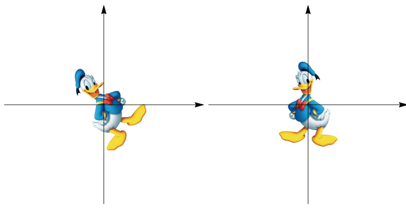
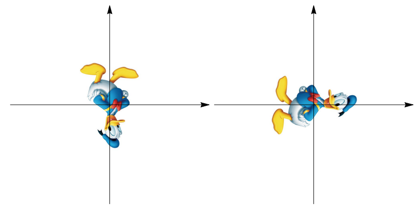


LINEAR ALGEBRA

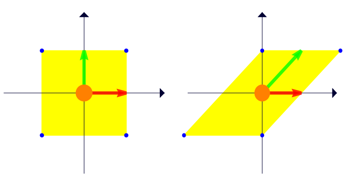
MATH 21B



Geometric Transformations

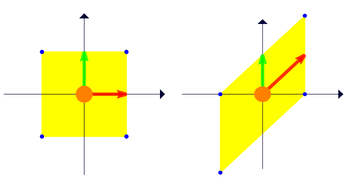
5.1. With linear transformations $T(\vec{x}) = A\vec{x}$ we can rotate, reflect, scale, twist or project. We look here at a few examples. Rotations and reflections are **orthogonal transformations**, preserving length and angle. Together with dilations, we have **similarity transformations** as seen in geometry. Projections are important role in statistics. Shears play a role in continuum mechanics for example. In each case, we link the geometric action with the algebra by looking at the columns of A.

5.2.



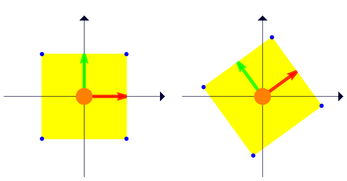
$$\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

Shear along x axes



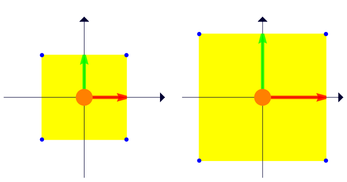
$$\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

Shear along y axes



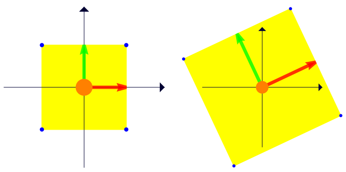
$$\begin{vmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{vmatrix}$$

Rotation



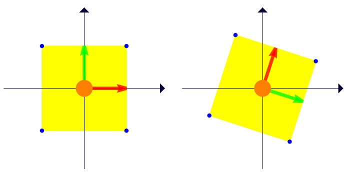
$$\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

Dilation



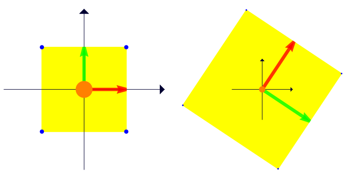
$$\begin{vmatrix} a & -b \\ b & a \end{vmatrix}$$

Rotation Dilation



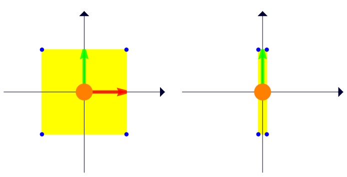
$$\begin{vmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{vmatrix}$$

Reflection at Line



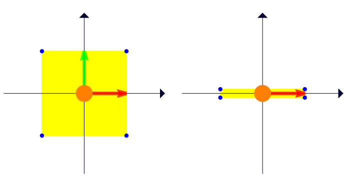
$$\begin{vmatrix} a & b \\ b & -a \end{vmatrix}$$

Reflection Dilation



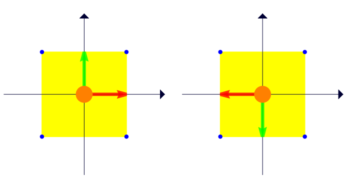
$$\begin{vmatrix} a^2 & ba \\ ab & b^2 \end{vmatrix}$$

Projection onto $\begin{bmatrix} a \\ b \end{bmatrix}$



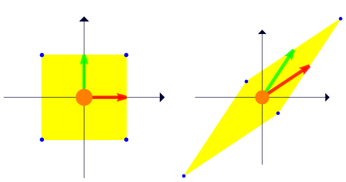
$$\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$$

