The *Suàn shù shū* 筭數書 'Writings on reckoning':

A translation of a Chinese mathematical collection of the second century BC, with explanatory commentary

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Introduction

The *Suàn shù shū* 筭數書 is an ancient Chinese collection of writings on mathematics approximately seven thousand characters in length, written on 190 bamboo strips. It was discovered together with other writings in 1983 when archaeologists opened a tomb at Zhāngjiāshān 張家山 in Húběi 湖北 province. From documentary evidence this tomb is thought to have been closed in 186 BC, early in the Western Hàn 漢 dynasty. The occupant of the tomb - whose name is unknown to us - appears to have been a minor local government official, who had begun his career in the service of the Qín 秦 dynasty, but started work for the Hàn in 202 BC: see Péng Hào (2001) 11-12. The *Suàn shù shū* is anonymous, in the sense that we do not know the name of the person who assembled this material. A few sections of text are however marked with the common surnames Wáng 王 and Yáng 楊; whether these persons were merely scribes or were the actual authors of mathematical material is not clear.

The work discussed here was not the only one deposited in this tomb: in addition to material containing administrative regulations there were also writings on medicine and therapeutic gymnastics, all of which have been published and widely discussed elsewhere. The *Suàn shù shū* itself is certainly the oldest Chinese excavated text with substantial mathematical content. Moreover, it is considerably older than any other Chinese mathematical text now extant. Its role in the history of East Asian mathematics is comparable to that of the Ahmose (or Rhind) papyrus in the history of the mathematics of the ancient cultures bordering on the Mediterranean (see Chace (1979) and Gilling (1972)). The importance of the *Suàn shù shū* for the history of world mathematics is therefore indisputable.

I present here a translation of this material into English, together with a detailed commentary. This translation is based on the critical edition of the text which I have prepared on the basis of the published photographs of the strips in Zhāngjiāshān (2001). I have naturally prepared this publication with all the care and attention I could manage; however, after having carried out such a complex task it seems a good idea to pause long enough to allow specialist colleagues to suggest improvements and make criticisms, before committing the result to publication in final form. Another important consideration is that at the time this translation was completed scholars were awaiting the publication by Karine Chemla and Guō Shūchūn 郭书春 of a new French translation of the earliest Chinese mathematical text known to us before the discovery of the *Suàn shù shū.* That is the *Jiŭ zhāng suàn shù* 九章算術, 'Mathematical procedures under nine headings' or 'Nine chapters on mathematical procedures' - commonly known amongst Western scholars as the 'Nine Chapters'. This work is usually thought to have reached its final form around the first century AD, and has a number of parallels of content with the *Suàn shù shū*. I cite the Nine Chapters frequently in my commentary,and I have no doubt that my translation will benefit from comparison with the work of Chemla and Guō. Subsequent revisions of points of detail may take some time, although I believe that the version I offer here is already a generally fair and accurate representation of the nature and content of the *Suàn shù shū* collection.

So far as introductory material is concerned, I have confined myself here to the minimum discussion of context necessary to relate the *Suàn shù shū* to what is already known of ancient Chinese mathematics. I have said even less by way of intercultural comparison. Such matters will be more fully discussed in a substantial article I have prepared for journal publication. Meanwhile, given that it is already over twenty years since the tomb at Zhāngjiāshān was opened, I see no reason why historians of mathematics in general should not have access to this material without further delay. I therefore use the opportunity presented by the inauguration of the *Needham Research Institute Working Papers* series to make my work available for consultation and criticism.

I should like to offer my warm thanks to colleagues who have helped me bring this project to its present stage, whether through practical help or through scholarly advice. In alphabetical order, these certainly include Susan Bennett, Karine Chemla, Catherine Jami, Geoffrey Lloyd, Niu Weixing, and Nathan Sivin. I hope that all readers of the present publication will let me have their suggestions for improvement.

The historical context of the *Suàn shù shū*

Historians of mathematics who are not specialists in China may find it useful to read a few words of general historical introduction to the world in which the *Suàn shù shū* was compiled. There are of course many books on Chinese history that can be consulted for more detail, such as the relevant volumes of the authoritative *Cambridge History of China*.

