

LINEAR ALGEBRA

MATH 21B

CHECKLIST

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Main Skills

- Do Gram-Schmidt using inner product.
- Find Fourier coefficients of a piecewise smooth function.
- Find the sum of the squares of the Fourier coefficients.
- Solve the heat and wave equation.

Definitions

- Inner product $\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x) dx$.
- Norm of a function $\sqrt{\langle f, f \rangle} = \|f\|$ is a length or magnitude.
- Fourier series $f(x) = a_0/\sqrt{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$.
- Fourier basis $1/\sqrt{2}, \cos(nx), \sin(x)$ for piecewise smooth functions on $[-\pi, \pi]$.
- Coefficients of even part $a_0 = \langle f, 1/\sqrt{2} \rangle, a_n = \langle f, \cos(nx) \rangle$
- Coefficients of odd part $b_n = \langle f, \sin(nx) \rangle$.
- Sine series have coefficients $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) dx$.
- Heat equation $f_t = \mu D^2 f$ solution $\sum_{n=1}^{\infty} b_n e^{-n^2 \mu t} \boxed{\sin(nx)}$
- Wave equation position $f_{tt} = c^2 D^2 f$ solution $\sum_{n=1}^{\infty} b_n \cos(nct) \boxed{\sin(nx)}$
- Wave equation velocity $f_{tt} = c^2 D^2 f$ solution $\sum_n \frac{b_n}{nc} \sin(nct) \boxed{\sin(nx)}$

Words of wisdom

- Think about a function as a vector, Think about D as a matrix.
- "Odd functions have sin-Fourier series." "Even functions have a cos series."
- Fourier coefficients are coordinates of a function in a special basis.
- For partial differential equations, use $\boxed{\sin(nx)}$ terms only.
- The differential equations $f'(t) = \lambda f(t)$ is needed in the heat part.
- The differential equation $f''(t) = -c^2 f(t)$ is needed in the wave equation.
- Oh, PDE, oh PDE, solved easily with Fourier.

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¹Since second midterm.