Projection onto x Axes



$$\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$$

Reflection at Origin

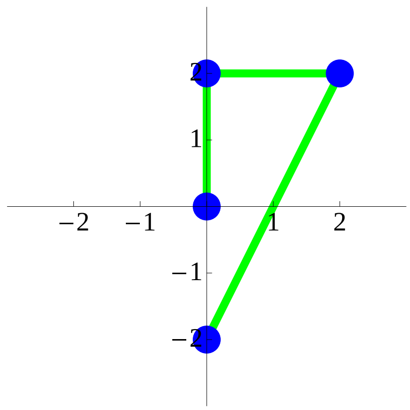


$$\begin{vmatrix} \cosh(\alpha) & \sinh(\alpha) \\ \sinh(\alpha) & \cosh(\alpha) \end{vmatrix}$$

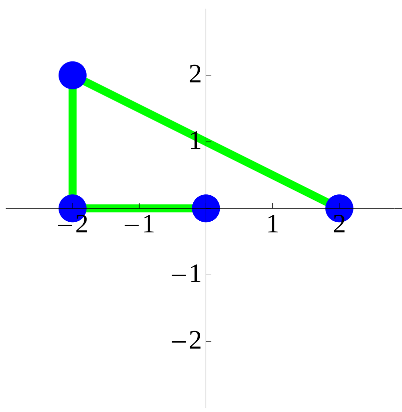
Lorentz Boost

Matching

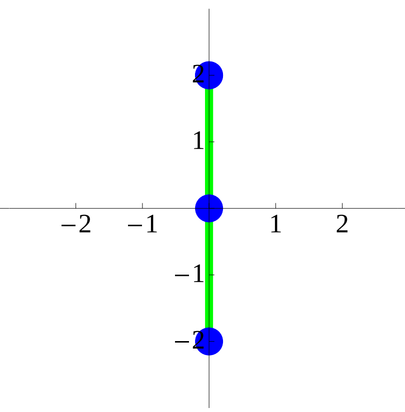
A polygonal figure through the points $(0,0)$,$(0,2)$, $(2,2)$ and $(0,-2)$ is exposed to a linear transformation $T(x) = Ax$, where A is a 2×2 matrix. Match the figures in the picture with the matrices



Enter 1)-6)	The matrix
	$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
	$A = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$
	$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
	$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
	$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
	$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$



1



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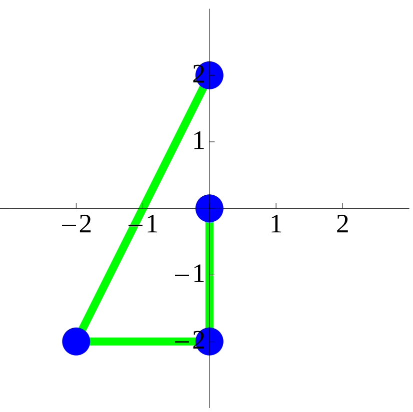

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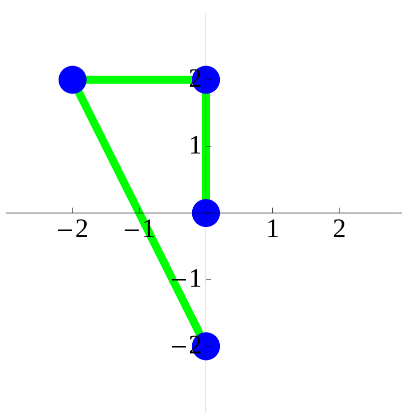

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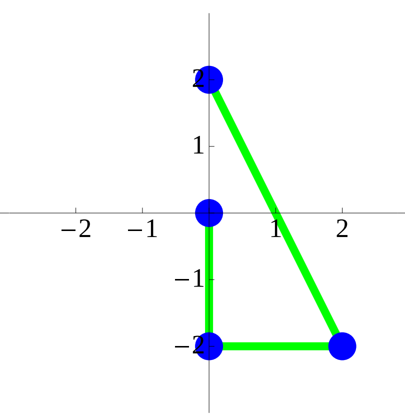

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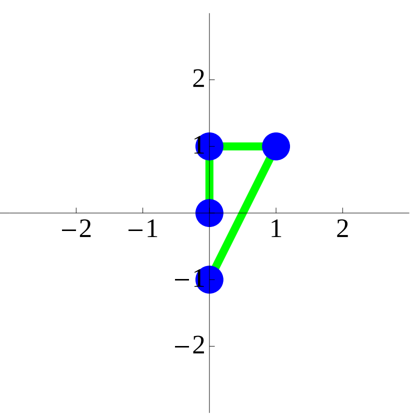

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