China is often spoken of, not least by Chinese people, as having a culture whose roots are very ancient. This is however true only in the sense that amidst the major states of the modern world it is China that has the best claim for some effective cultural continuity through the changes of the last two millennia. We may dispute how dark the so-called 'Dark Ages' of western Europe really were, but pre-modern China certainly had no period of radical cultural, social and political disruption comparable to what happened (for instance) in post-Roman Britain.

But notwithstanding that, China as a unified and centrally governed imperial state is not very old compared to (say) the cultures of ancient Mesopotamia. The first emperor who unified the Chinese world under his government took the throne of his short-lived Qín 秦 dynasty in 221 BC. For the first time most of what is now modern China was governed by a civil service responsible to one man at the centre of things. For the preceding five centuries there had been no entity with a realistic claim to hegemony over the multitude of aristocratically governed feudal states into which China was divided. When we do go back far enough to find such a claim being realistically maintained, the hegemony of the rulers of the Western Zhōu 周

kingdom (c. 1045 BC – 771 BC) was nothing like as strong, fine-meshed and direct as that which the Qín inaugurated. And whereas the rule of the Zhōu kings over most of their territory was exercised indirectly through hereditary feudal vassals, the career officials appointed by Qín as governors of provinces and magistrates of counties reported direct to the emperor, and continued in office only subject to his approval.

So the world of the Qín was a new world, a world in which China as we see it today is for the first time discernible. After the Qín, the Hàn 漢 dynasty (206 BC to AD 221) ruled for more than four centuries. This dynasty is normally spoken of as divided into the Western (or Former) and Eastern (or Later) Hàn, separated by the interlude of the abortive Xīn 新 dynasty set up by a former Hàn minister from AD 9 to AD 23; these names are derived from the shift of capital from Cháng'ān (modern Xī'ān 西安) in the west to Luòyáng 洛陽 in the east.

Under the Hàn emperors a new class, the scholar-official gentry, grew up to serve the needs of the empire for competent and ideologically reliable administrators. The qualifications for their jobs included literacy, numeracy, and familiarity with an increasing mass of government regulations, but also the mastery of a core curriculum of ancient texts embodying the moral basis by which the emperors claimed to rule - what are now called the Confucian classics. The rich grave-goods in their tombs are witness to the prosperity brought to them by service to the emperor. For us today, the most significant point about Hàn funerary practices is that for at least the first two centuries of the dynasty officials were often buried with collections of books to take into the next world. Now before such collections began to be excavated in recent decades, there were only two ways of studying ancient Chinese literature. One was to rely on the texts transmitted to us initially through scribal copying and then from about 1000 AD through printing. The other was to look at the lists of titles in ancient bibliographies, in many cases referring to books now utterly lost. But now we can read the some of the actual writings that Hàn scholars read two thousand years ago. The *Suàn shù shū* is one of those.

The nature and origins of the *Suàn shù shū*: a sketch

At the time of completion of this material, I have in draft a major article on the question of the origin, nature and significance of the *Suàn shù shū.* Pending the publication of this, I shall give here only an outline sketch of the main points in my views of these questions, omitting most detailed argument and citation of evidence.

(a) Content

Firstly, let us consider a summary of the contents of the present collection, as tabulated below. The 'Groups' identified here are formed on my own editorial initiative. It must be recalled, however, that the ordering of the strips as presented in my text and translation is not necessarily close to the ancient order, but follows the arrangement proposed by modern Chinese editors from Péng Hào onwards on the basis of the topics covered. As a result my groupings are only useful as a modern analysis of the content of an ancient collection that may have been quite differently ordered. I have added in the final column a note of the sections of the Nine Chapters which parallel parts of the *Suàn shù shū* .

