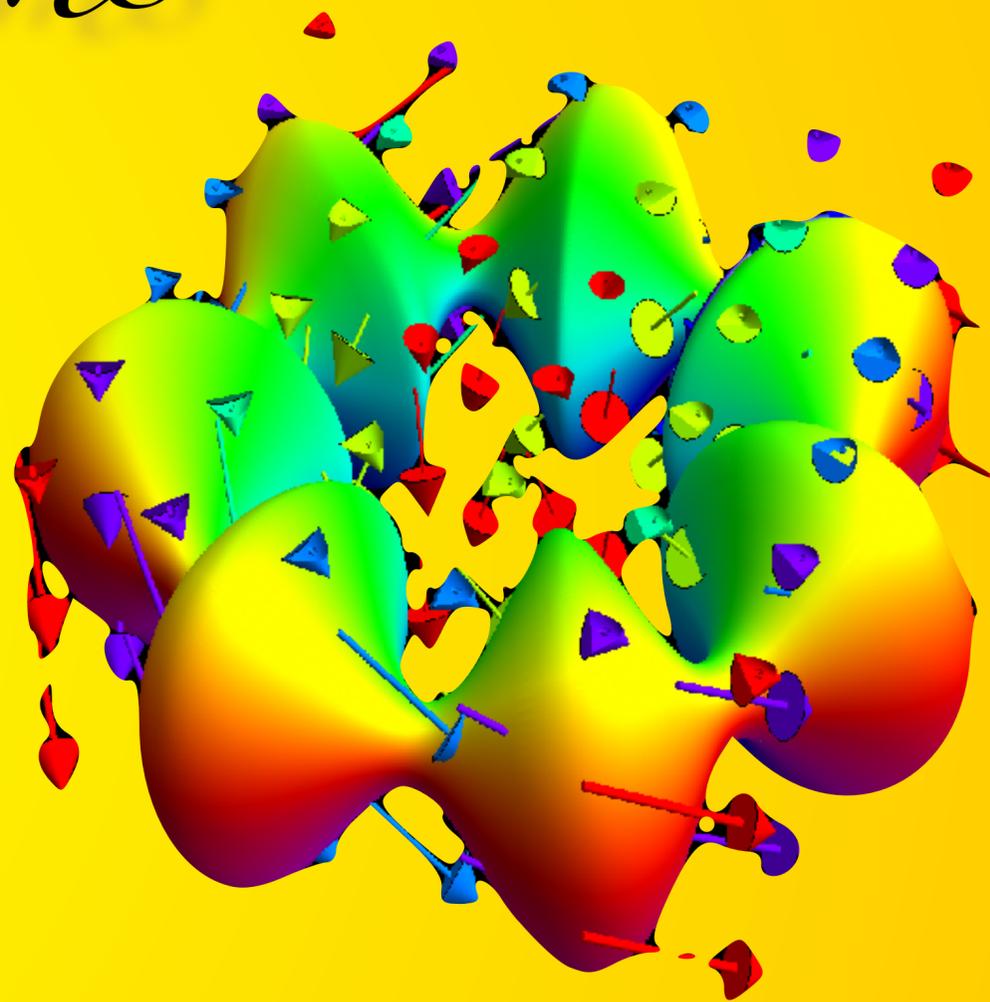
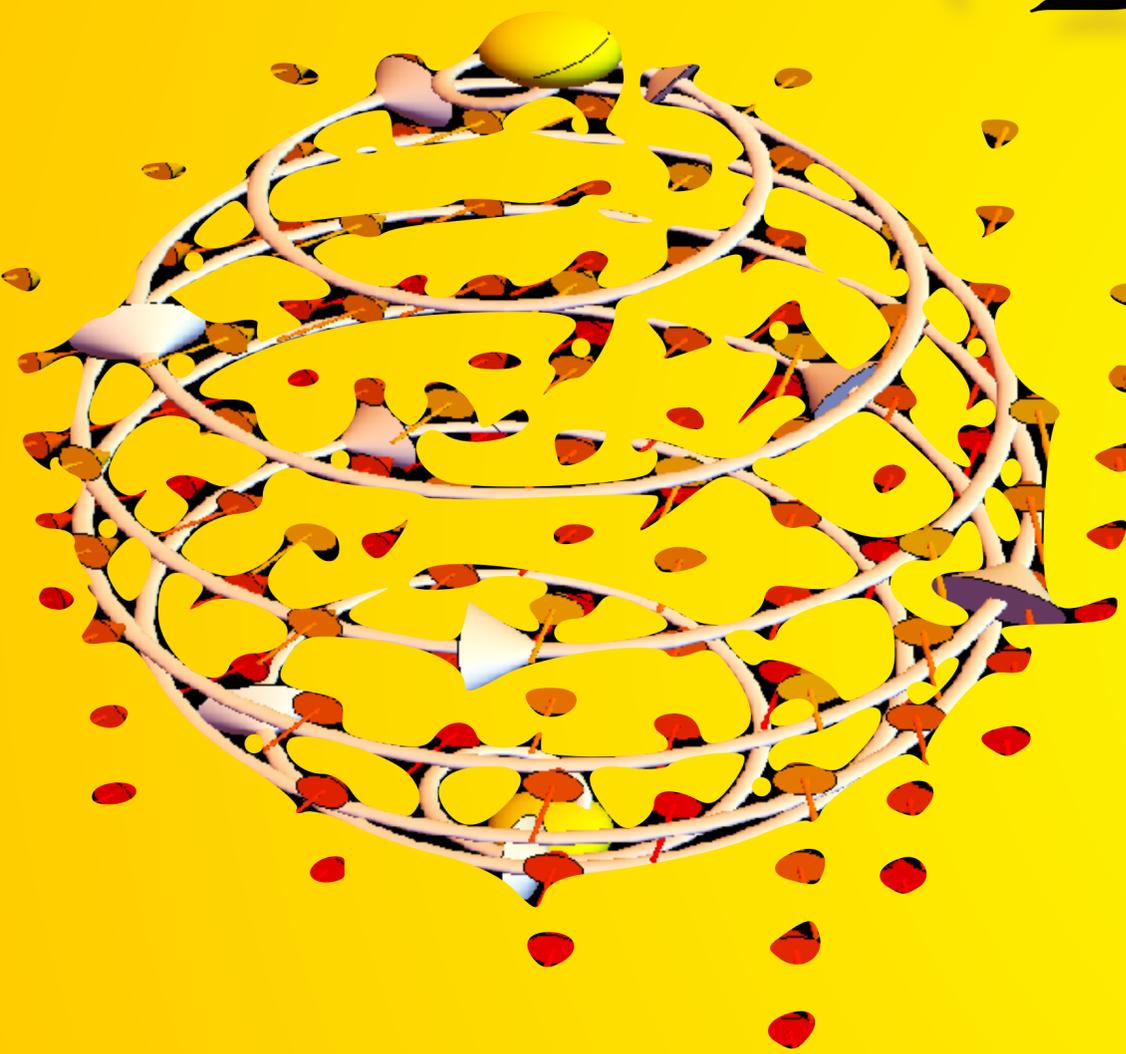
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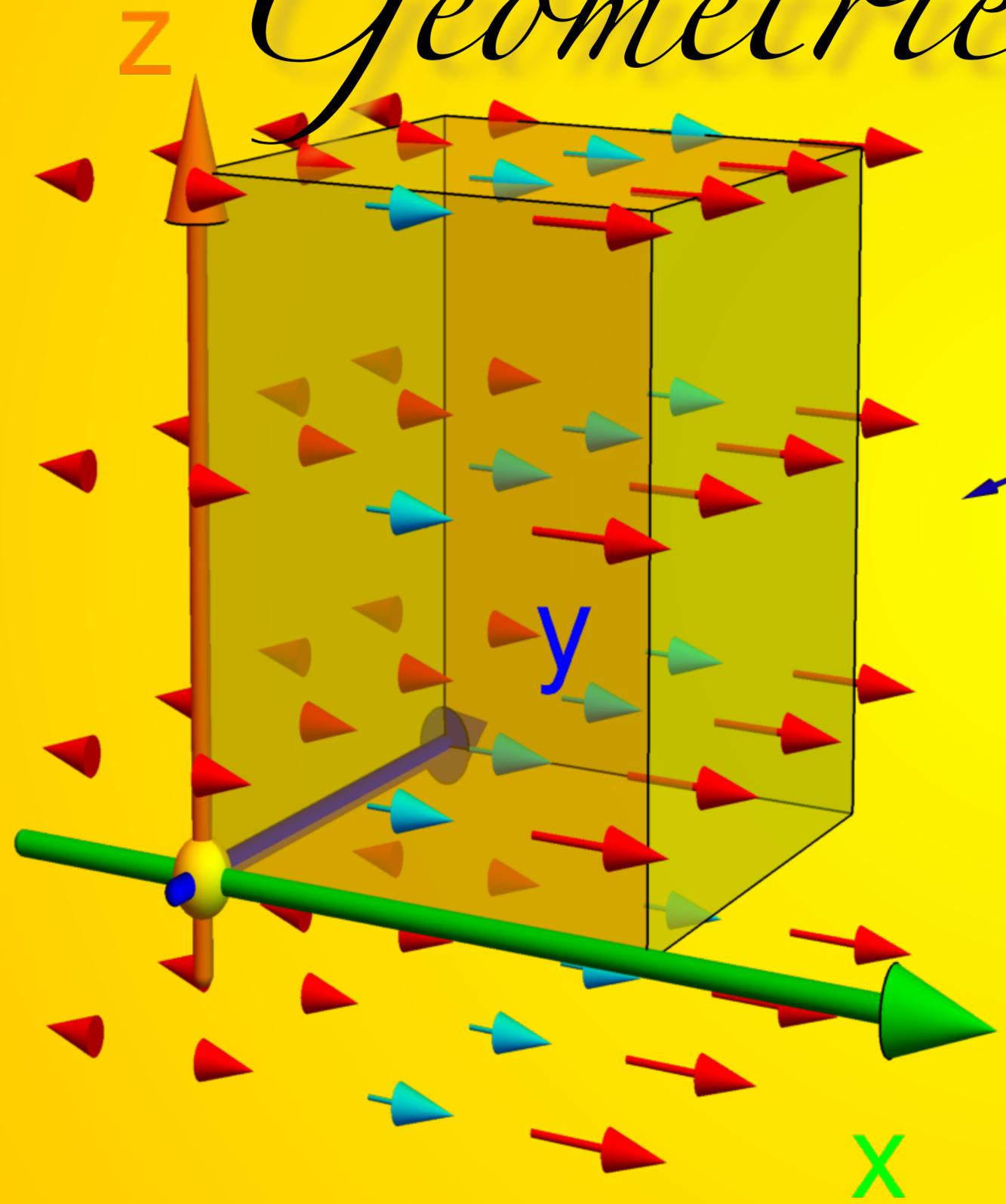
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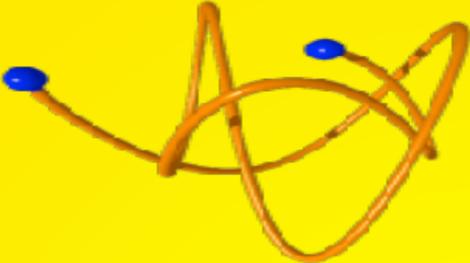
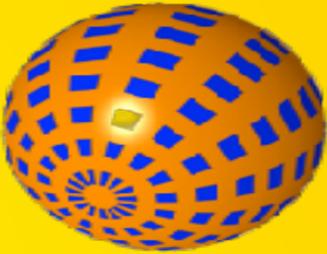
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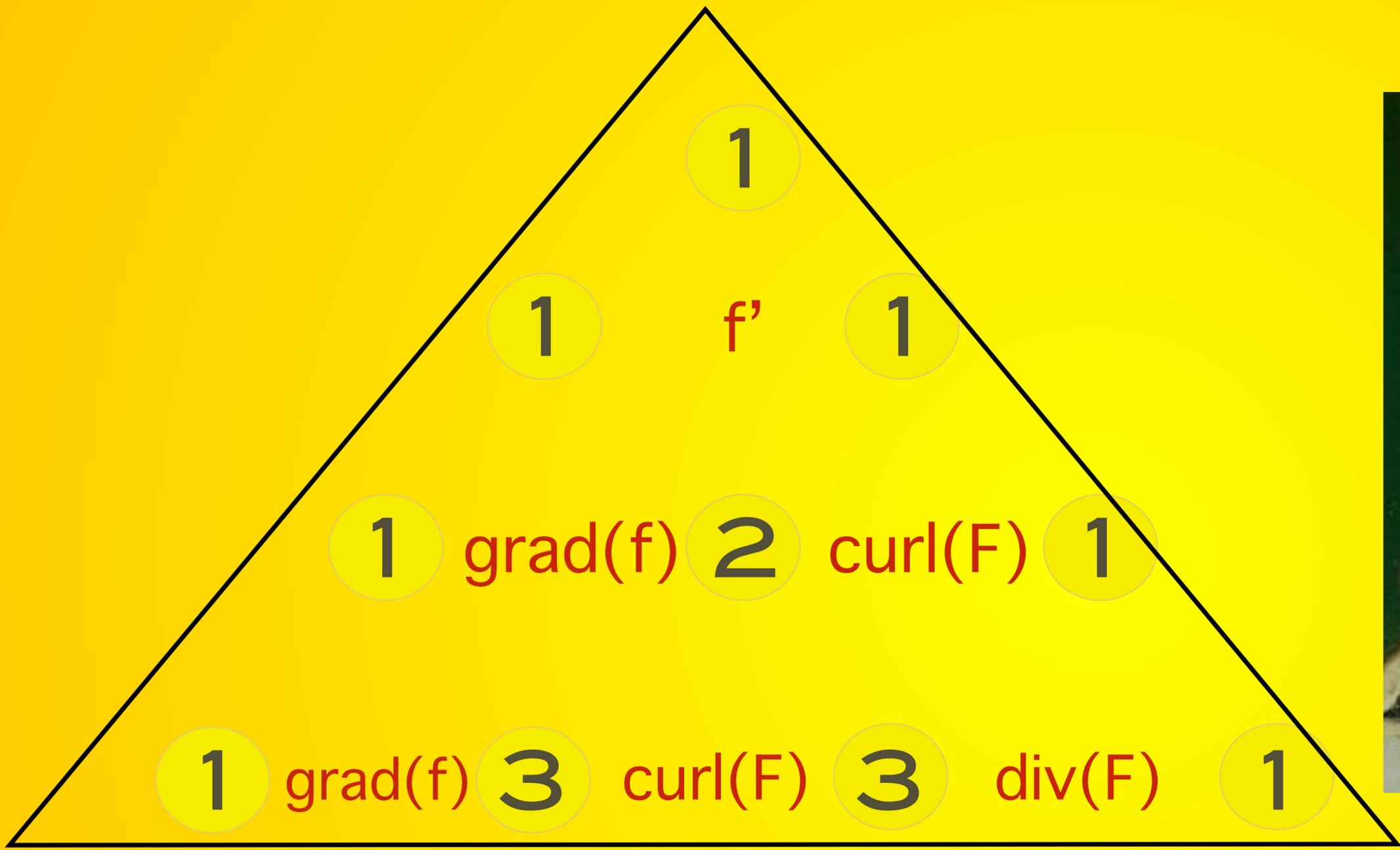
The Theorems



Geometries and Fields

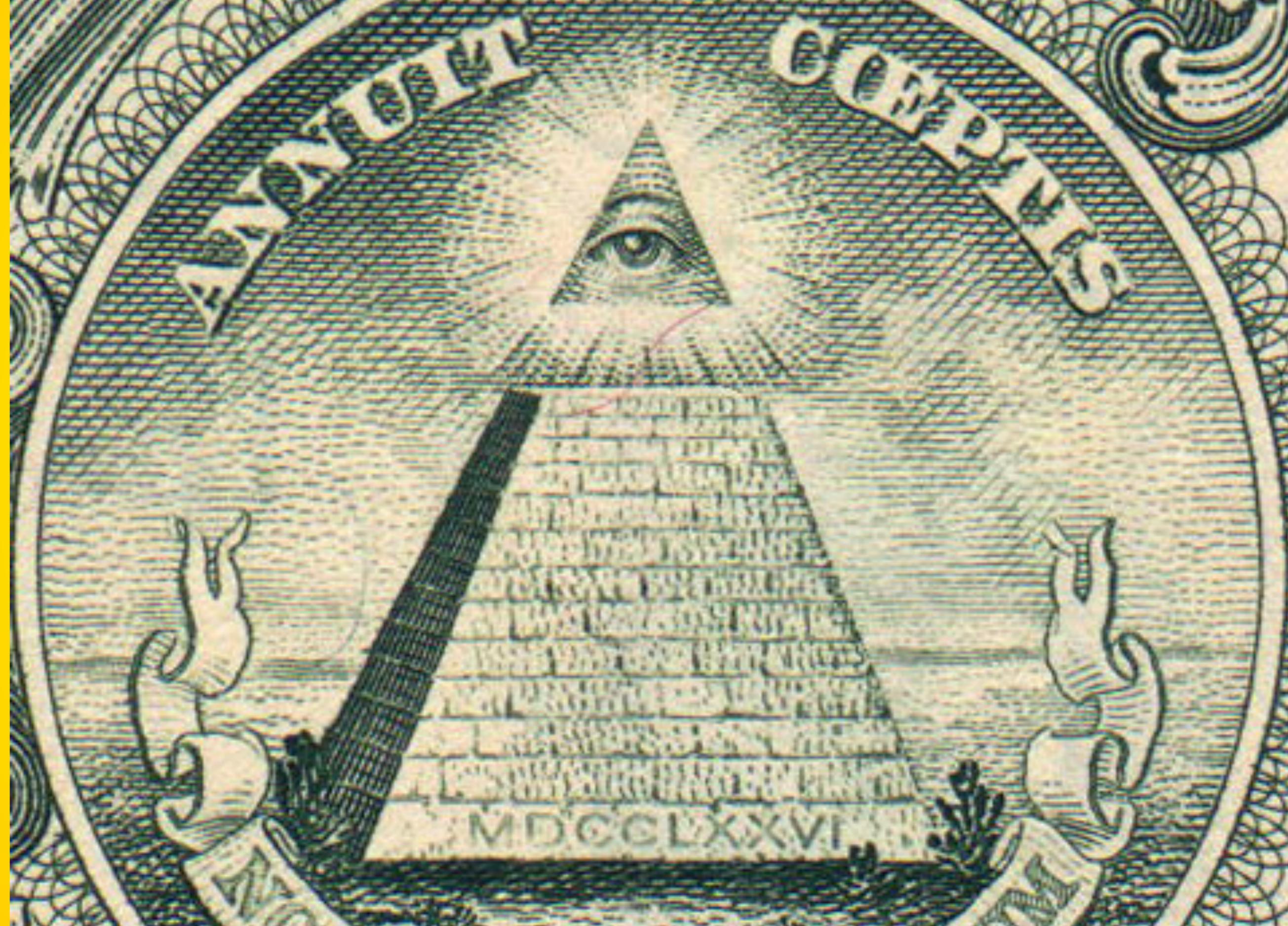


			
DIM=1	FTC		
DIM=2	FTLI	GREEN	
DIM=3	FTLI	STOKES	GAUSS

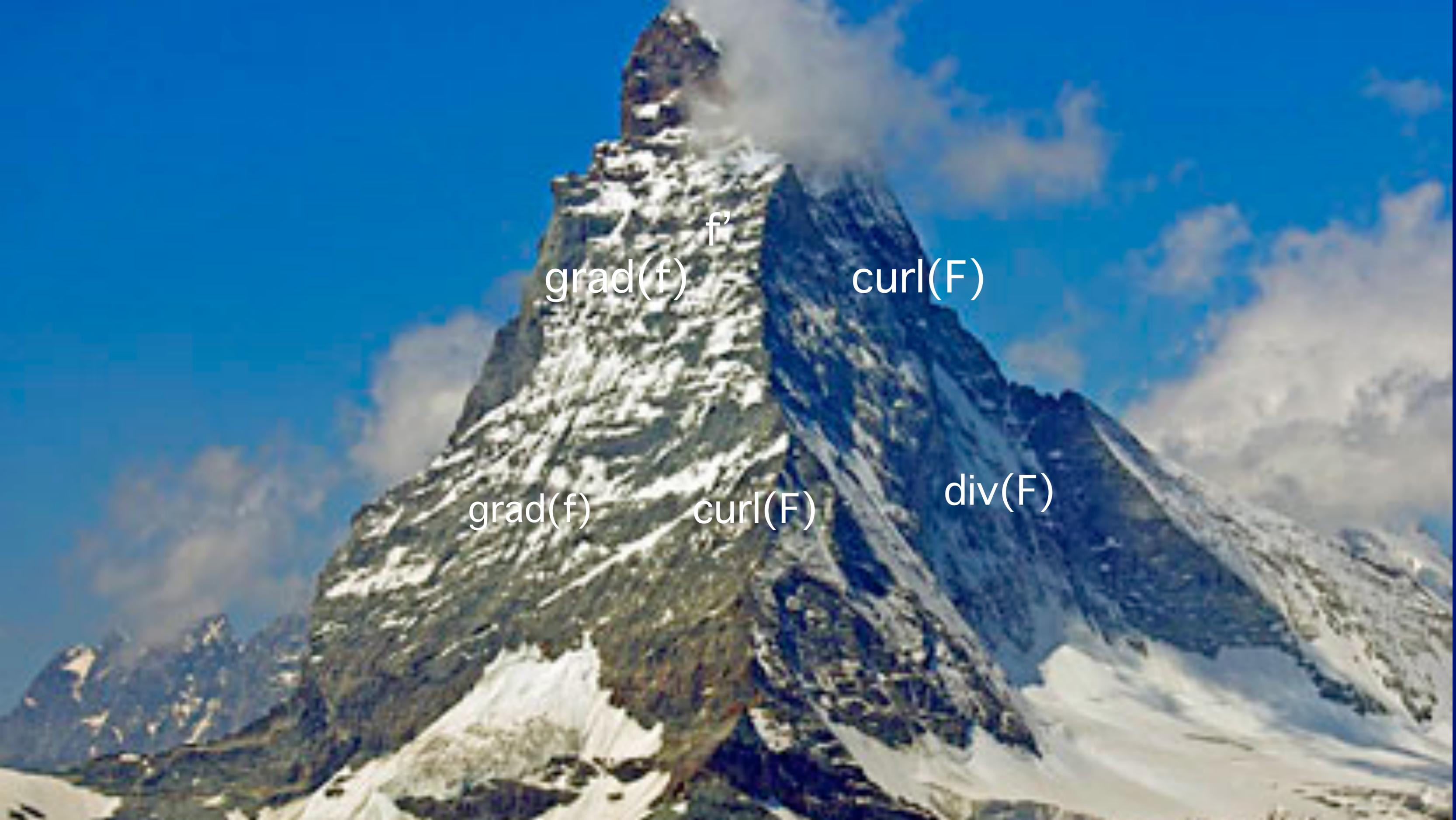


1 4 6 4 1

On every Dollar bill



And mountain



grad(f)

f'

curl(F)

grad(f)

curl(F)

div(F)



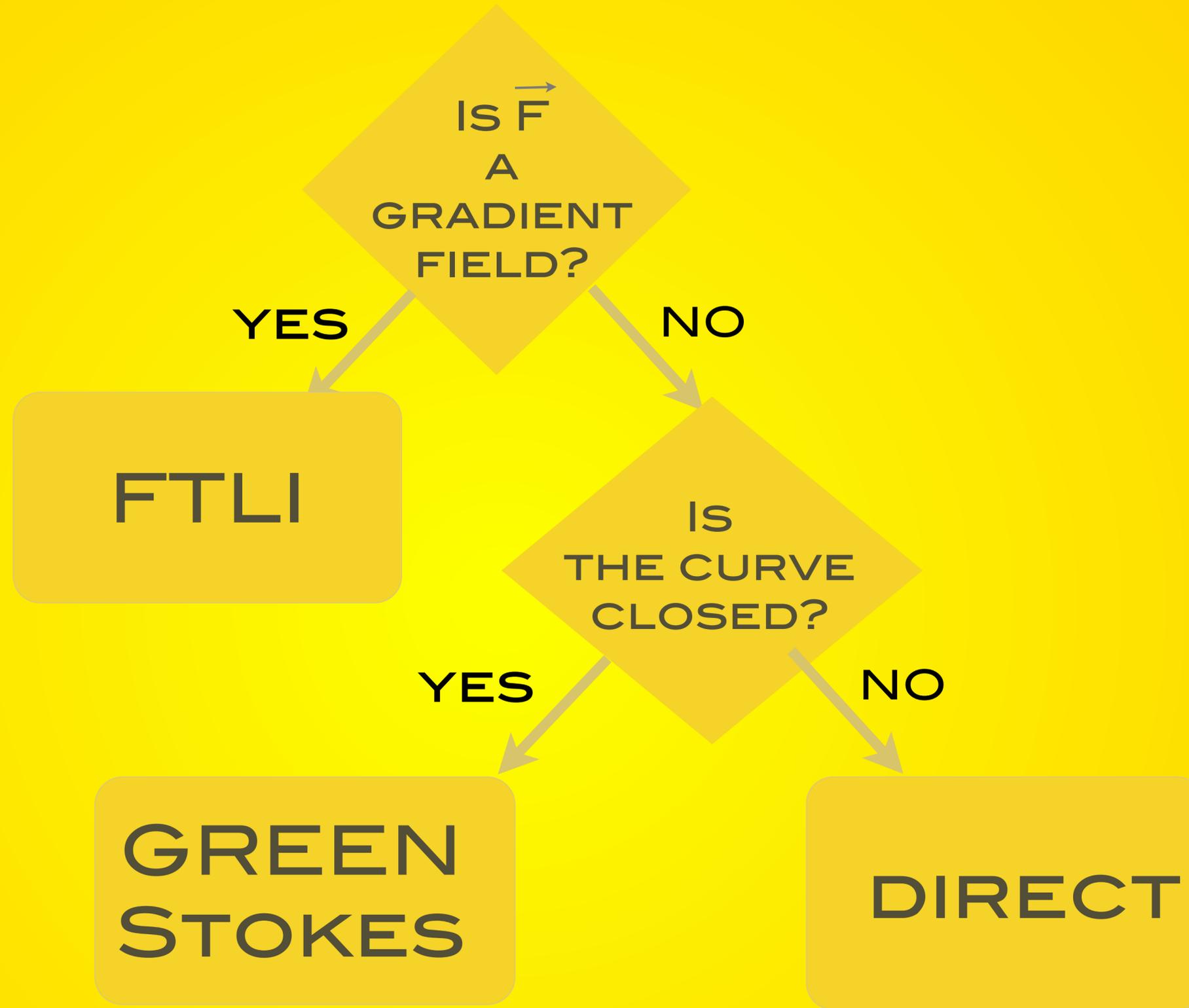
SWISS MILK CHOCOLATE WITH HONEY AND ALMOND NOUGAT

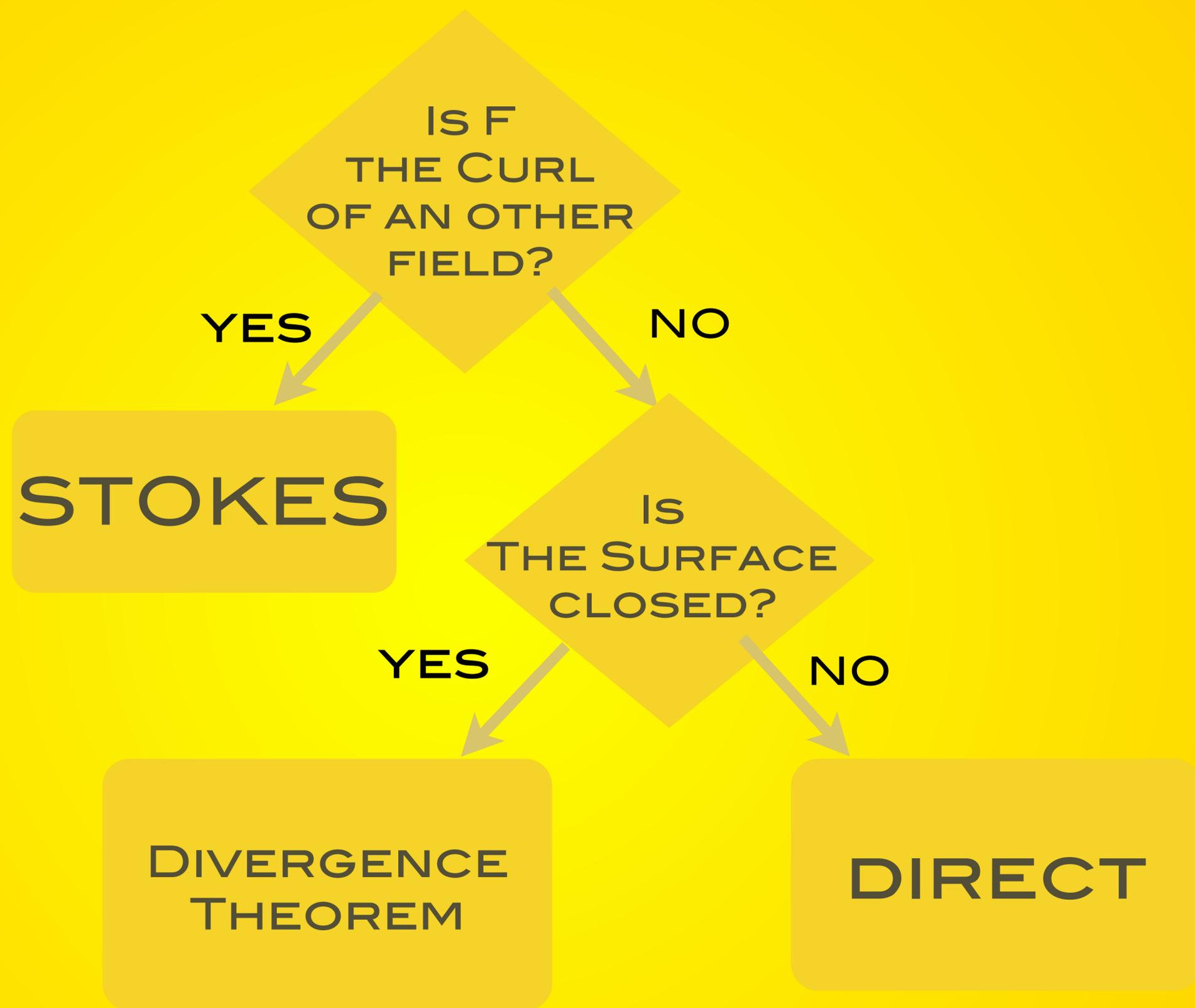


TOBLERONE[®]

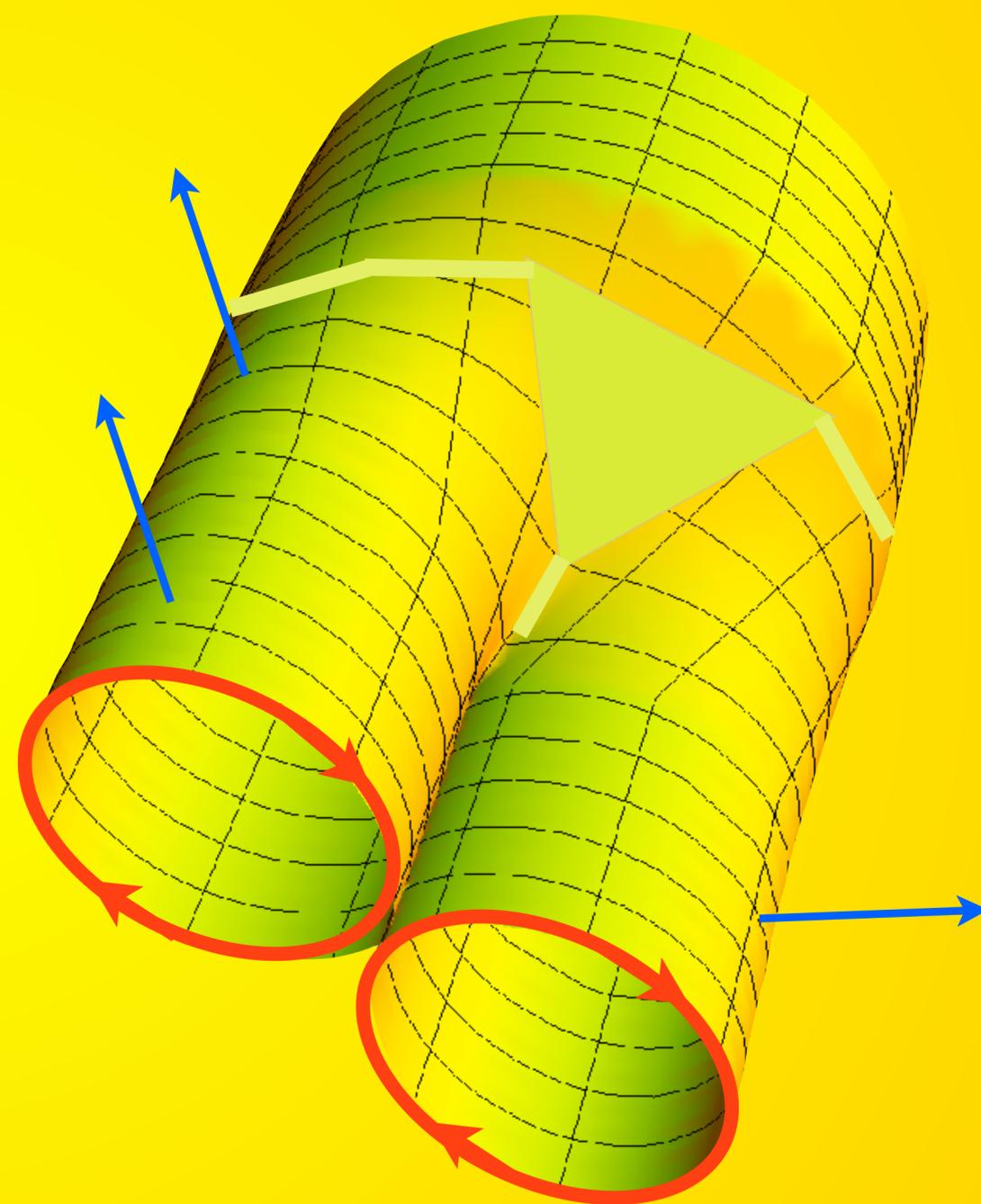
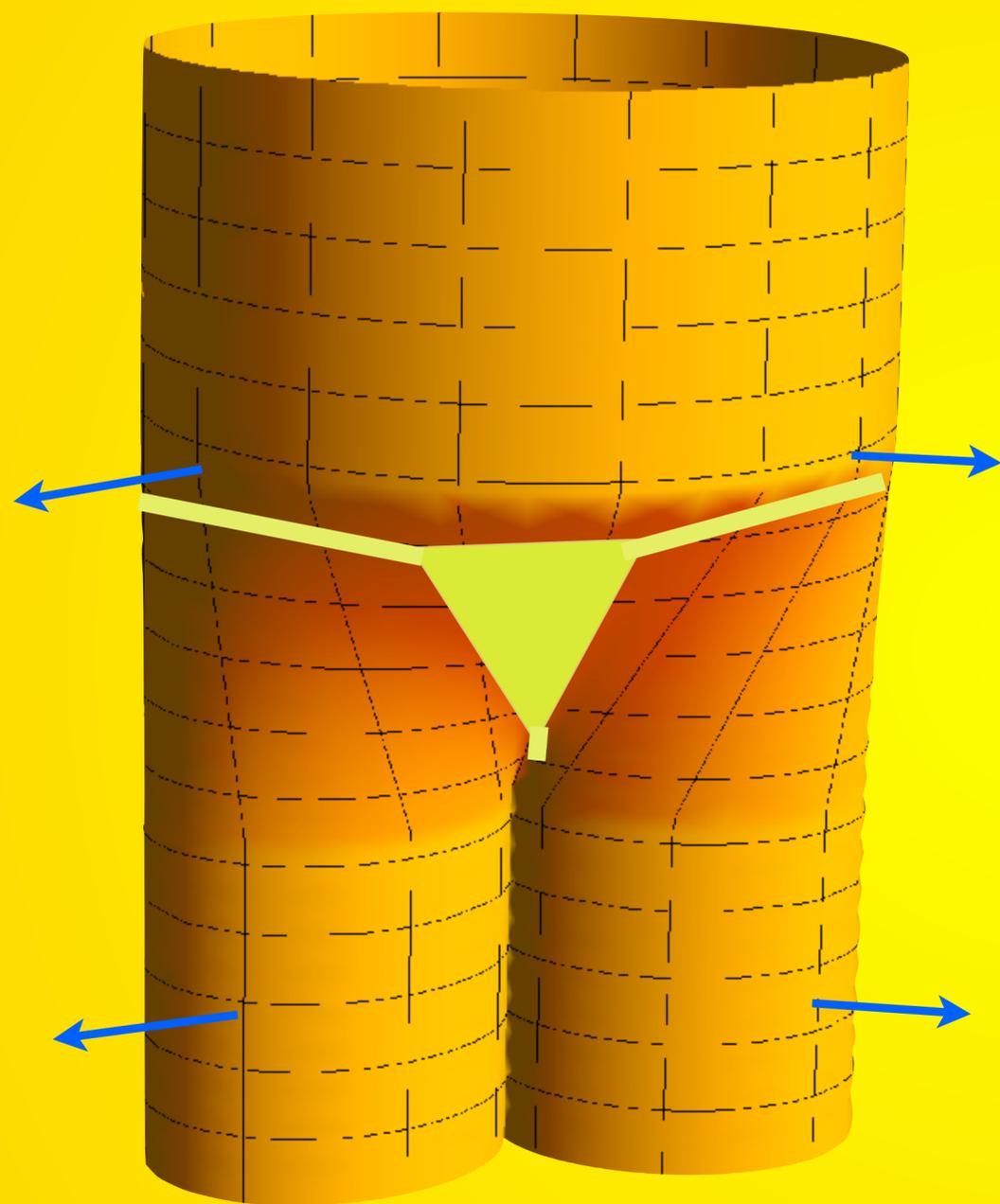
OF SWITZERLAND

Which Theorem?





Orientation



Objects in Last Part

Objects

☐ CURVES

☐ SURFACES

☐ SOLIDS

Fields

☐ SCALAR FIELD

☐ PLANE FIELD

☐ SPACE FIELDS

Scalar Integrals

☐ SCALAR LINE

☐ SCALAR SURFACE

☐ VOLUME INTEGRALS

Vector Integrals

☐ LINE INTEGRALS

☐ FLUX INTEGRALS

☐ TRIPLE INTEGRALS

Derivatives

☐ GRAD

☐ CURL

☐ DIV

Theorems

☐ FTLI

☐ GREEN, STOKES

☐ DIVERGENCE

Top Mistakes in 21a

Math TV

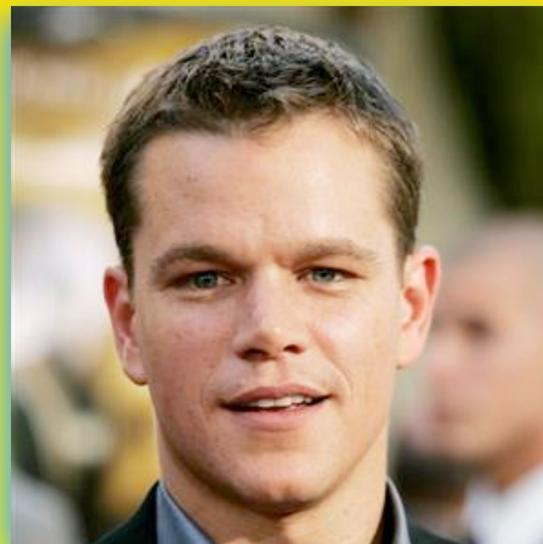
Professor Ed Burge

- 1) NONLINEAR LINEARIZATION
- 2) DIVISION BY ZERO IN LAGRANGE
- 3) NOT TRYING AN INTEGRAL THEOREM
- 4) LACK OF FIGURE IN INTEGRATION
- 5) MIXUP OF DIMENSION FOR GRAPHS
- 6) FORGETTING THE -1 IN CROSS PRODUCT
- 7) USE $Pdx+Qdy$ FOR LINE INTEGRALS
- 8) FORGETTING INTEGRATION FACTOR
- 9) WRONG ORIENTATION
- 10) FINAL RESULT IS NOT WRITTEN

History



1777-1855 CARL FRIEDRICH GAUSS



1793-1841 GEORGE GREEN



1819-1903 GEORGE STOKES



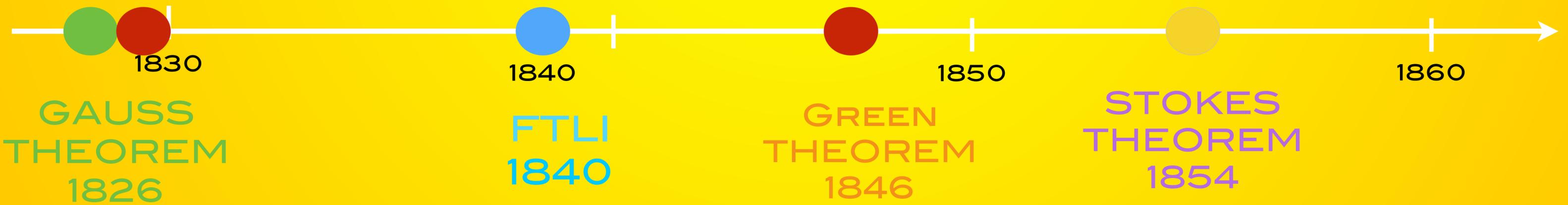
1775-1846 JEAN MARY AMPERE



1801-1862 MIKHAEL OSTROGRADSKY



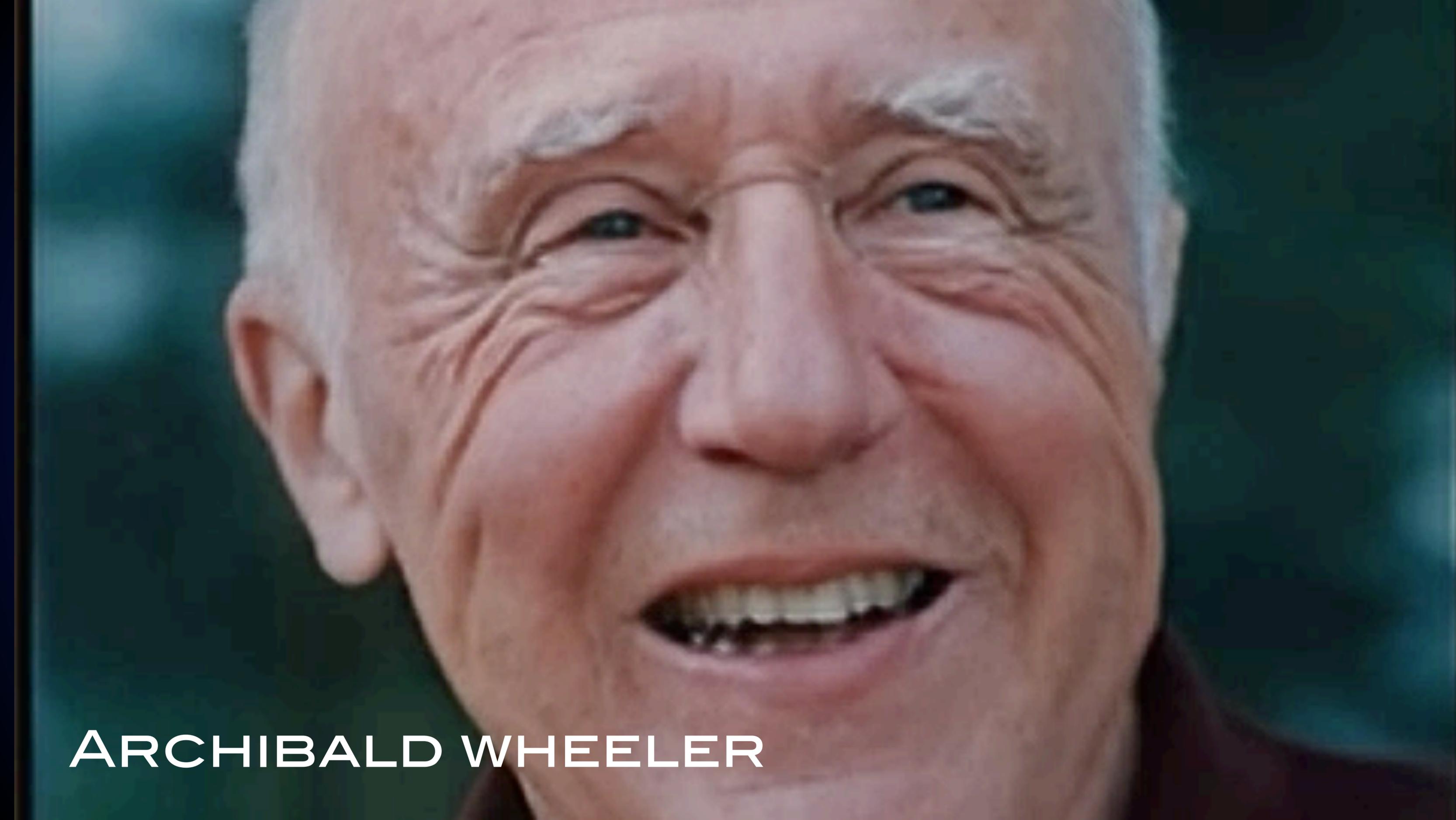
1789-1857 AUGUSTINE CAUCHY





ELIE CARTAN

Used in Relativity



ARCHIBALD WHEELER

Academic family



Erhard Weigel
1625-1699 [W]
[G], Popularizer of science



Gottfried Leibniz
1646-1716 [W]
[G], Invention of Calculus



Jacob Bernoulli
1654-1705 [W] [G],
Seperable differential equations



Johann Bernoulli
1667-1748 [W]
[G], L'Hopital rule



Leonhard Euler
1707-1783 [W]
[G], Euler formula



Joseph Lagrange
1736-1813 [W]
[G], Lagrange multipliers



Jean-Baptiste Fourier
1768-1830 [W] [G],
Fourier series



Johann Dirichlet
1805-1859 [W]
[G], Dirichlet principle



Rudolf Lipschitz
1832-1903 [W]
[G], Lipshitz condition



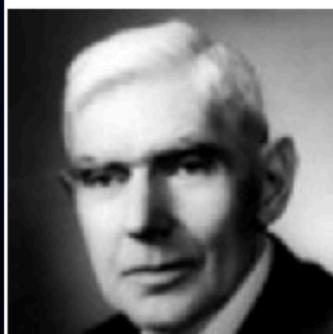
Felix Klein
1849-1925 [W]
[G], Erlanger program



Ferdinand Lindemann
1852-1939 [W] [G],
Pi Transcendence



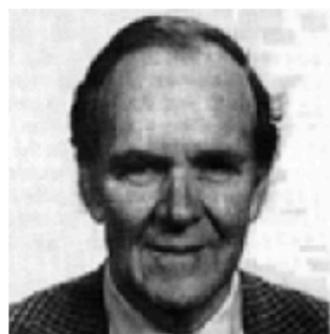
Arnold Sommerfeld
1868-1951 [W]
[G], Fine-structure constant



Karl Herzfeld
1892-1978 [W]
[G], Physical chemistry



John Wheeler
1911-2008 [W]
[G], Black hole



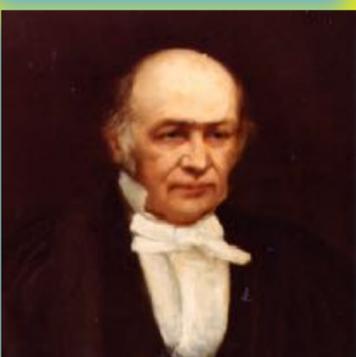
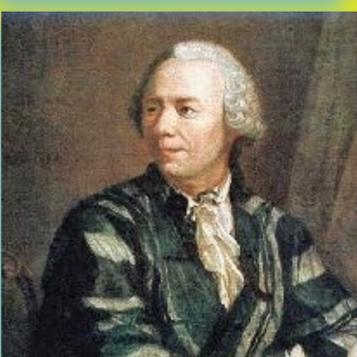
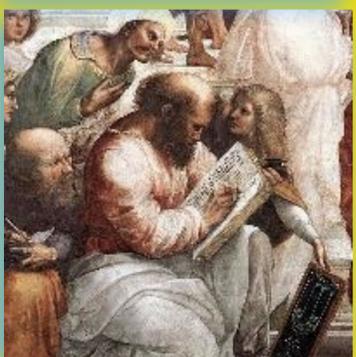
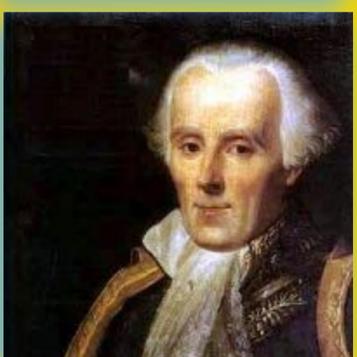
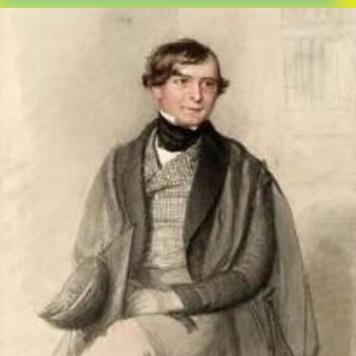
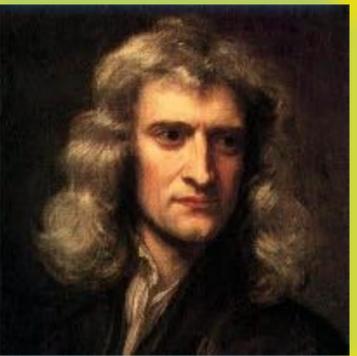
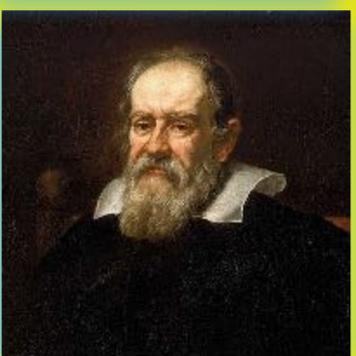
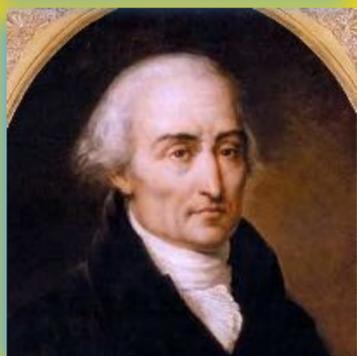
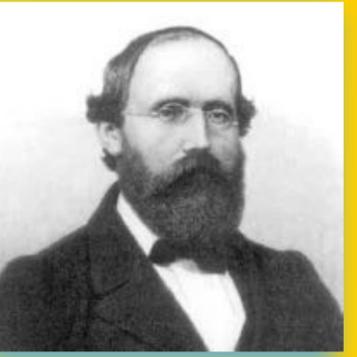
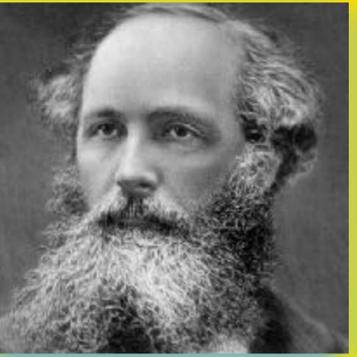
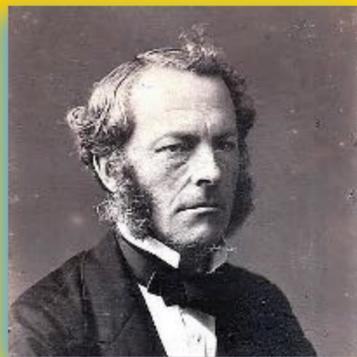
Arthur Wightman
1922- [W] [G],
Quantum field theory



Oscar Lanford
1940- [W] [G],
Feigenbaum fixed point



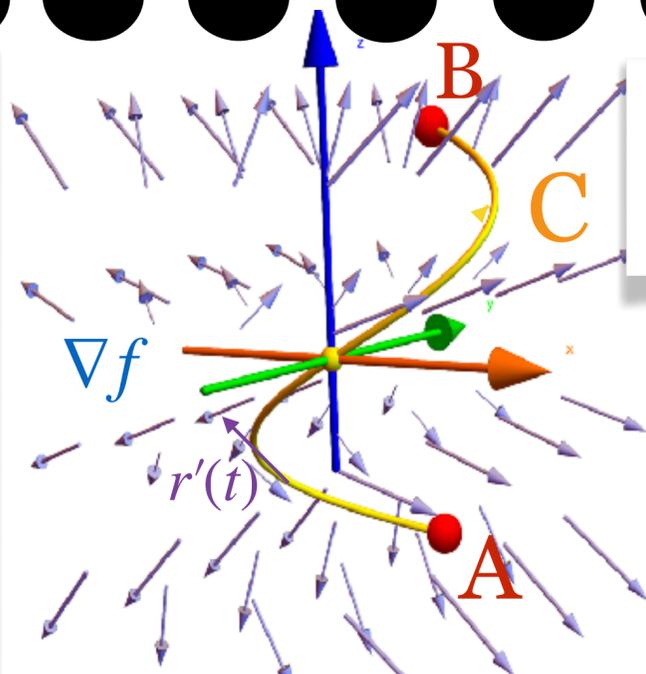
Oliver



Memorabilia

A) Stamps

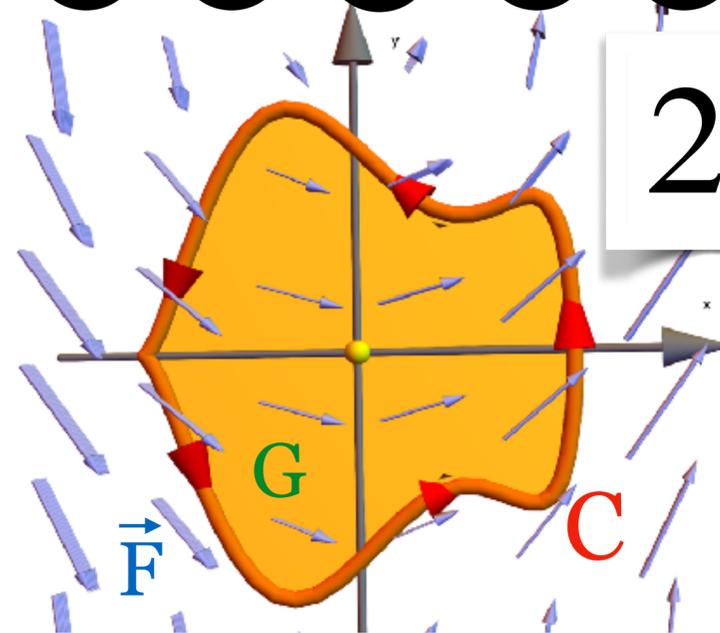
Line Integral Theorem



21

$$\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)$$

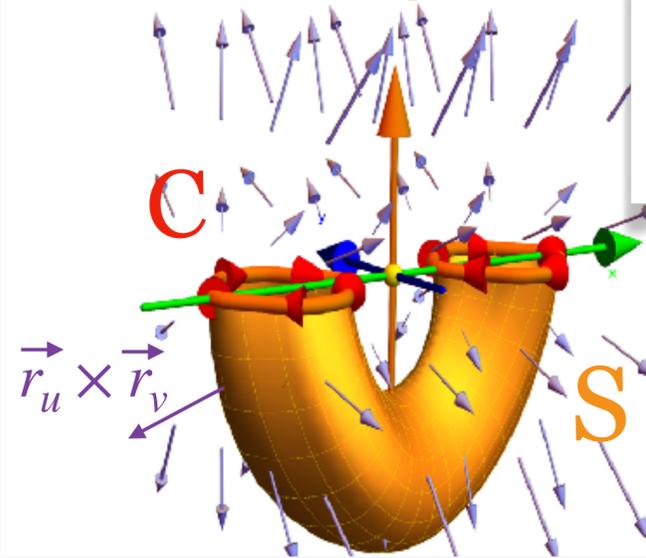
Green's Theorem



21

$$\iint_G \text{curl}(\vec{F}) \, dA = \int_C \vec{F} \cdot d\vec{r}$$

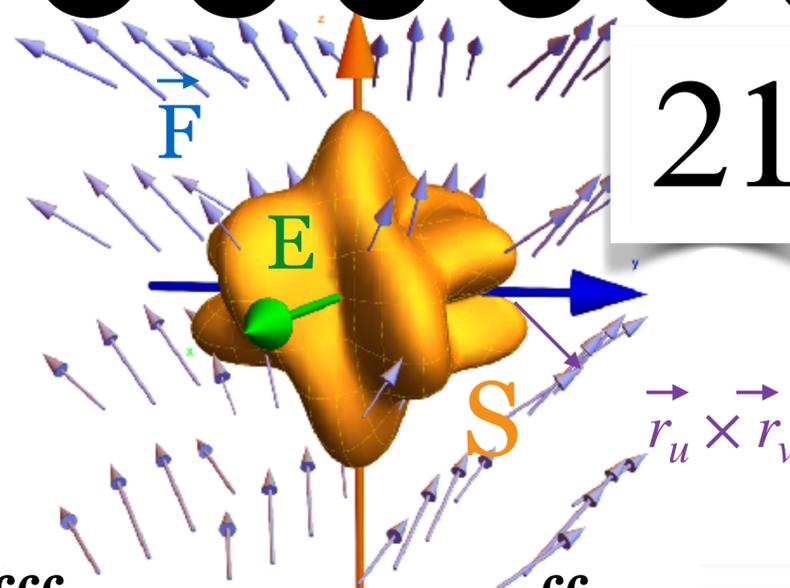
Stokes Theorem



21

$$\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$$

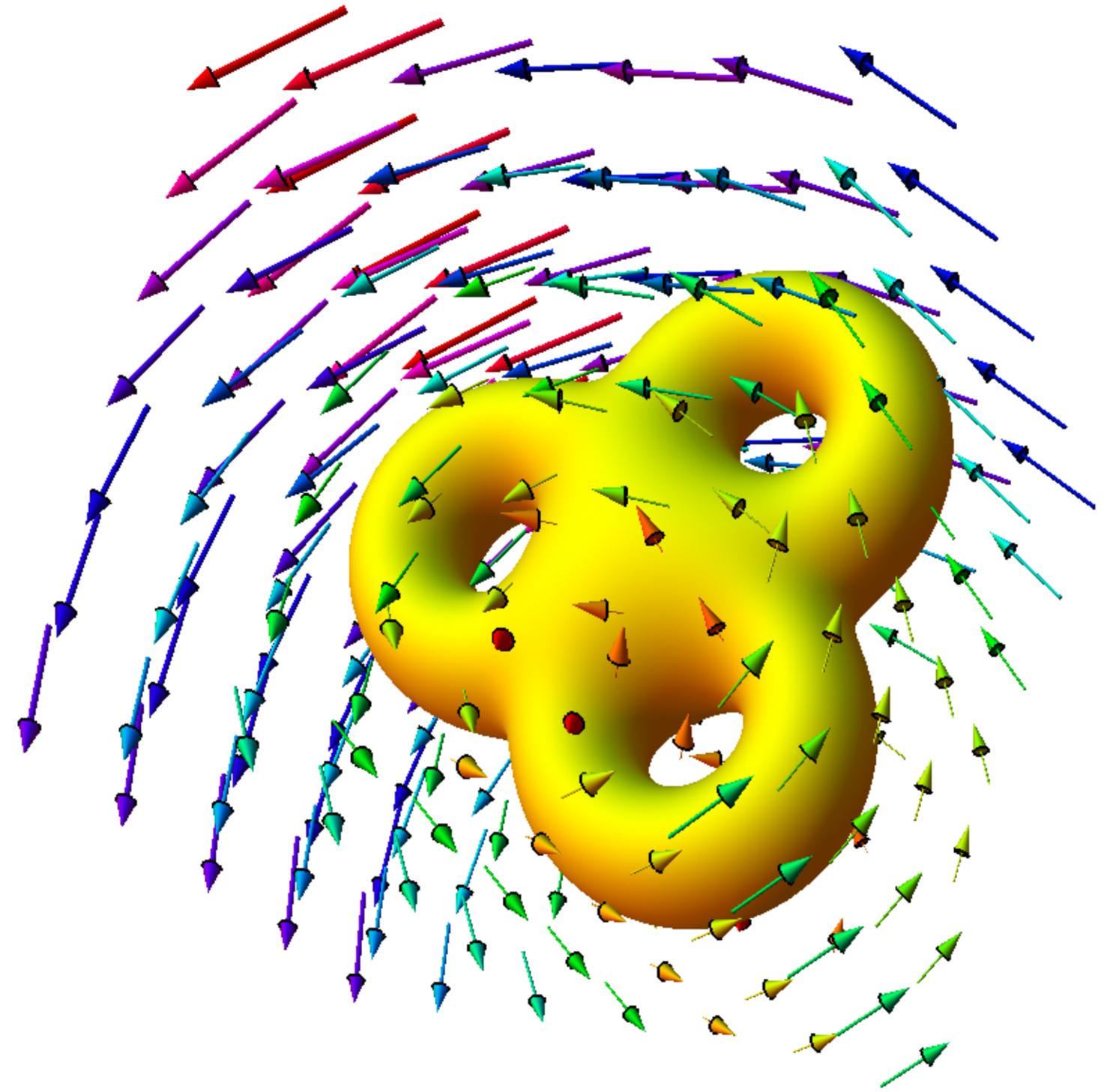
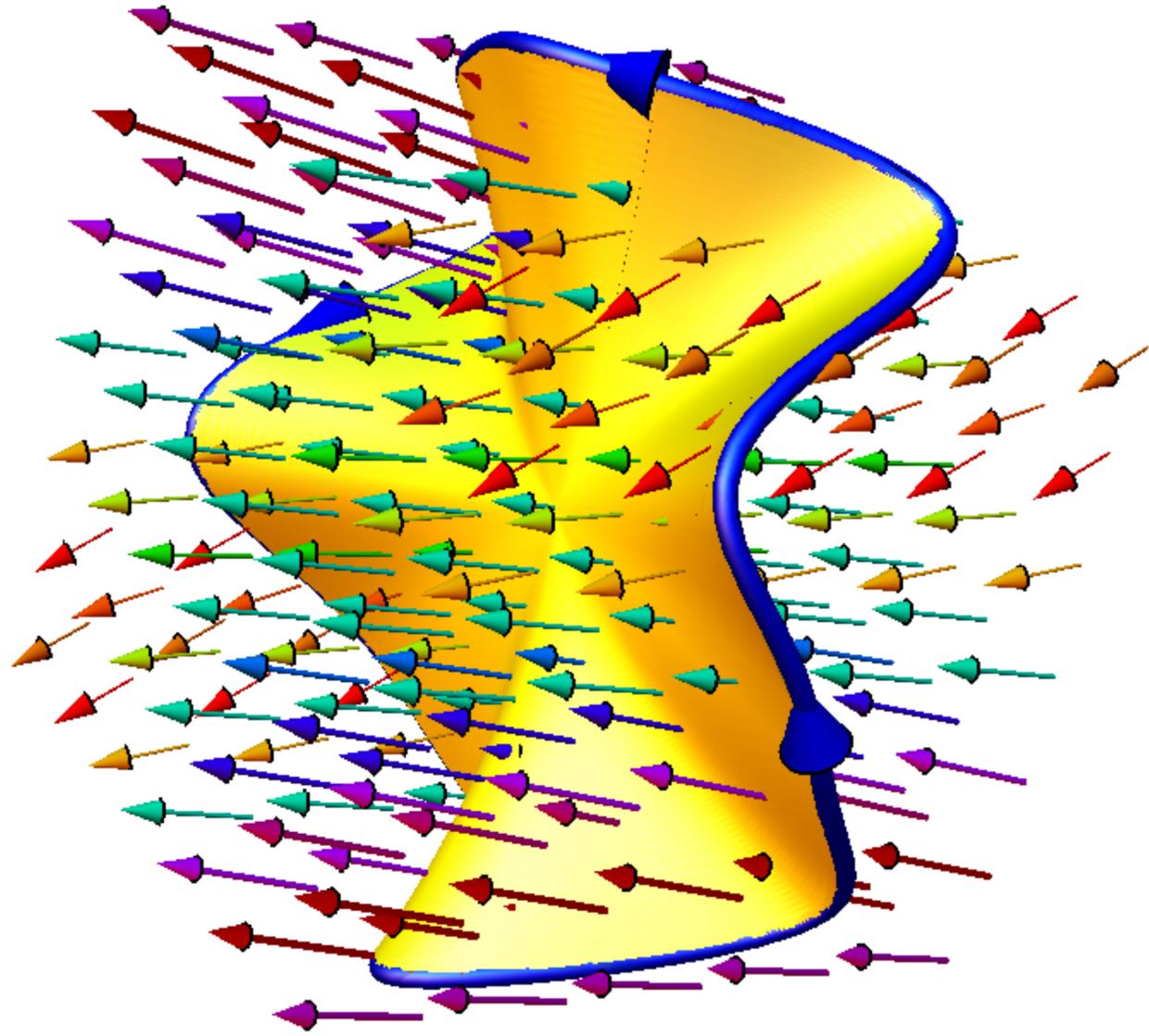
Divergence Theorem



21

$$\iiint_E \text{div}(\vec{F}) \, dV = \iint_S \vec{F} \cdot d\vec{S}$$

B) Tattoos



[Your Account](#) / [Order History](#) / [Order #IM9C6E3EA942](#)

Order Details

Ordered on Nov 30, 2018 | Order #IM9C6E3EA942

BILLING ADDRESS	SHIPPING ADDRESS	PAYMENT METHOD	ORDER SUMMARY
Oliver Knill 60 Crawford Street Arlington, Massachusetts 02474 United States	Oliver Knill 60 Crawford Street Arlington, Massachusetts 02474 United States	Credit Card 2563 Exp: 11/2022	Order Subtotal \$71.65 Sale Tax: \$0.00 <hr/> Order Total \$71.65
Order Status: Shipped/Picked up			Download Invoice

Shipped Items



70 of 1.5 x 3 Custom Tattoos

\$71.65

Artwork Type: Upload My Artwork

Artwork: Preview is not yet available [[Download Link: File .PNG \(File is being verified\)](#)]

Proof Charge: No

Customized In: USA [+0.13]

Production Time: Rush Production - 1 Day [+0.00]

Shipping Time: Standard Shipping - 5 Days [+0.00]

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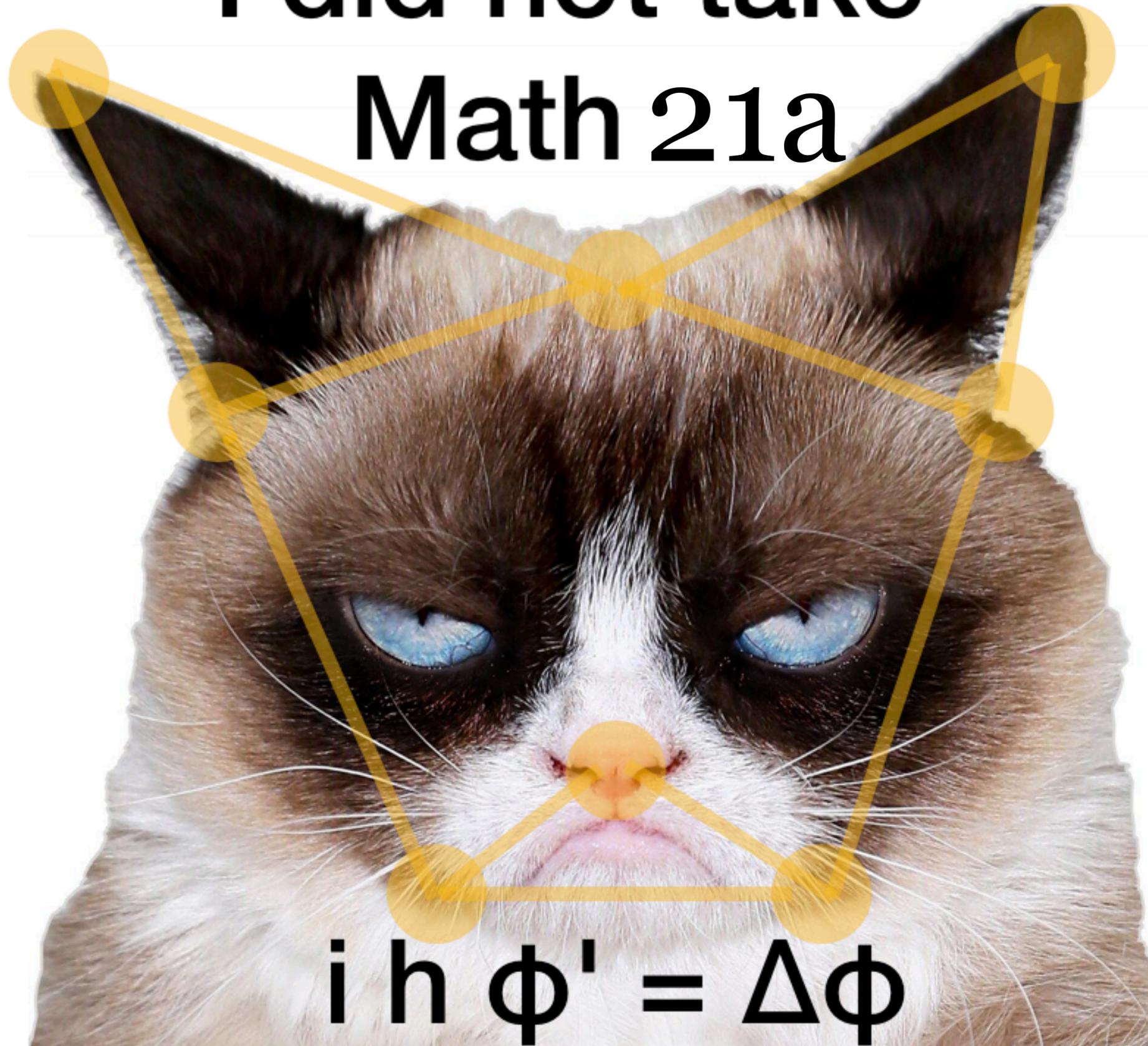
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C) Posters

I did not take
Math 21a



$$i \hbar \phi' = \Delta \phi$$

C) Wine Bottles

21 Cabernet Sauvignon

Math 21a

Knill Selection

2022

E) Car Sticker

I  [®]
MATH21

F) TV Show

Hell's Kitchen



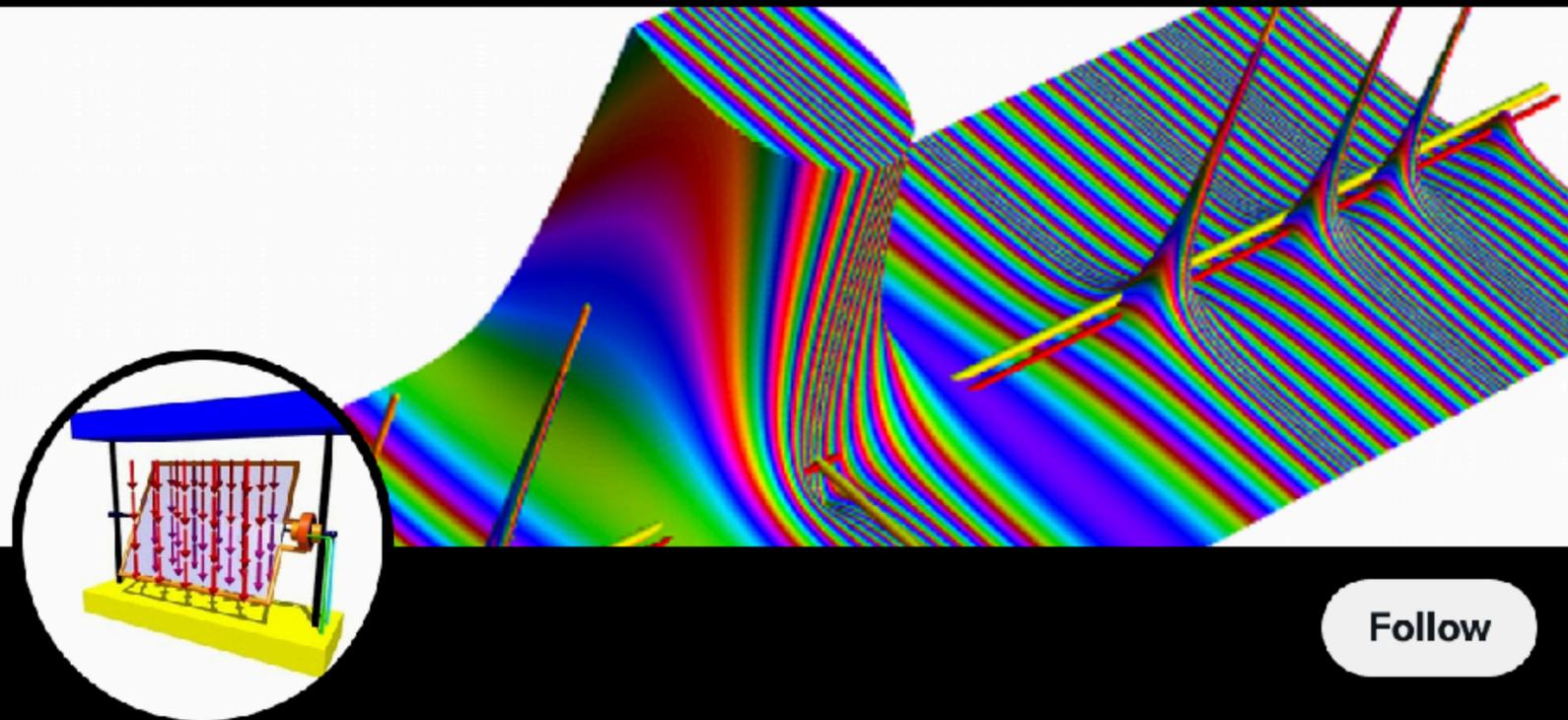


G) Twitter



Harvard Multivariable Calculus

224 Tweets



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Welcome to Math S 21a of Summer 2021. A welcome video: youtube.com/watch?v=iTr8xP...

The website is here:

people.math.harvard.edu/~knill/teachin...

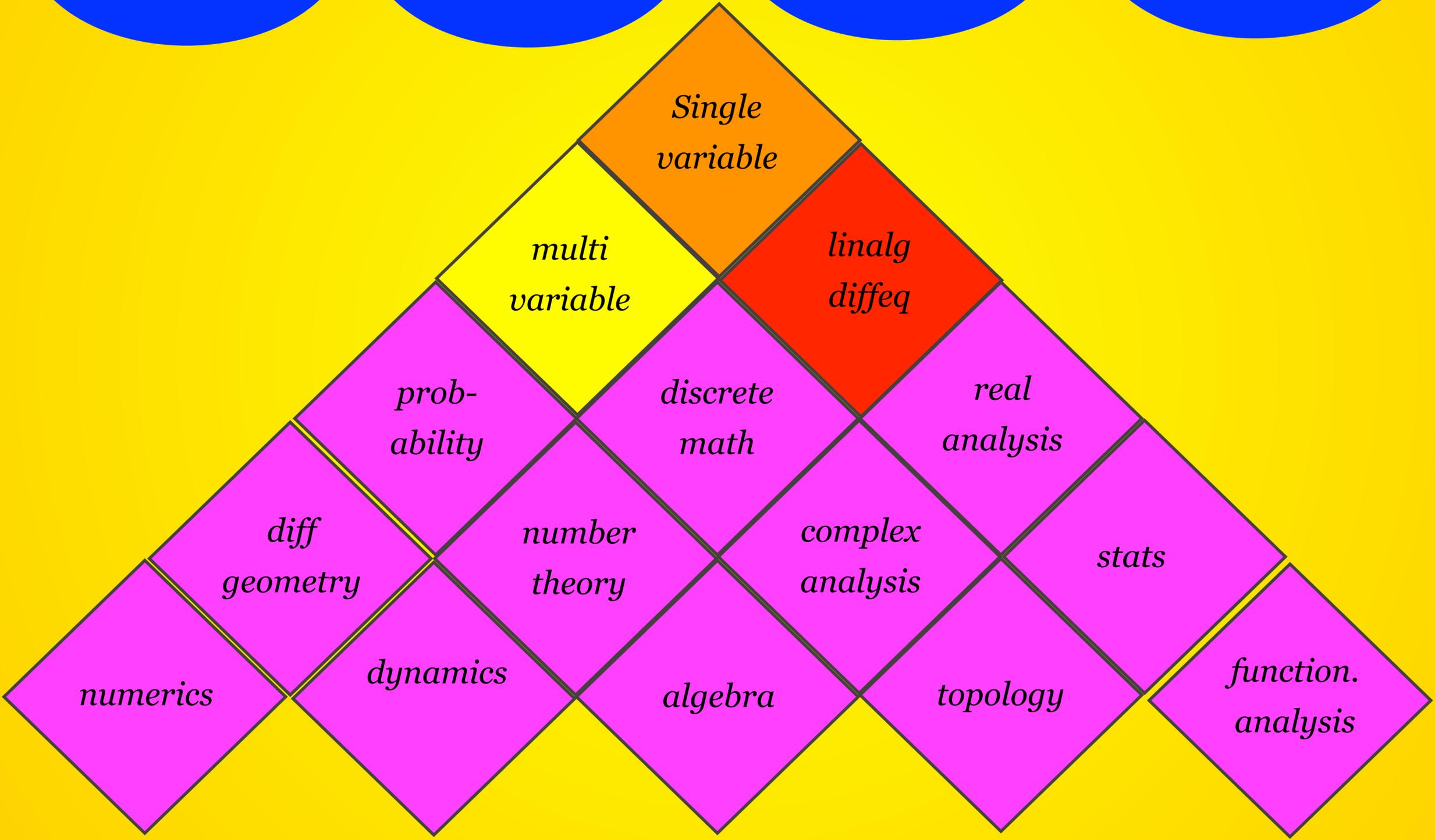
Beyond Calculus

*Computer
Science*

*Physics
Astronomy*

*Business
Economics*

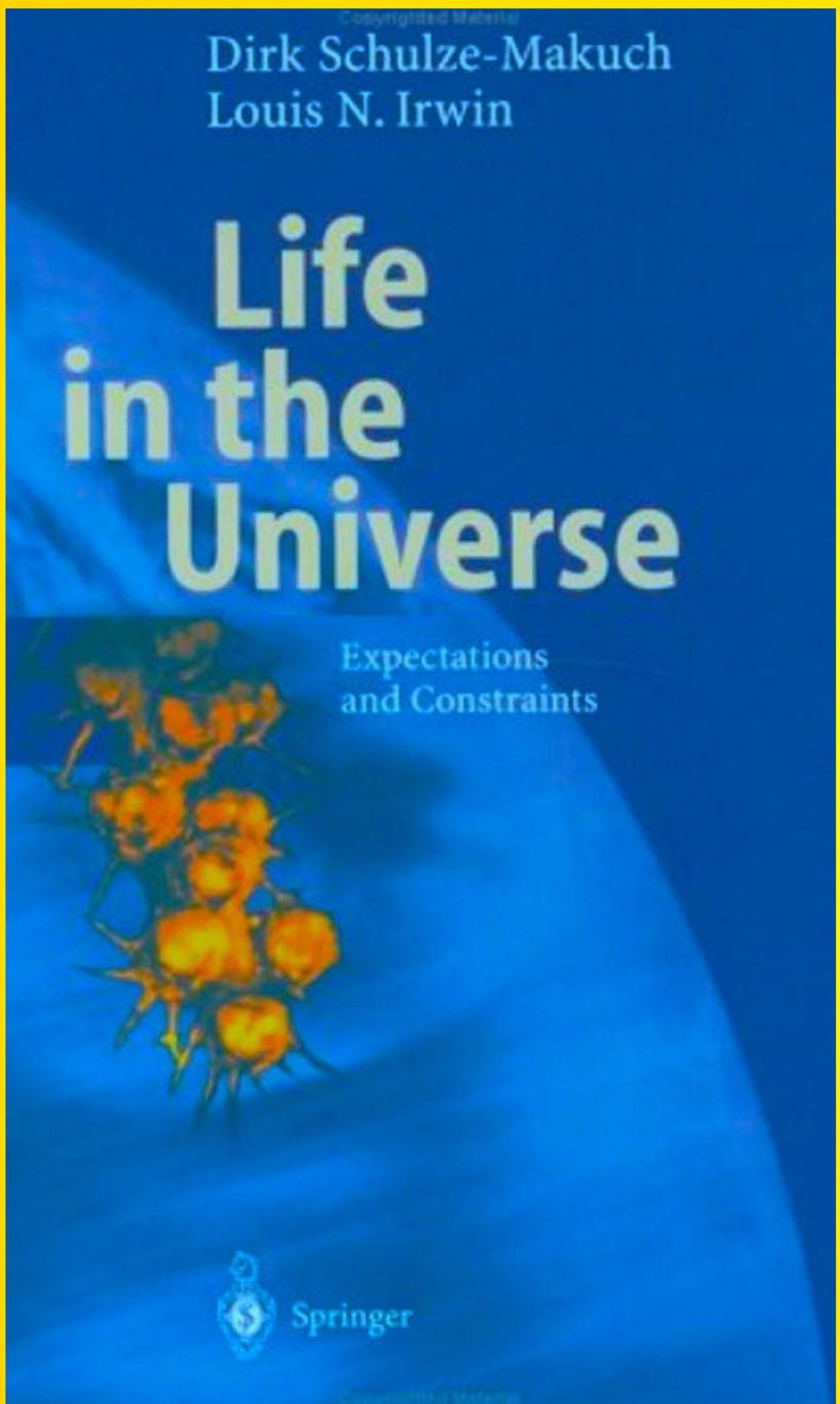
*BioChem
LifeScience*



Other ways for Calculus?



Contact



(1989) suggested that zeolites may also be used by silicon-based life as enzyme mimics. Still, the problem of a suitable solvent at these temperatures, as discussed for silicone, remains.

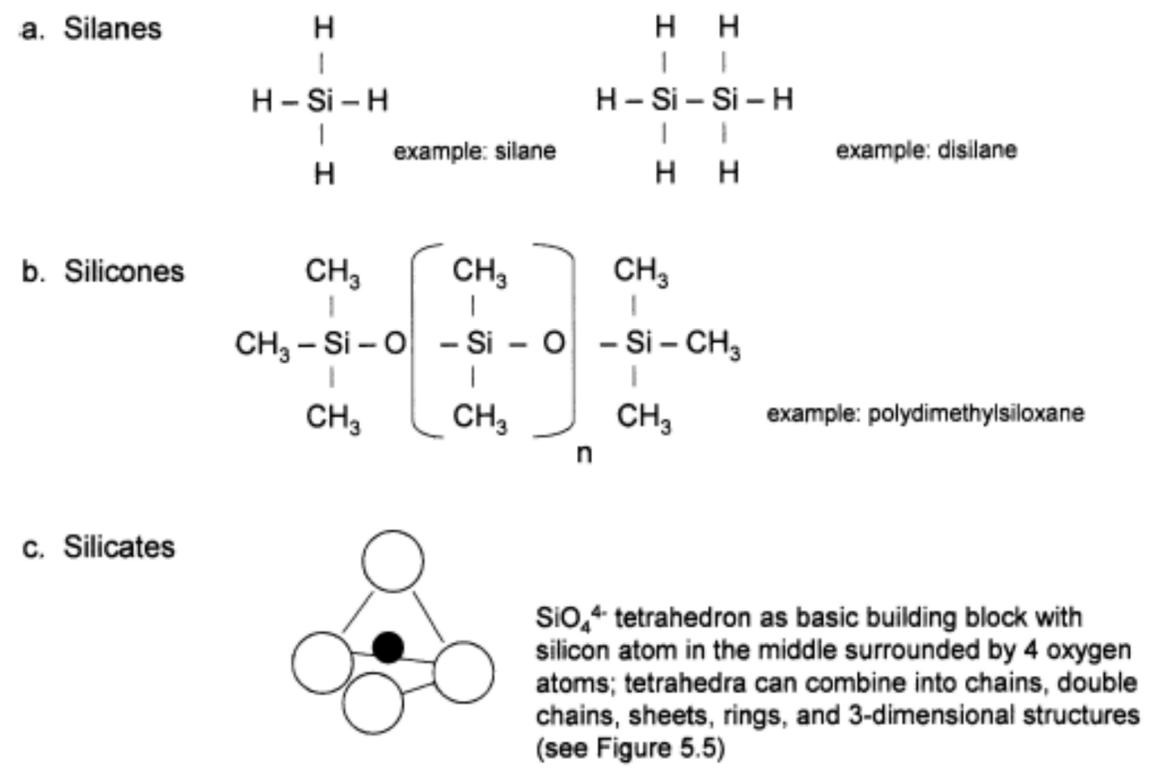
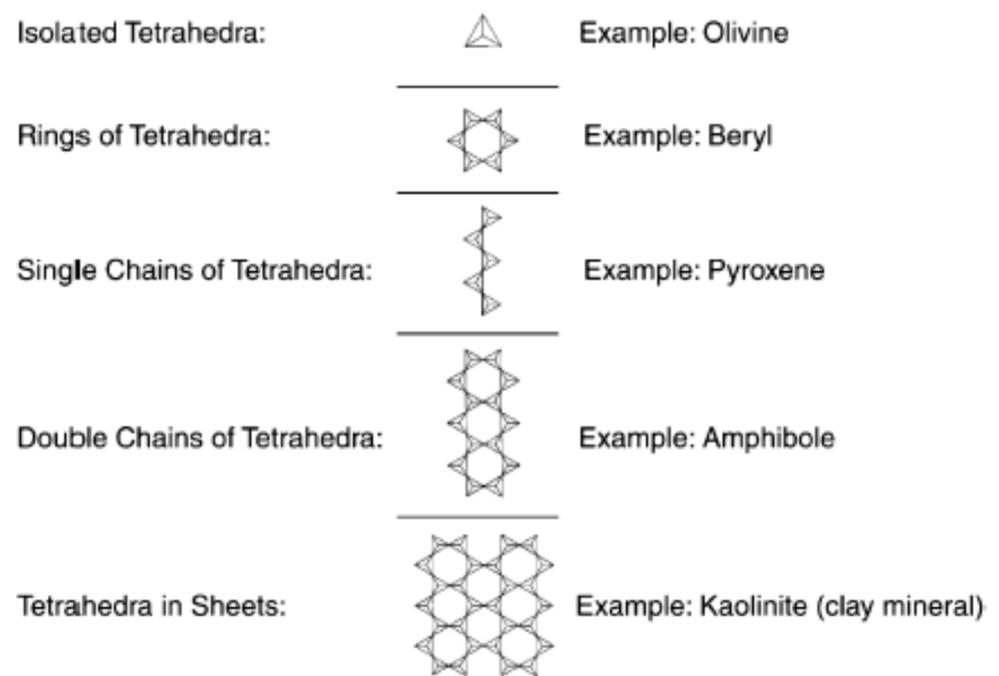


Fig. 5.4 Structure of (a) silanes, (b) silicones, and (c) silicates.

94 5 Building Blocks of Life



Fractal Calculus



BASED ON AN ORIGINAL IDEA BY
Brian Price and Mike Betar

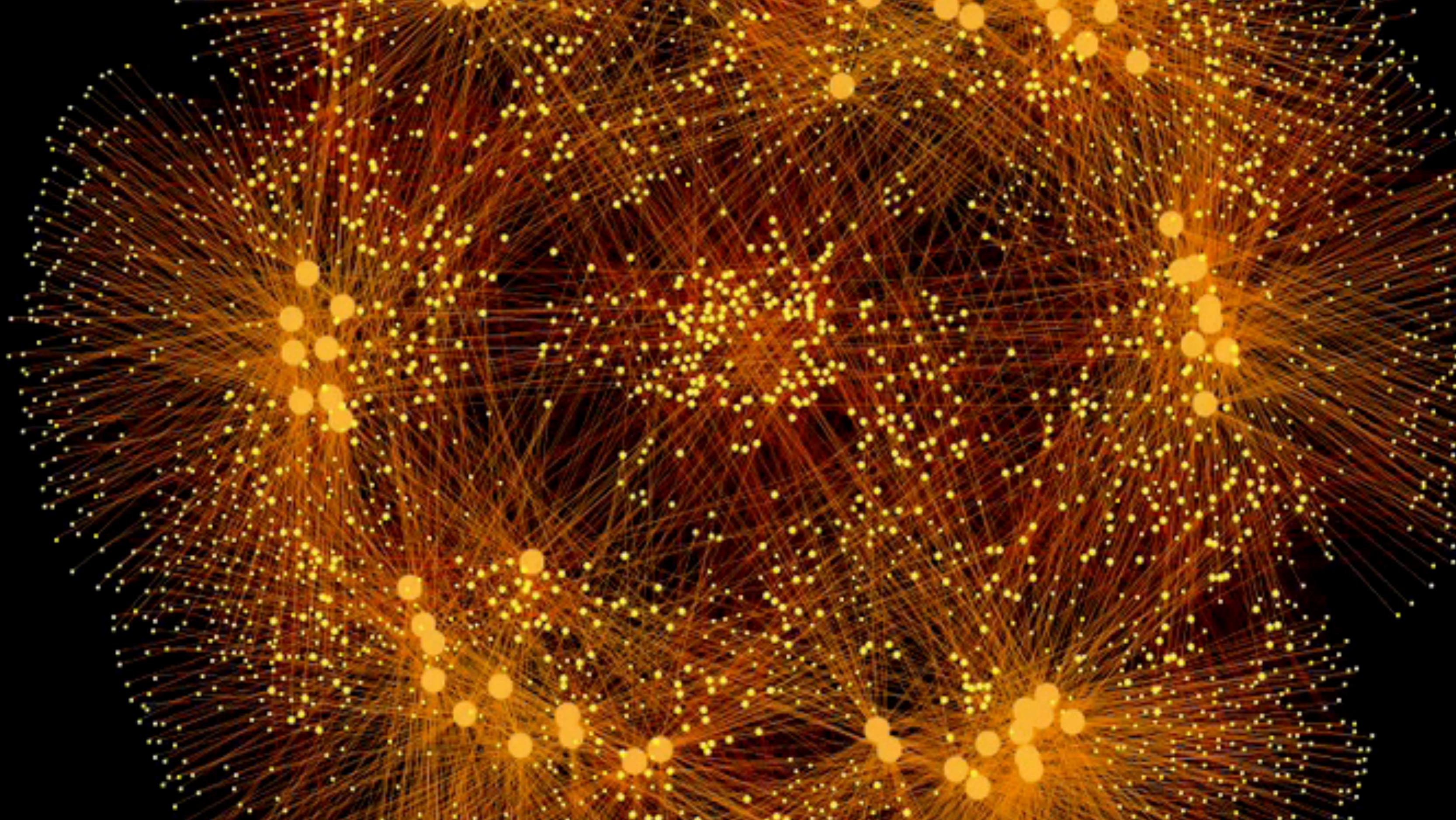
10^{-0}



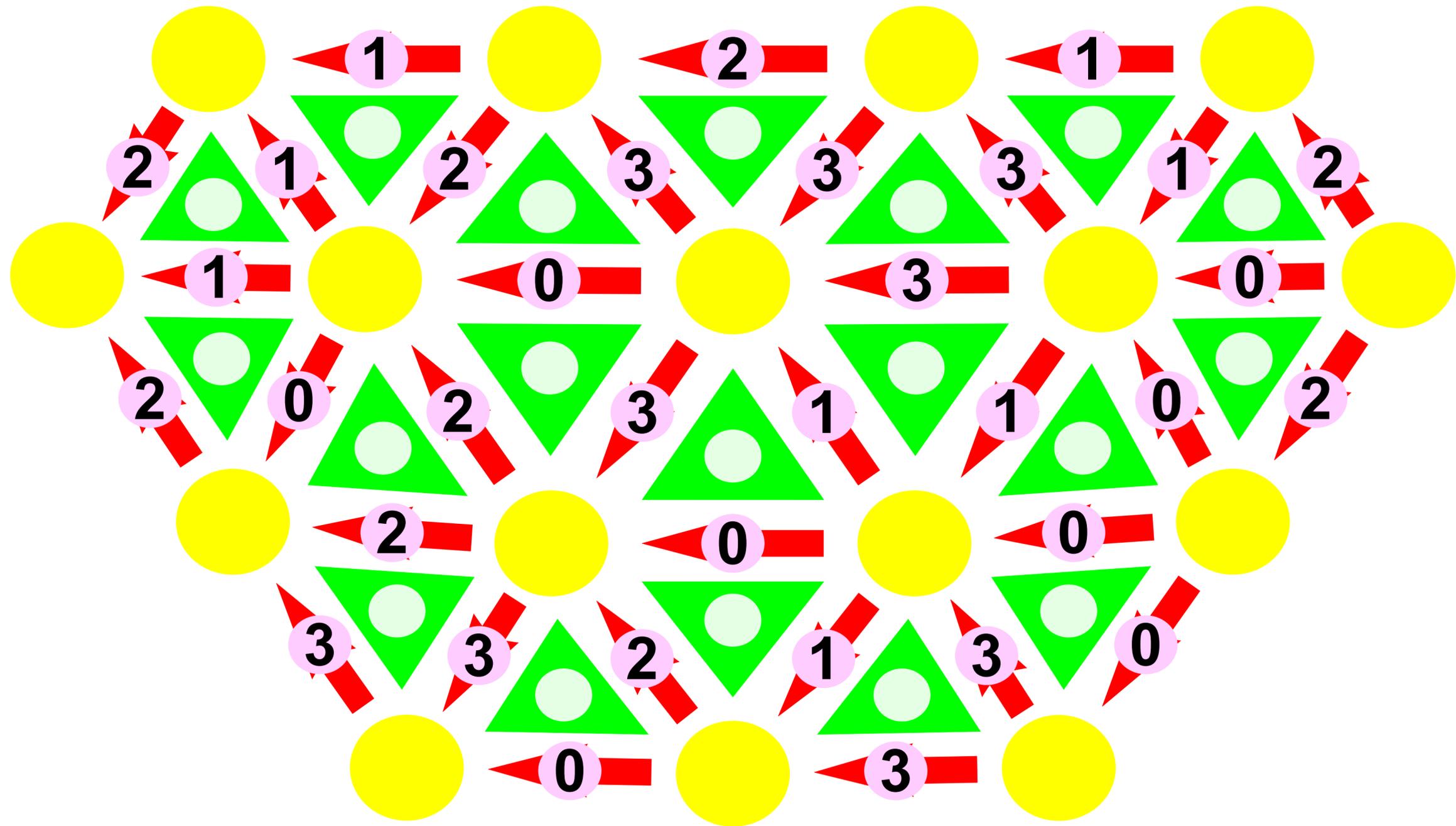
Source: tthsqe12 Youtube, 8 times sped up, added music by Angelis

In the Discrete

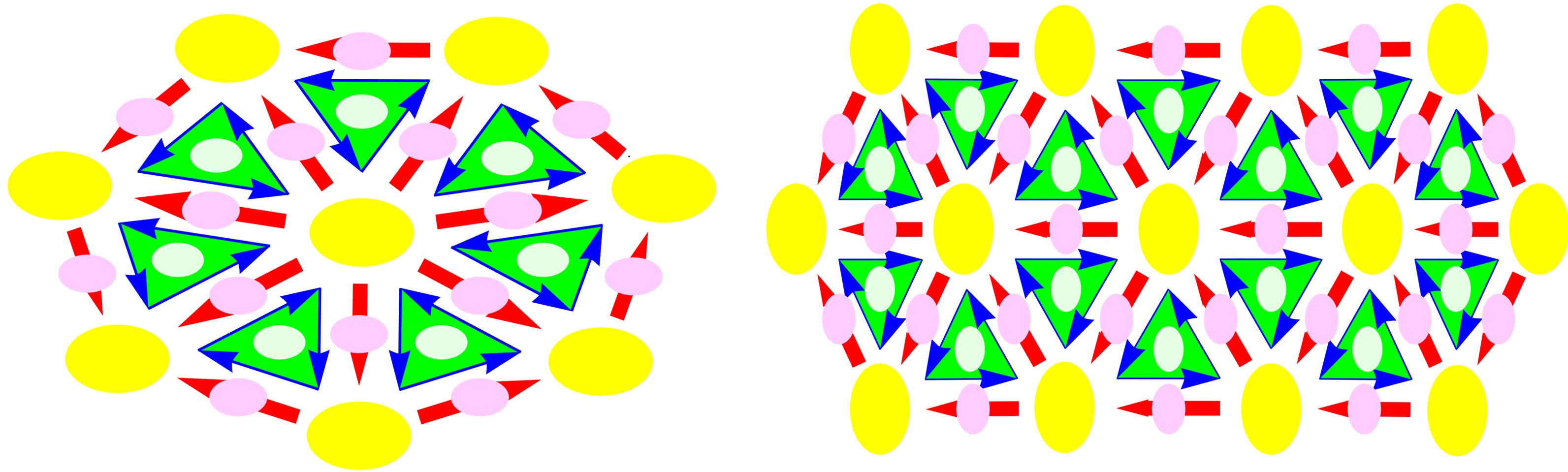




Discrete Calculus



Notions



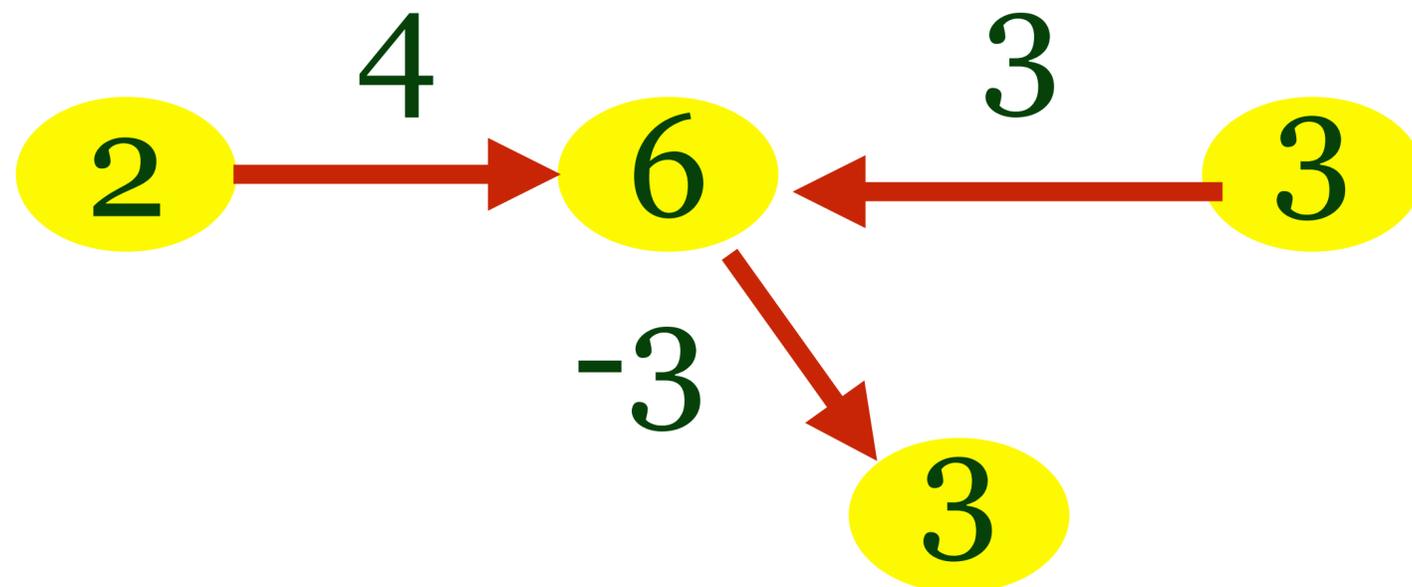
$G=(V,E)$ is a graph. V are vertices = nodes,
E are edges = connections. Also important:
triangles, tetrahedra etc (complete subgraphs)

Gradient

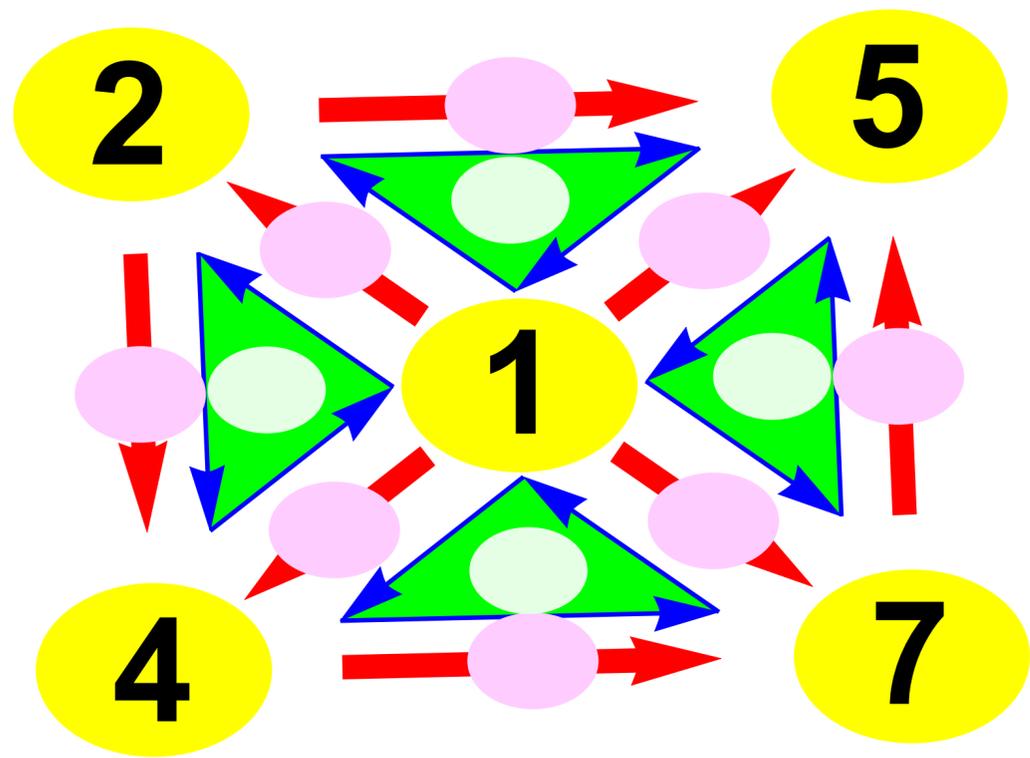


Gradient

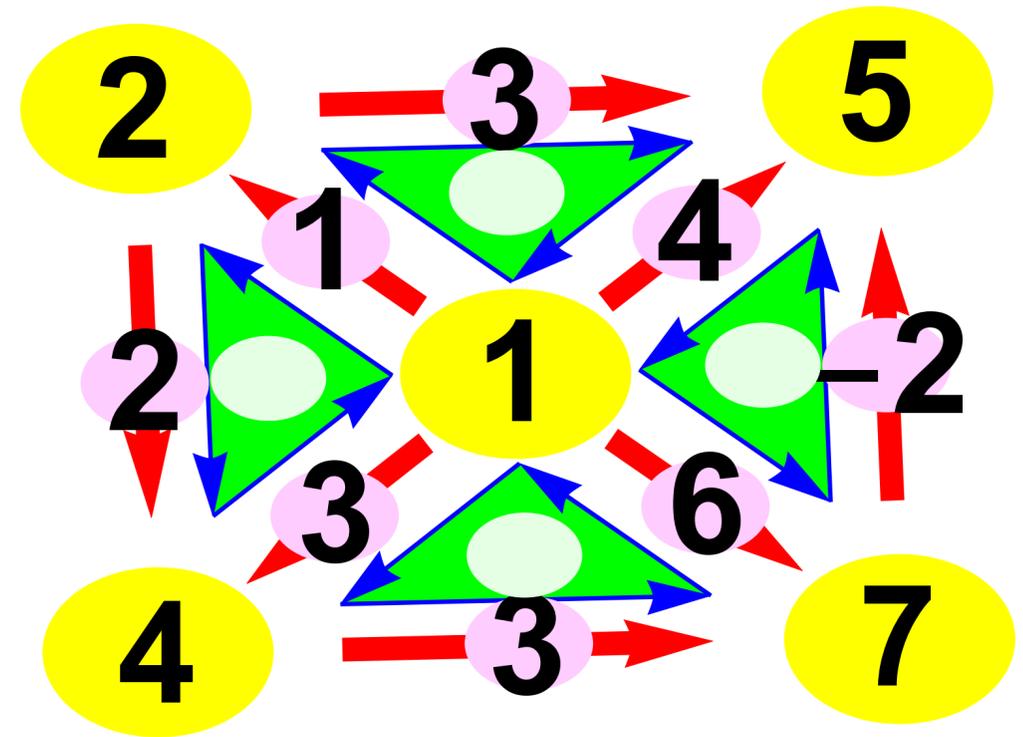
$$df(a,b) = f(b) - f(a) = 4$$



Example:



f

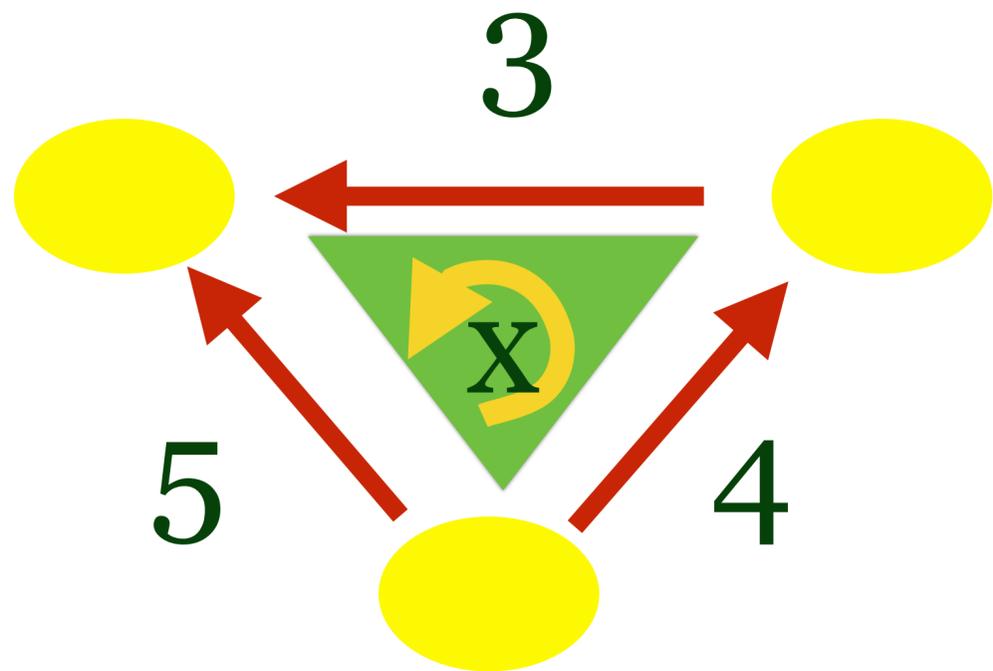


$F = df$

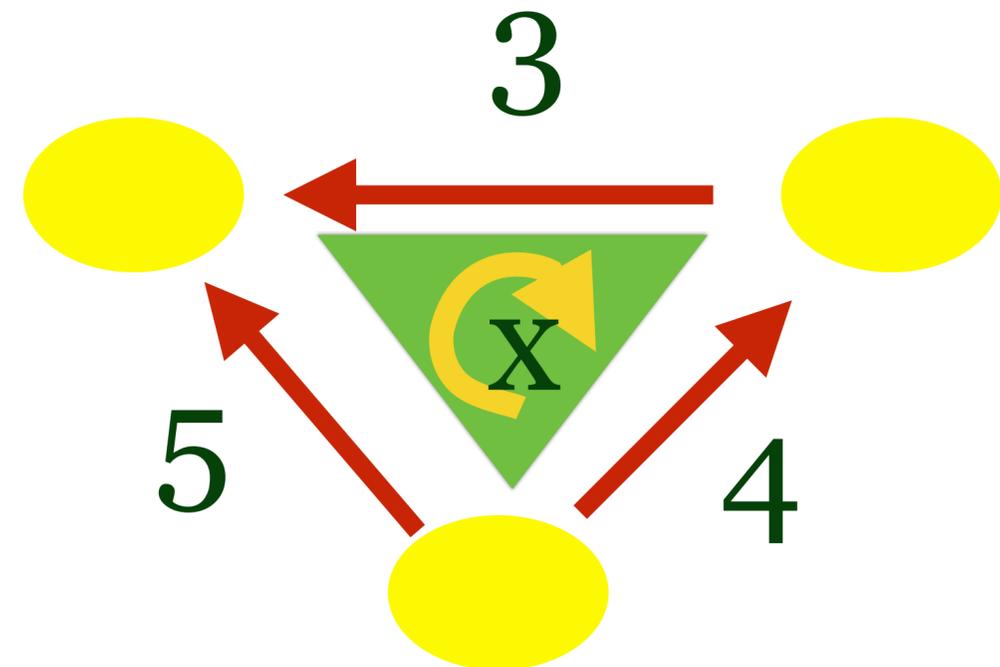
Curl

Curl

$$dF(a,b,c) = F(a,b) + F(b,c) + F(c,a)$$

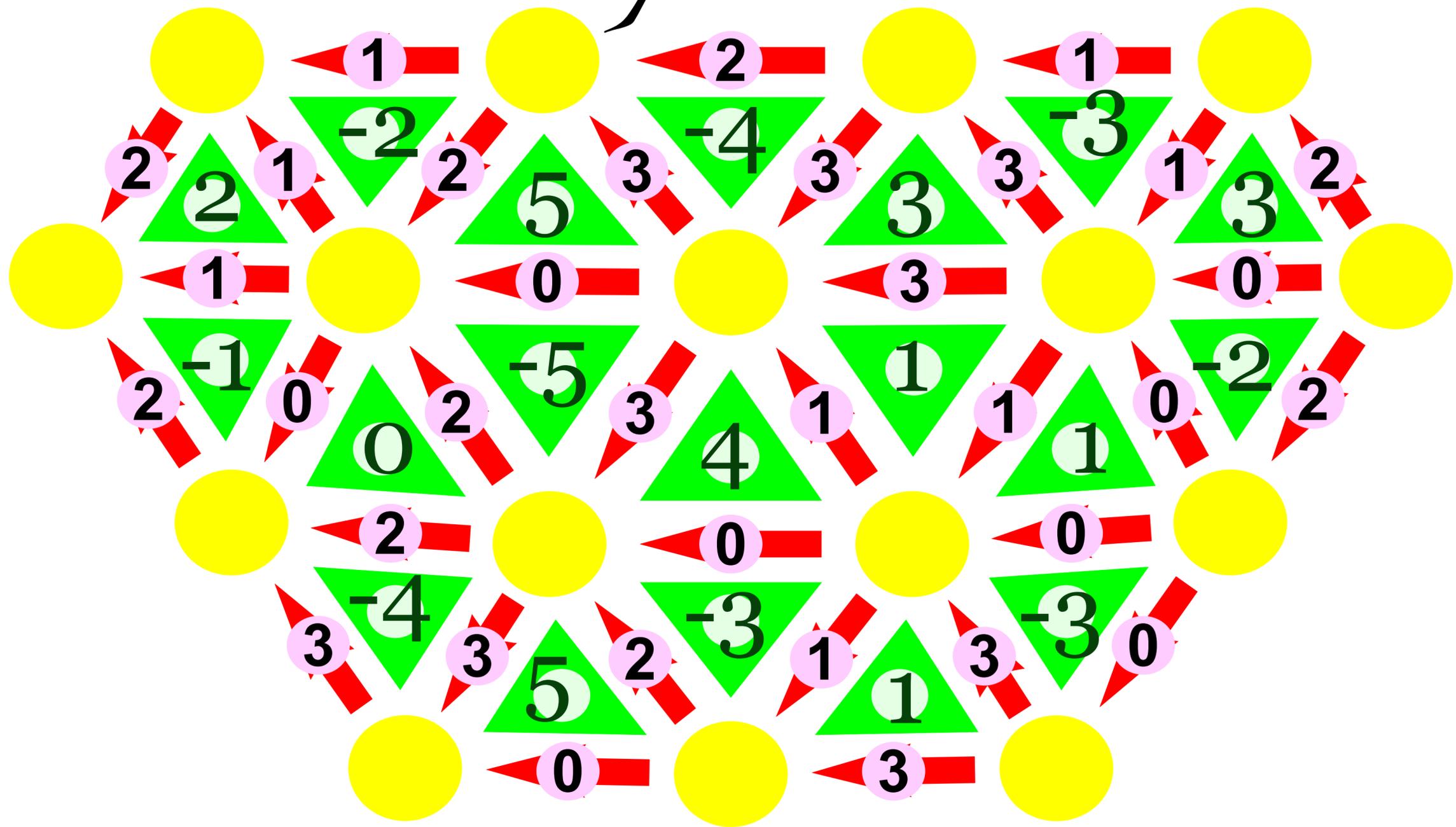


$$dF(x) = 4 + 3 - 5 = 2$$

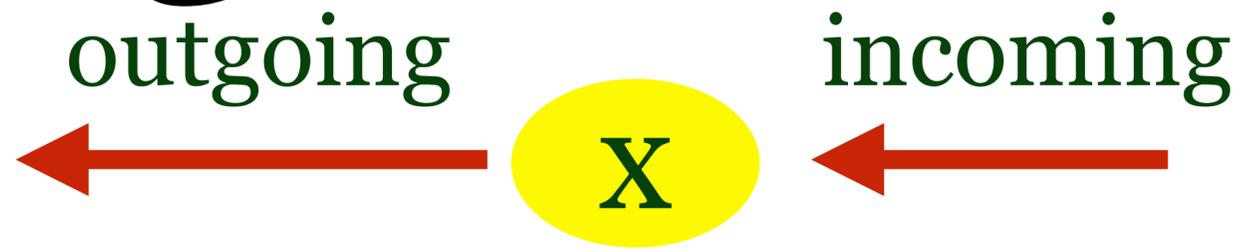


$$dF(x) = 5 - 3 - 4 = -2$$

Example

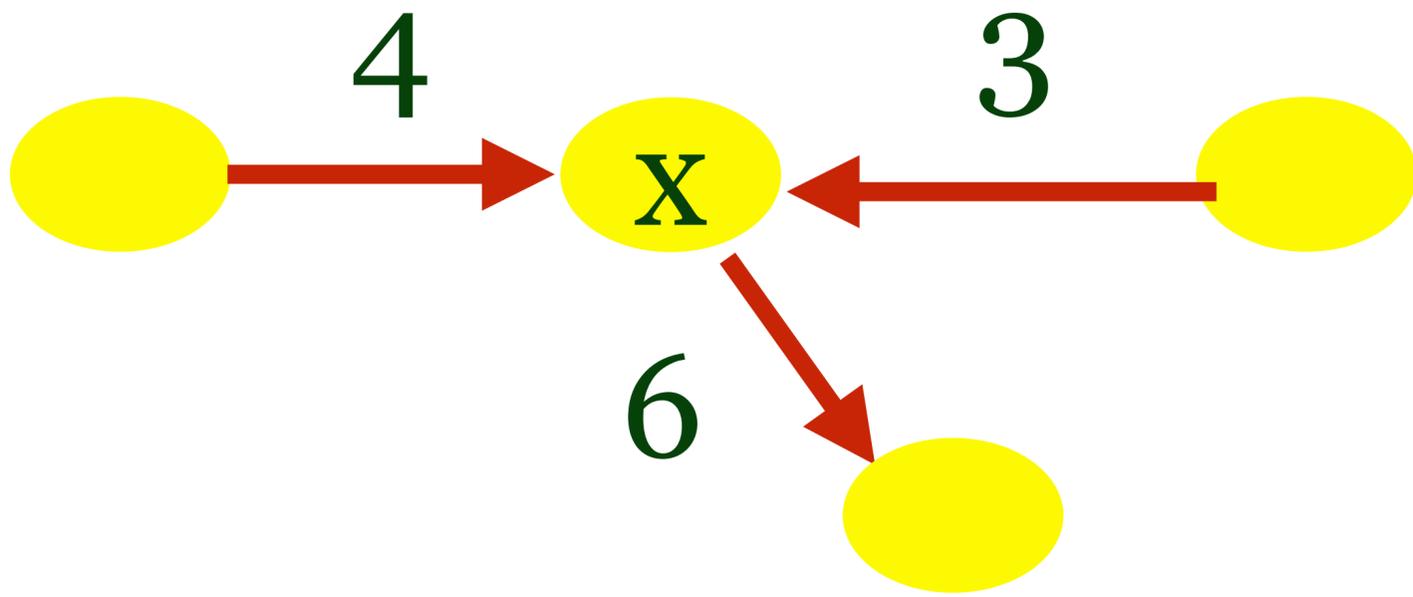


Dual gradient



Dual Gradient

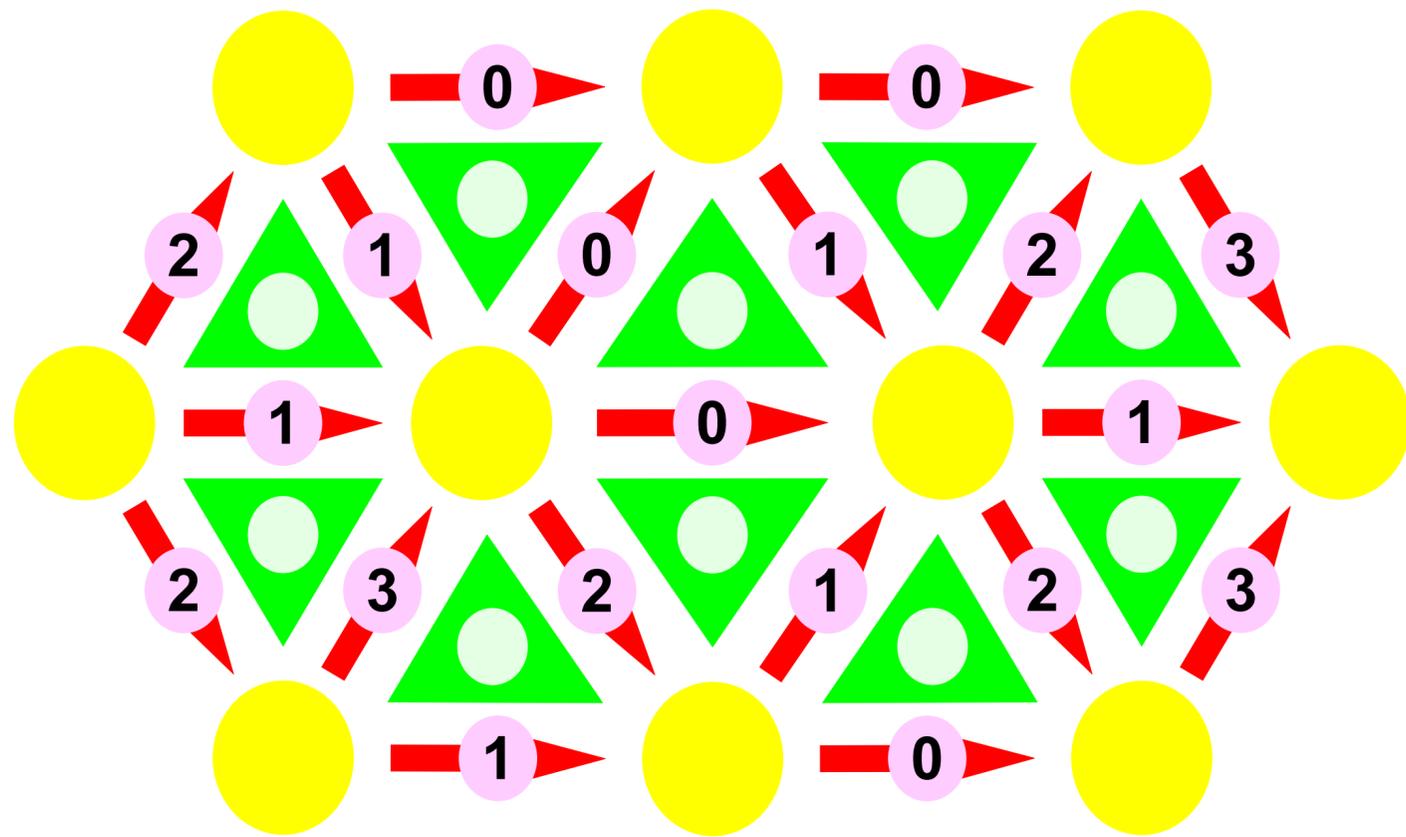
$$d^*F(x) = \sum_{\substack{e \\ \text{e coming in}}} F(e) - \sum_{\substack{e \\ \text{e going out}}} F(e)$$



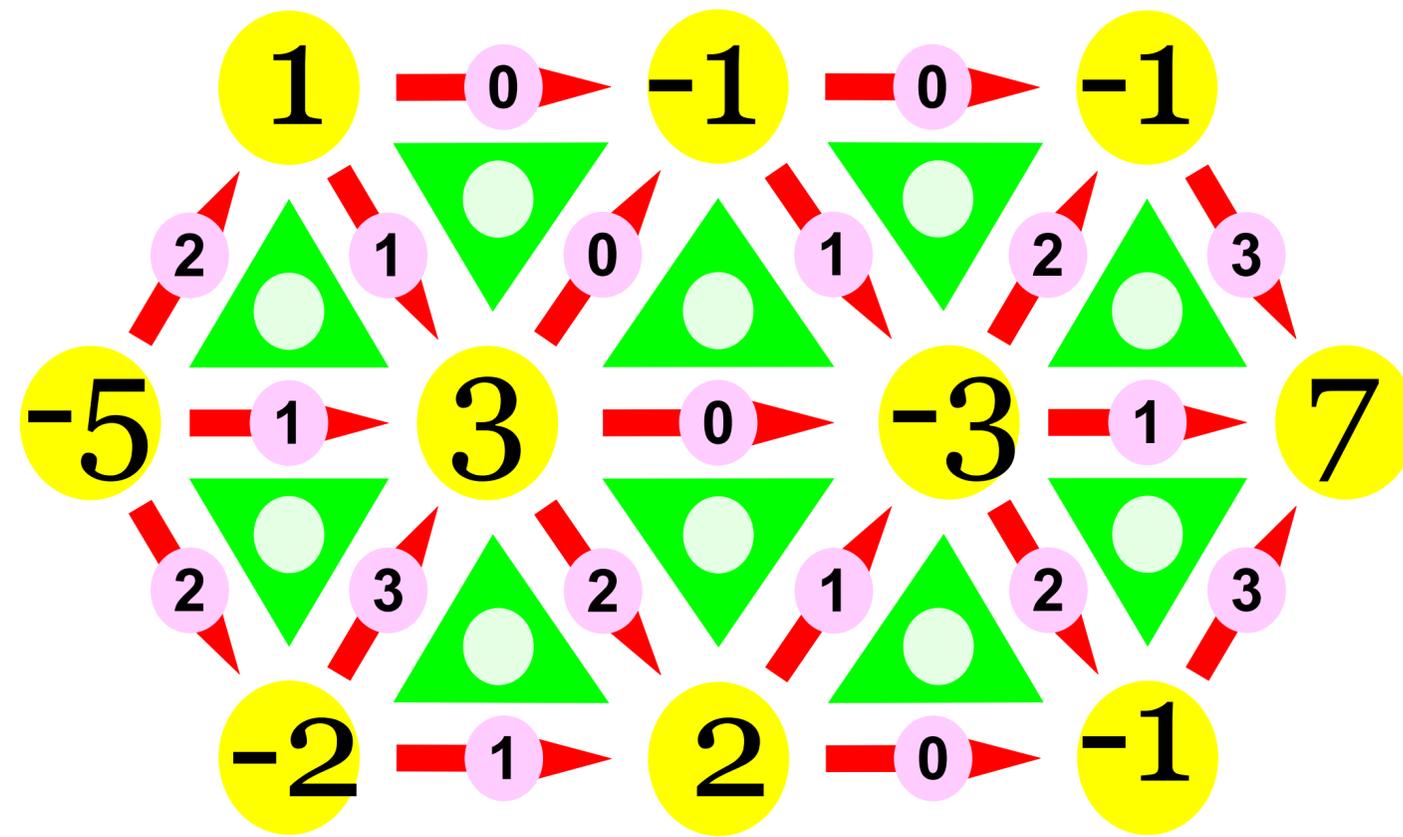
$$d^*F(x) = 4 + 3 - 6 = 1$$

Sums to zero

$$g(x) = d^*F(x)$$



F is a 1-form



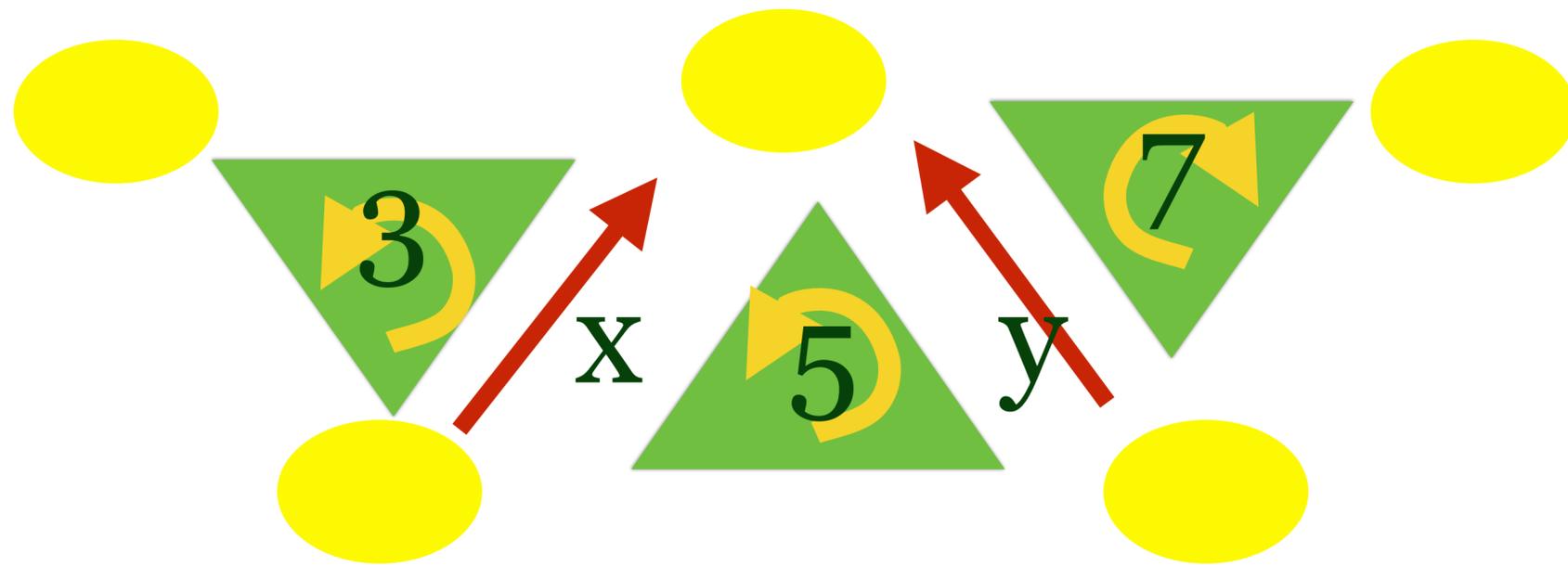
g

The sum of g values is 0

Dual Curl: 2-forms \rightarrow 1-forms

Dual Curl

$$dH(x) = \sum_{\text{orientation fit}} H(t) - \sum_{\text{orientation not fit}} H(t)$$

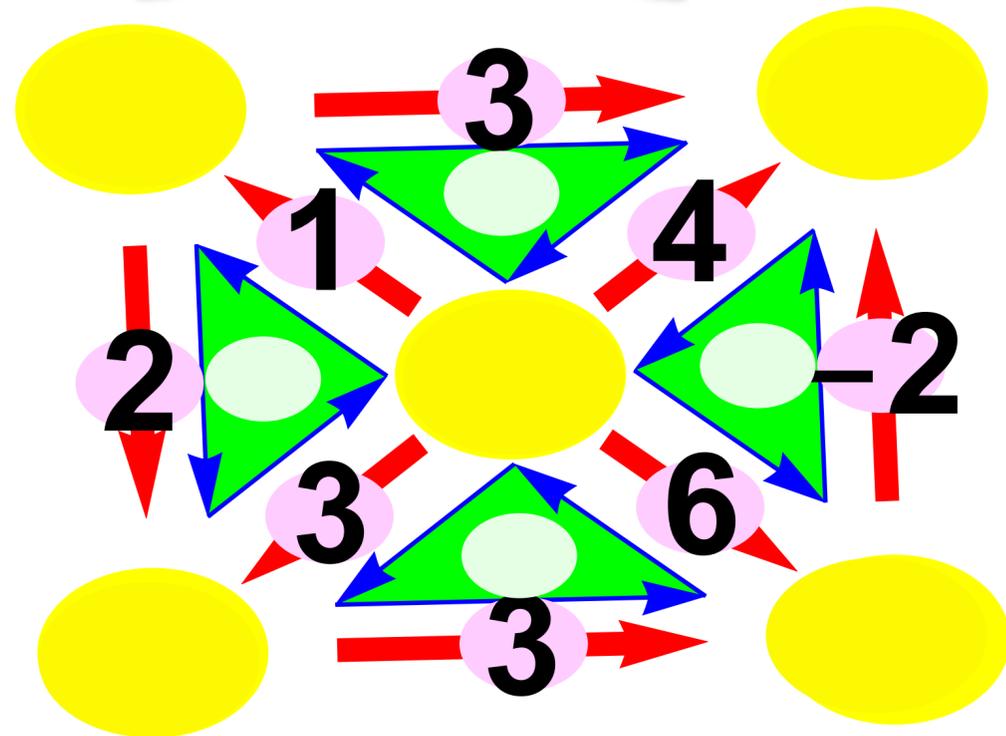
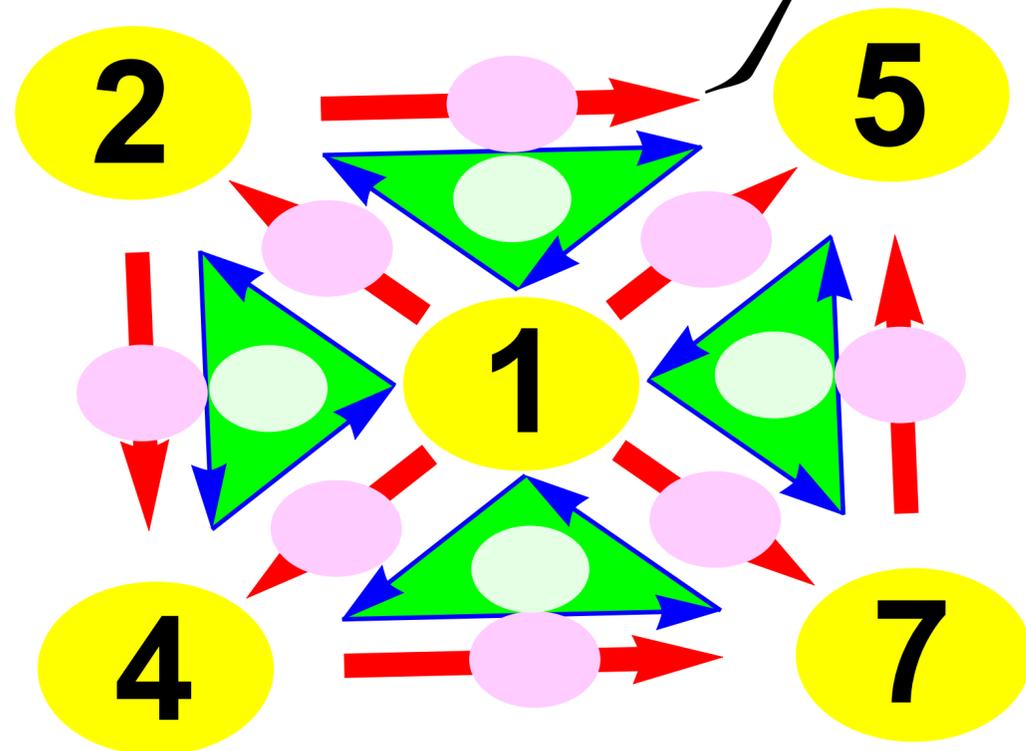


$$dH(x) = -5 + 3$$
$$dH(y) = 7 + 5$$

Laplacian

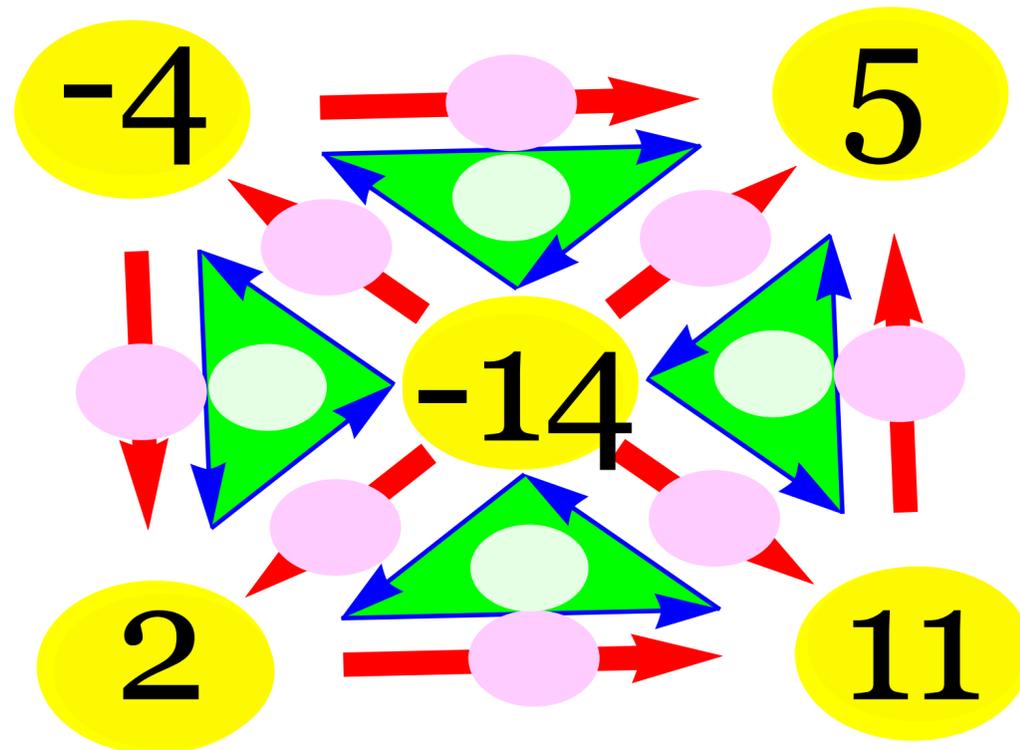
$$L = d^*d$$

$$= \text{div}(\text{grad})$$



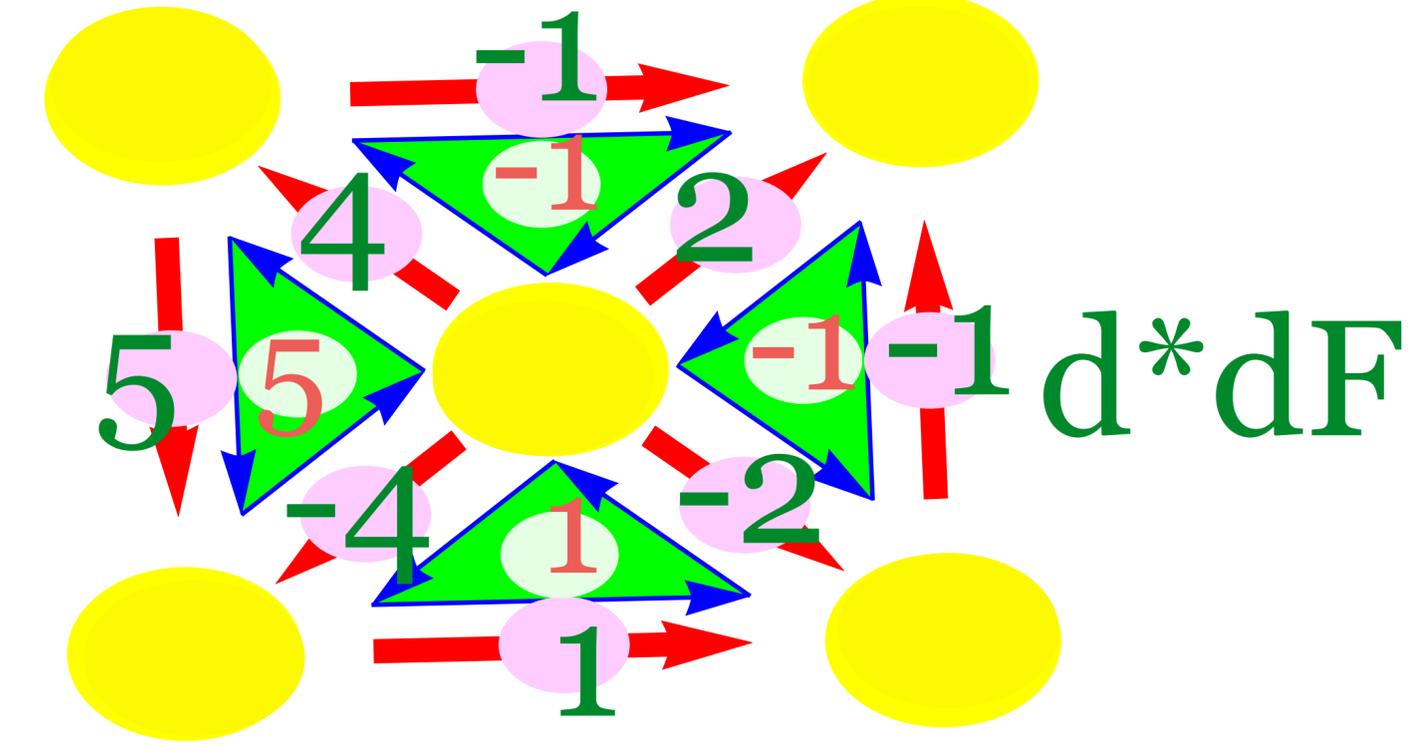
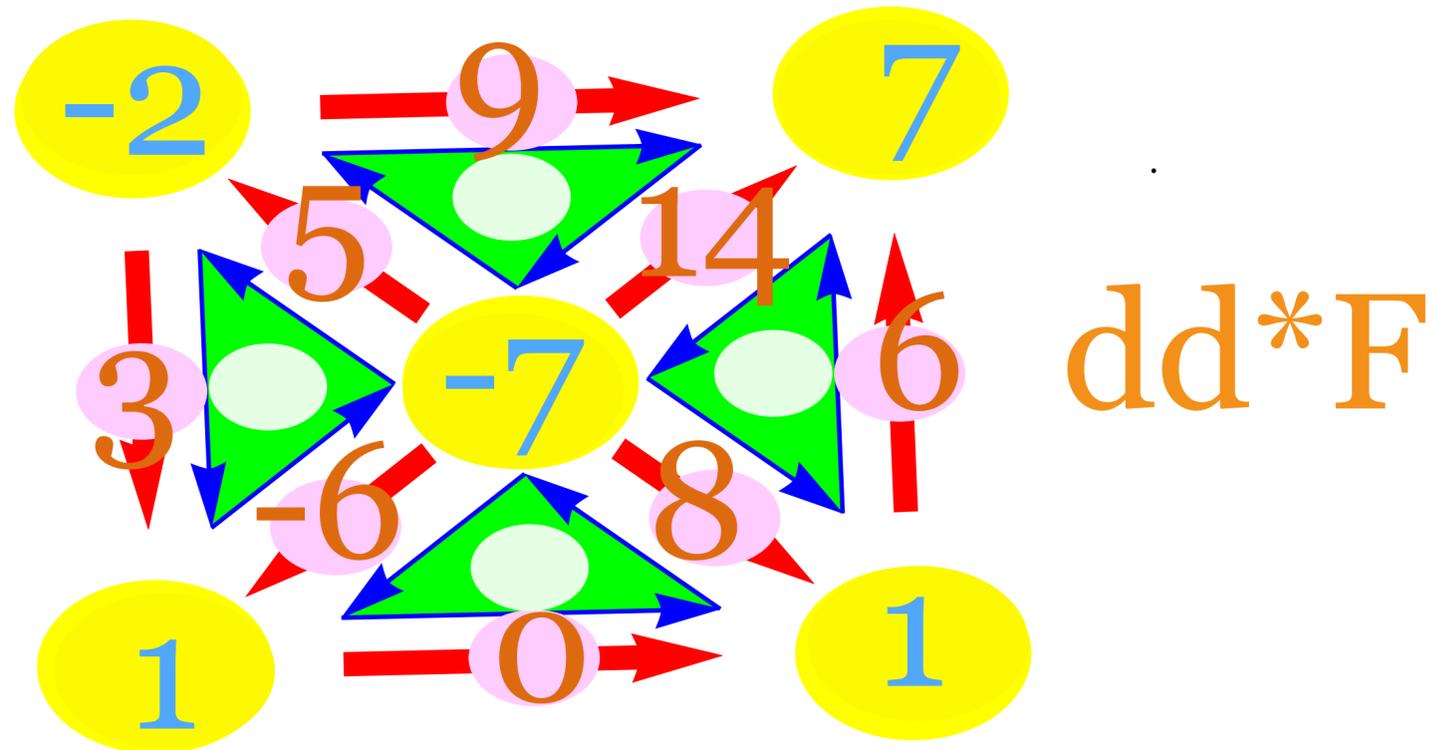
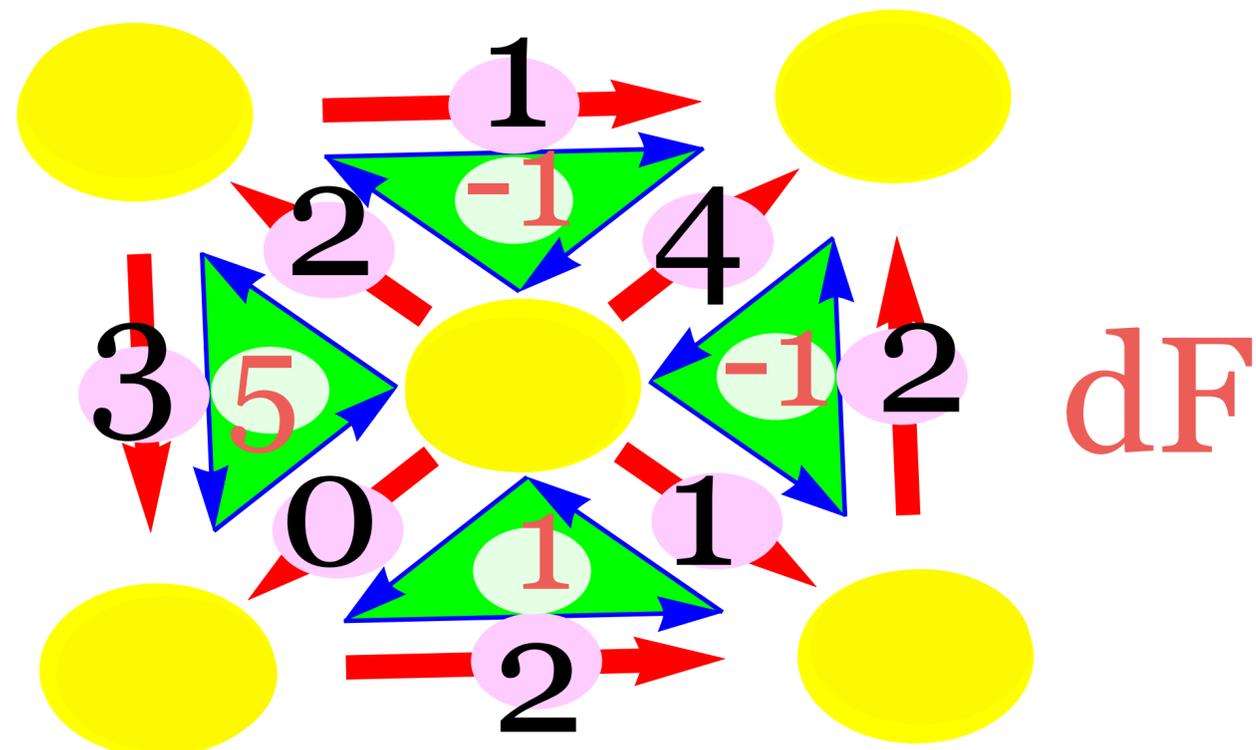
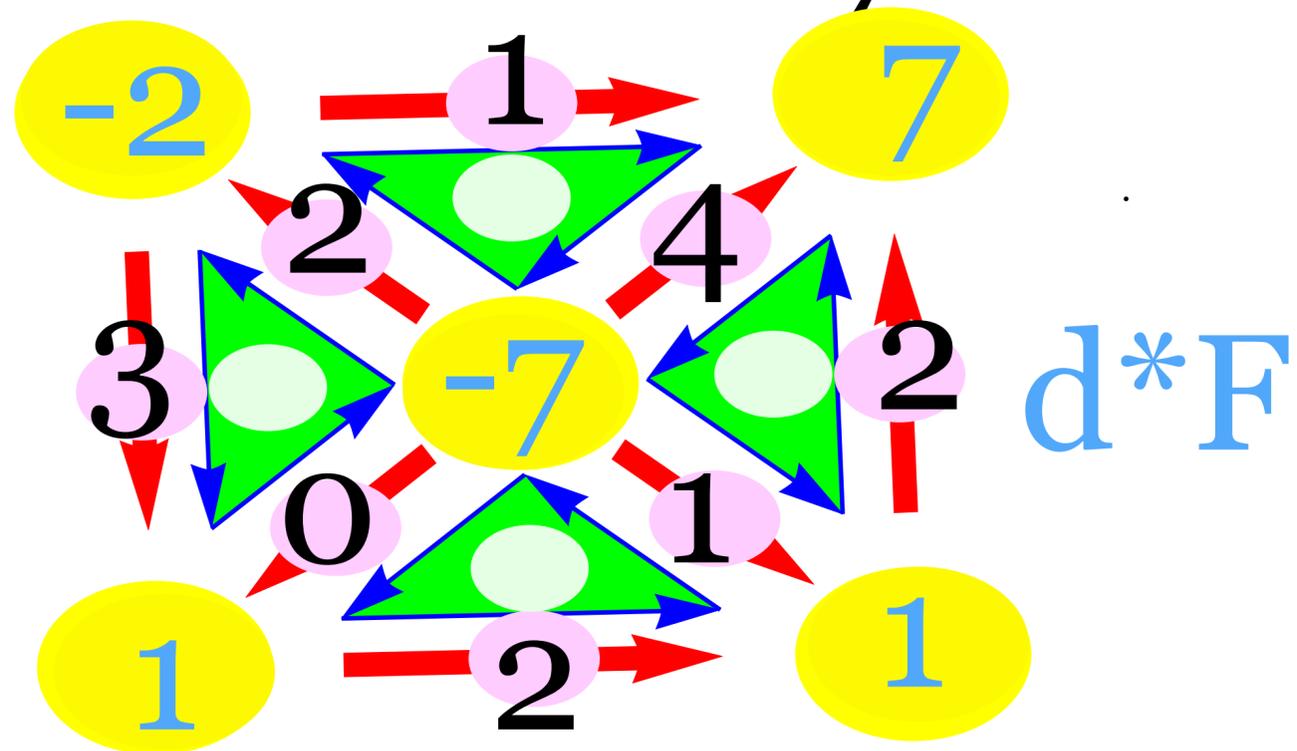
$$F = df$$

$$g = d^*df = Lf$$

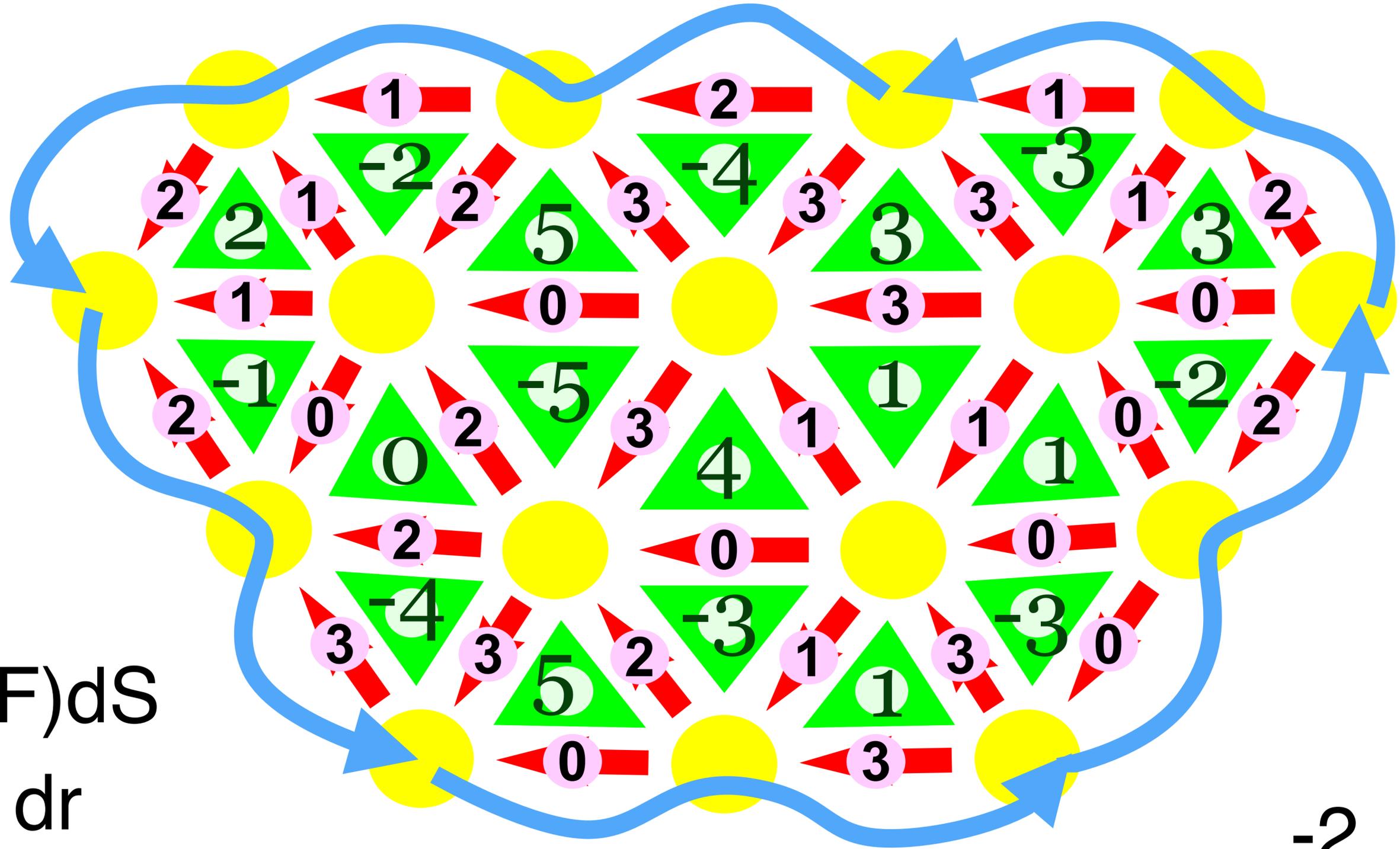


Form Laplacian

$$LF = (d^*d + dd^*)F$$



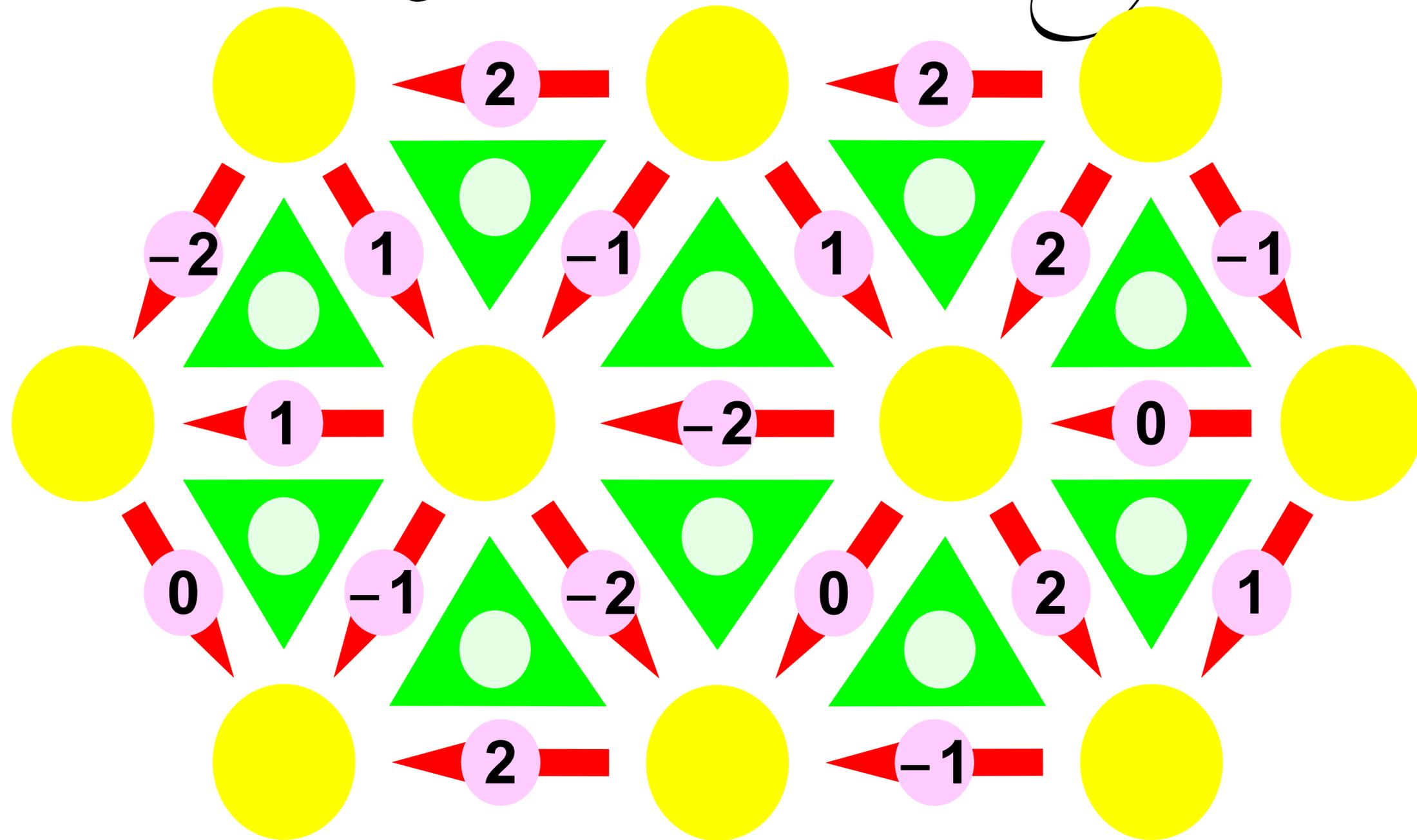
Stokes Theorem



$$\iint \text{curl}(F) dS = \int F dr$$

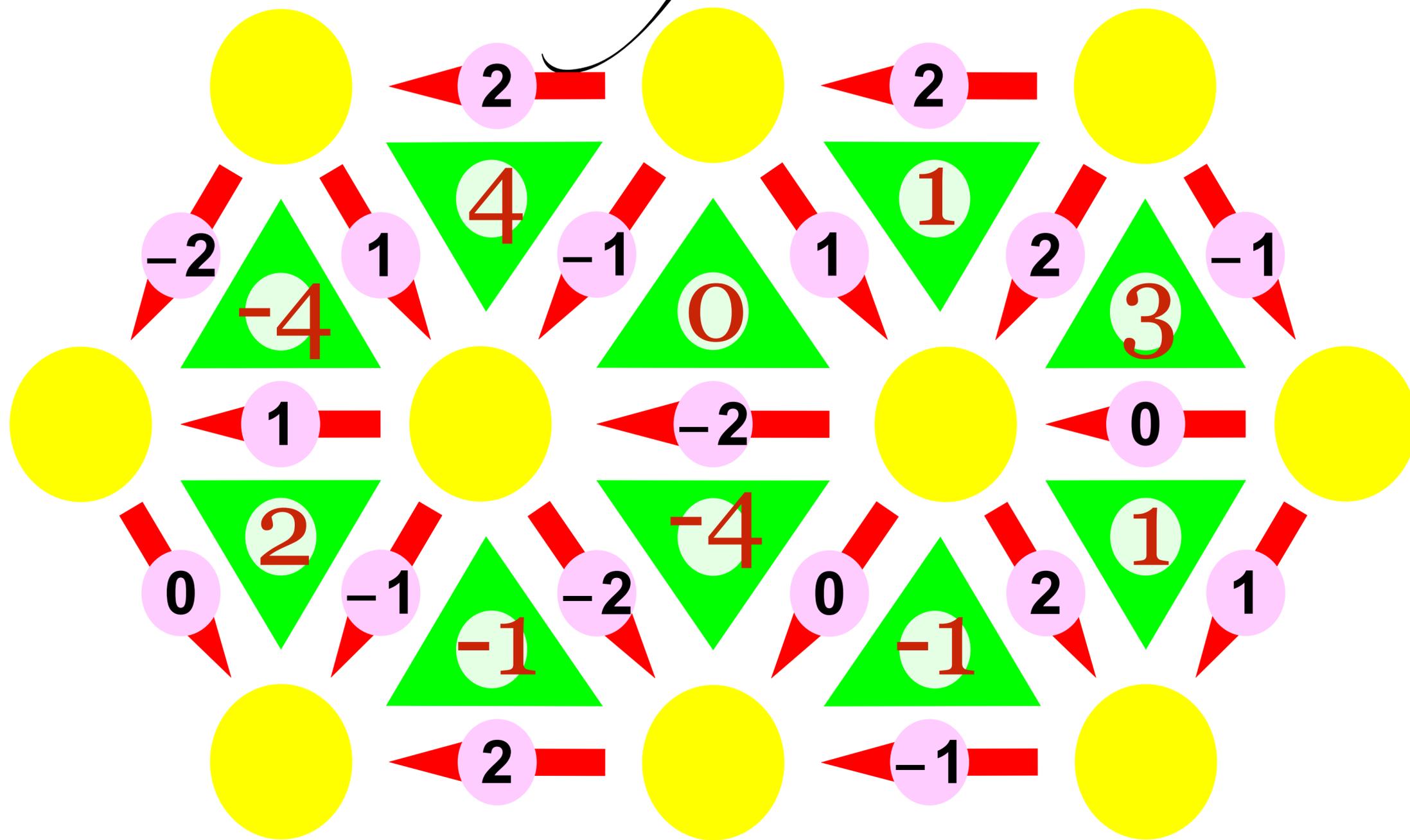
-2

Your Turn: first line integral



$$-2+1-1+1+2+2-2-0 = 1$$

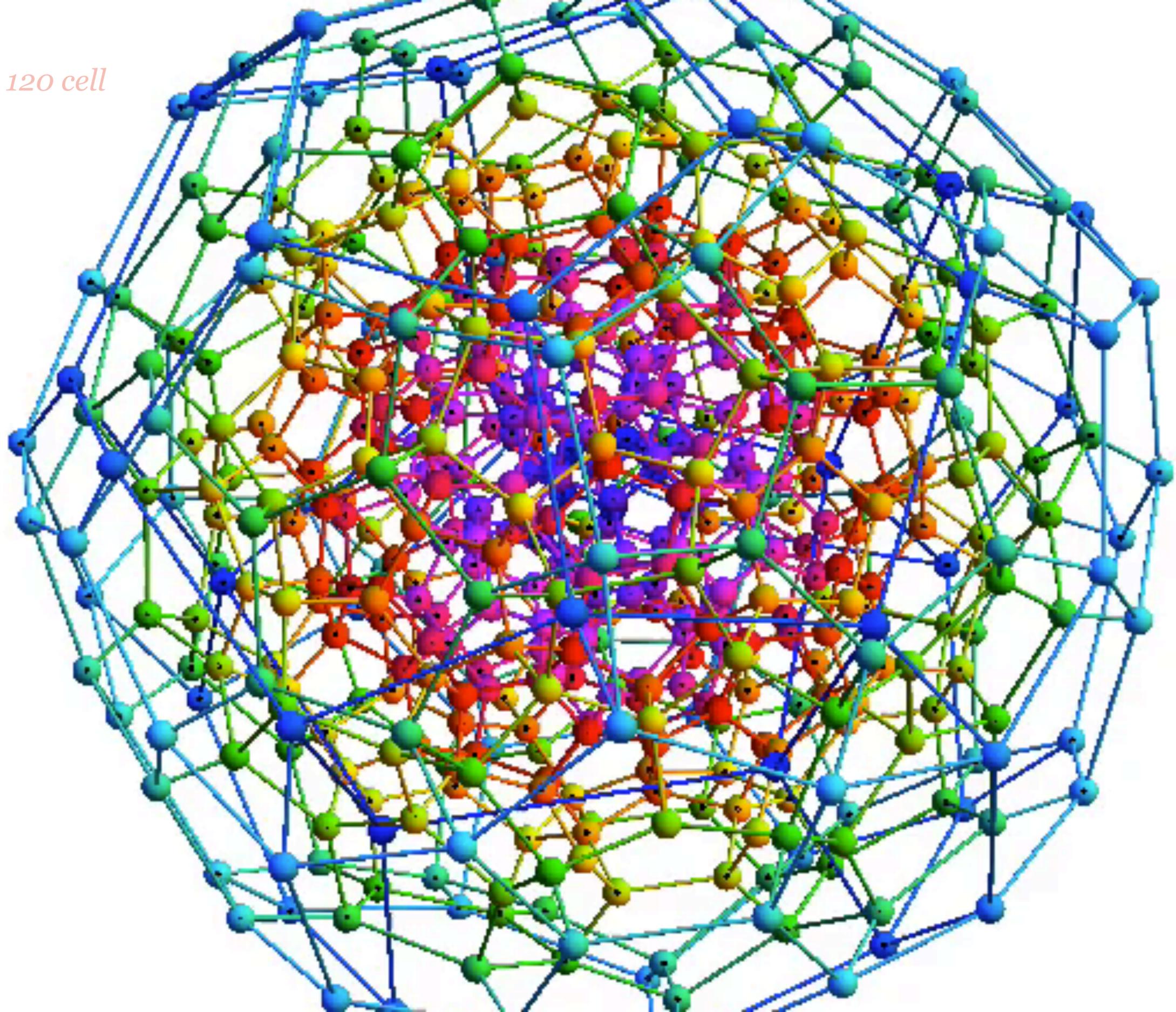
Sum of curl



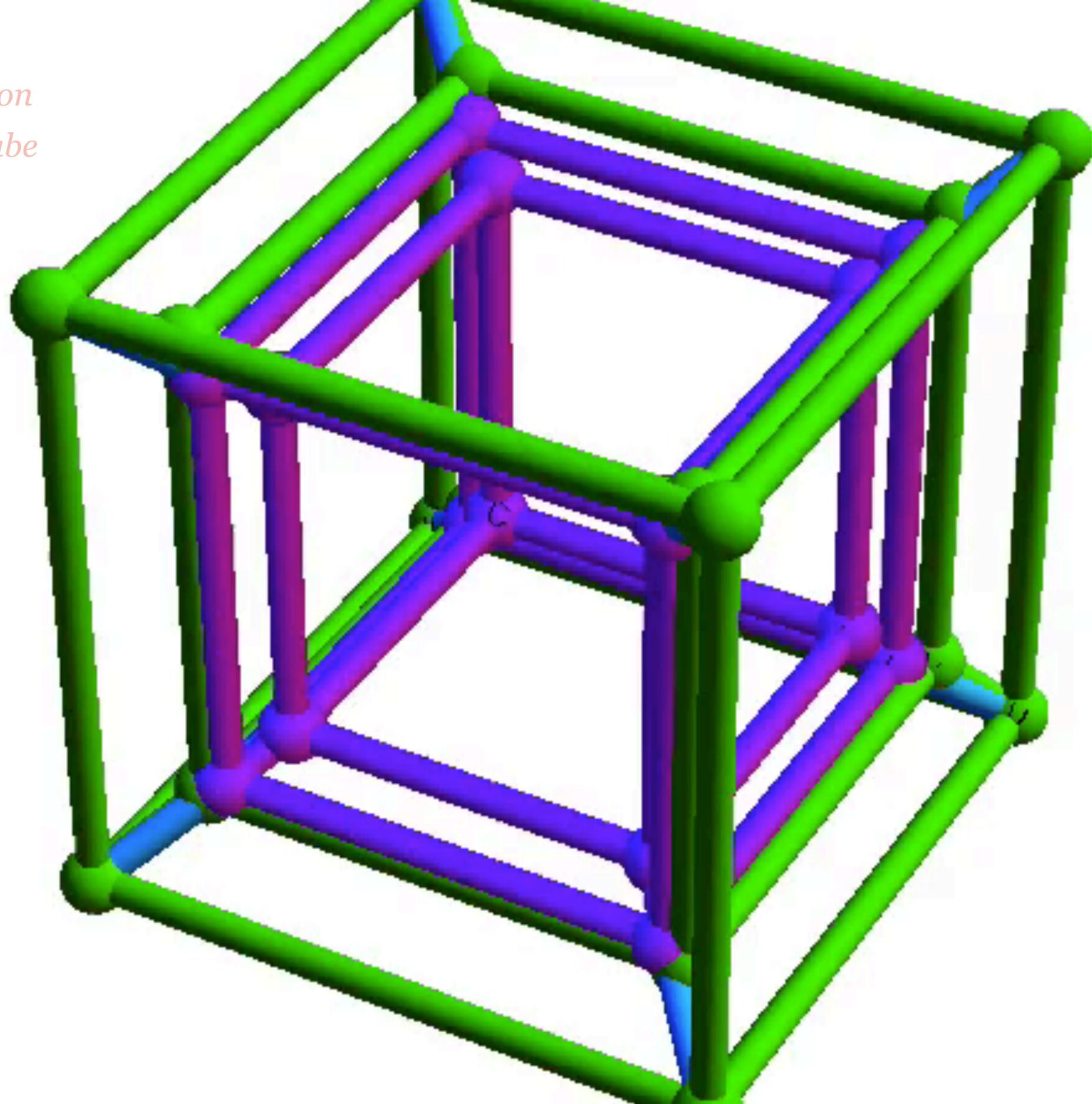
$$2-1-4-1+1+ \quad 1+3-0+4-4 = 1$$

Higher Dimensions

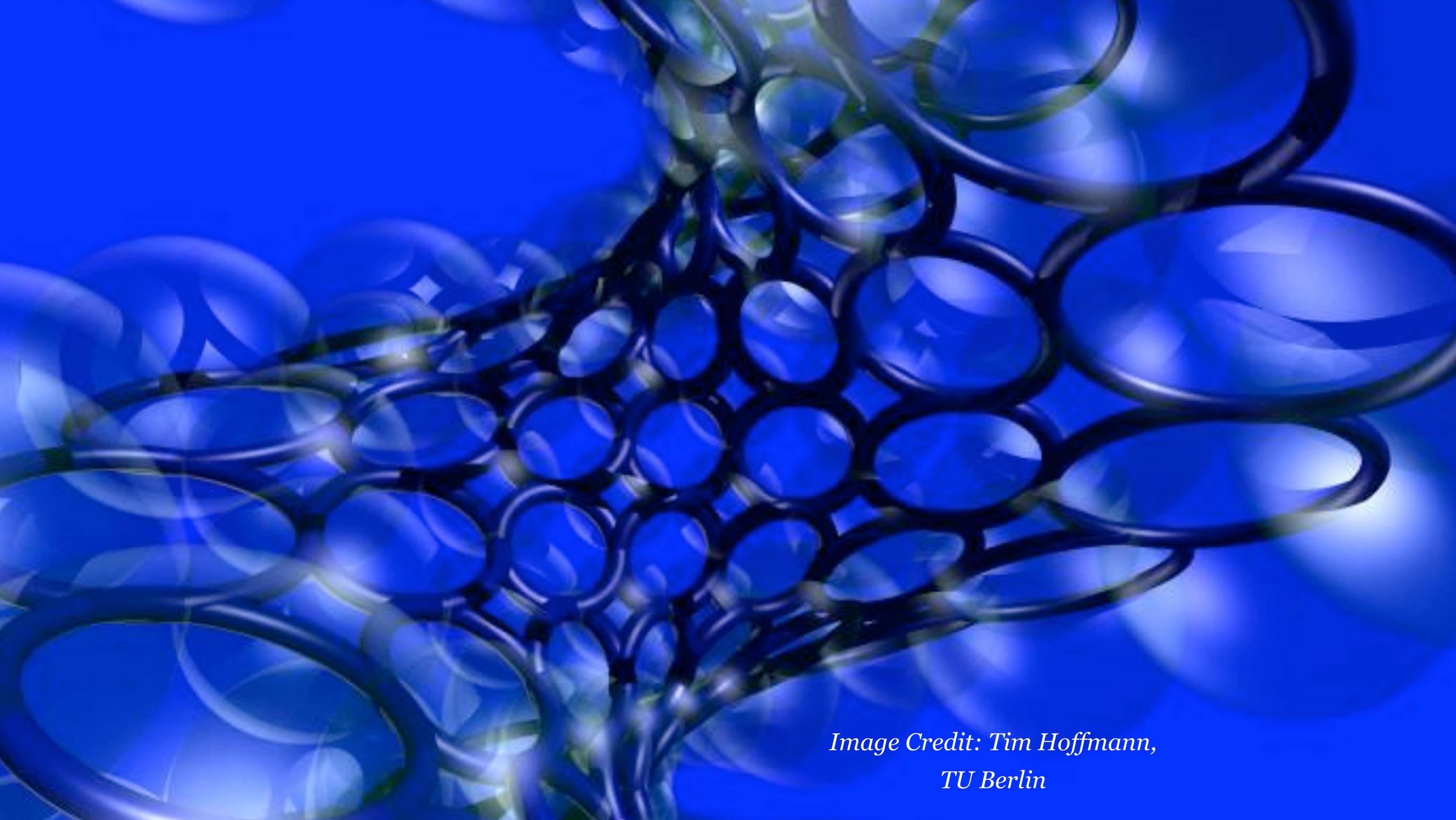
120 cell



*Polyteron
hypercube*

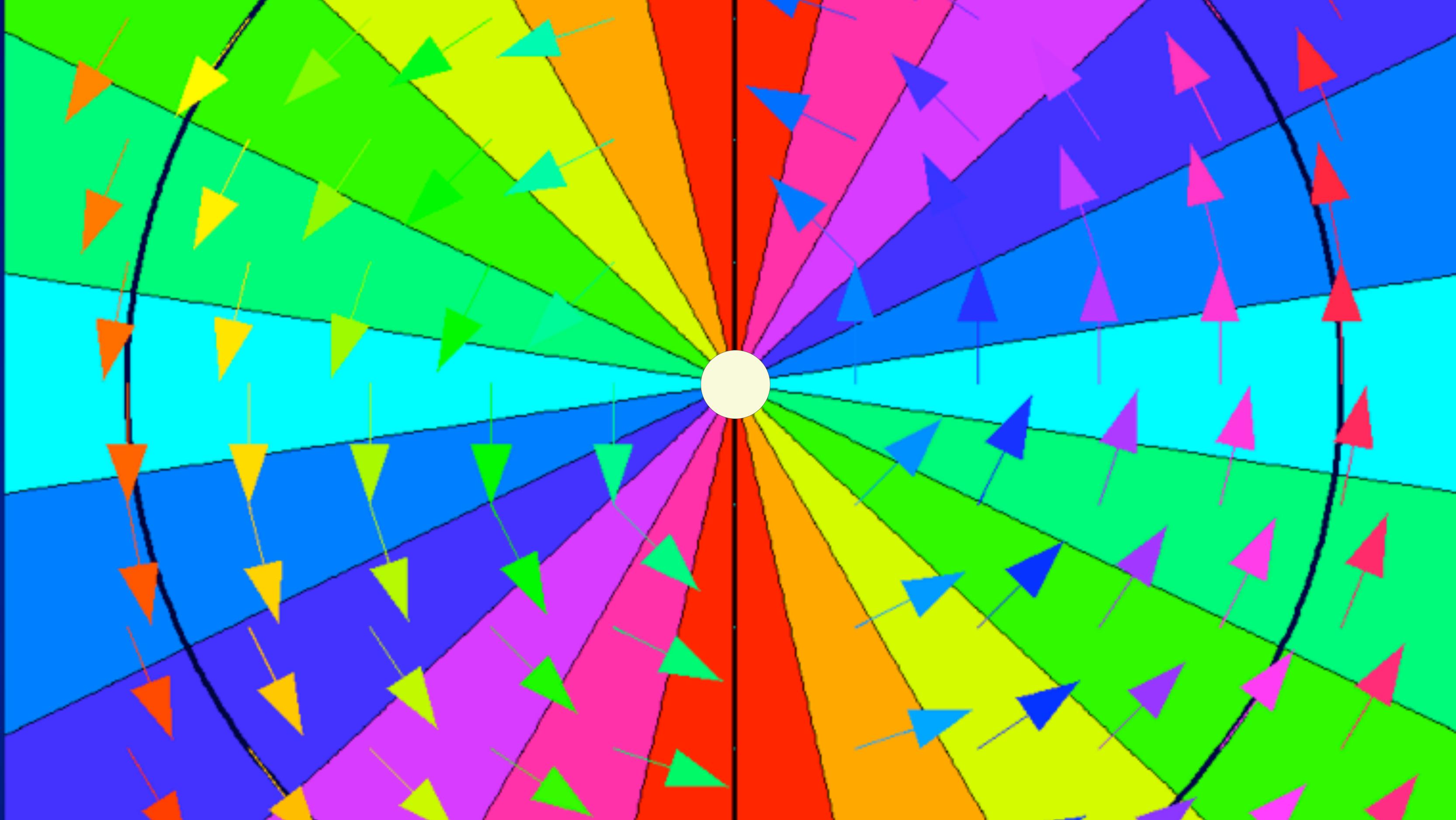


Infinite Dimensions



*Image Credit: Tim Hoffmann,
TU Berlin*

Generalized Functions



*Finally: Thanks to
our CAs!*



Tali Wong



Marco Hansel



Carol Zhang

A night cityscape with fireworks exploding in the sky. The fireworks are in shades of blue, purple, and yellow. The city buildings are illuminated with warm lights, and some windows are lit up. The sky is dark, and the overall scene is festive and celebratory.

Office hours: usual
lecture time and usual
OH time.