GROUP	SECTIONS	CONTENT	PARALLEL CONTENT IN NINE CHAPTERS
Group 1: Elementary operations	1-8	Multiplying fractions; simplifying fractions; adding fractions.	1: *Fāng tián* 方田
Group 2: Sharing in proportion; progressions	9-17	Division of a mixed number by a whole number; subtraction of a fraction from a mixed number, and of one fraction from another; division of a common pool of profit or liability in ratio of contrbutions; the case of contributions in geometrical progression; restoration of an original amount repeatedly diminished in a given proportion to produce a given result; value of given amount of commodity, given a unit price.	1: *Fāng tián* 方田 3: *Cuī fēn* 衰分 6: *Jūn shū* 均輸
Group 3: Wastage	18-19	Amount to be allowed to obtain a given amount of product in a process involving wastage; amount of wastage from a given initial amount.	3: *Cuī fēn* 衰分
Group 4: Sharing, contributions and pricing.	20-26	Sharing; reaching a total through contributions at different rates; cost of some quantity from price of a given amount; interest payable for given time on basis of monthly rate.	6: *Jūn shū* 均輸 3: *Cuī fēn* 衰分
Group 5: Changes in rates	27-29	Correcting error in tax rate; change in product from different amount of raw material; change in amount of tax given change in taxable amount.	---
Group 6: Rating by unit	30-33	Calculation of unit rate from price of given amount; ingredients in mixture of given total amount.	2: *Sù mǐ* 粟米

Group 7: Wastage and equivalents	34-37	Allowance for wastage in drying; exchange of one commodity for another.	2: *Sù mǐ* 粟米 3: *Cuī fēn* 衰分
Group 8: Allowing for mistakes	38-39	Dealing with use of incorrect tax rate by changing nominal area of field.	--
Group 9: Converting grains	40-47	The use of standard ratios to calculate amount of one type of grain equivalent to another type, or amount of product when grain is processed; problems of sharing and mixing involving grains.	2: *Sù mǐ* 粟米
Group 10: Rationalising and checking tasks	48-51	Calculation of a rate of unit production from rate at which parts of production task are completed; expected production of processed from raw silk; checking time taken for journey from cyclical days.	6: *Jūn shū* 均輸
Group 11: Rule of false position	52-54	Use of Rule of False Position to solve problems of sharing and mixtures, and extraction of approximate square root.	7: *Yíng bù zú* 盈不足
Group 12: Shapes and volumes	55-61	Calculation of the volume of various 3-dimensional shapes.	5: *Shāng gōng* 商功
Group 13: Circle and square	62-64	Relative dimensions of a square and its inscribed circle.	(But note the brief reference in the *Zhōu bì*, in Liú and Guō 2001, 35-36)
Group 14: Sides and areas with mixed numbers	65-69	Calculation of unknown side of rectangle, given area and one side; divisions involving the sum of several different unit fractions; multiplication of mixed numbers; interconversion of area units.	1: *Fāng tián* 方田 4: *Shǎo guǎng* 少廣

Although the content of the *Suàn shù shū* is clearly quite closely related to that of the Nine Chapters (a topic that will be resumed later), the two texts are far from identical in coverage. In the first place, we may note that certain broad mathematical topics included in the Nine Chapters are completely absent from the *Suàn shù shū*. From the table it is evident that there are no major parallels with the last two of the Nine Chapters:

8: *Fāng chéng* 方程; this deals with what in modern terms would be called the solution of sets of linear equations in several unknowns, using methods equivalent to determinants.

9: *Gōu gǔ* 句股; all the problems in this chapter involve applications of the Pythagoras theorem.

There are no indications of a knowledge of such techniques in the *Suàn shù shū*. Other significant omissions are also evident. Taking the Nine Chapters in order, we may identify major absences from the *Suàn shù shū* as follows.

Chapter	Section of Nine Chapters	Omitted by *Suàn shù shū*
1	方田 *Fāng tián* 'Rectangular fields"	Mean value of a group of fractions; areas of plane figures other than rectangles and circles (such as triangles); areas of spherical caps.
2	粟米 *Sù mǐ* 'Millet and rice'	Problems of finding combinations of prices of goods purchased in one lot.
3	衰分*Cuī fēn* 'Proportional distribution'	No major omissions.
4	少廣 *Shǎo guǎng* 'The lesser breadth'	Algorithm for extraction of square roots; algorithm for extraction of cube roots; volume of sphere.
5	商功 *Shāng gōng* 'Consultations on works'	Volumes of *yángmǎ* 陽馬 and *biēnáo* 鱉臑 (special forms required for systematic application of Liú Huī's volume dissection techniques); volume of square pyramid (although the more complex *chútōng* 芻童, a frustum of a rectangular pyramid, is treated, as is the circular cone).
6	均輸 *Jūn shū* 'Equitable transport'	The conspicuous omission here is the basic concept of *Jūn shū* "equitable transport" itself, that is the administrative technique of apportioning tax liability by taking account of population and the distance over which the delivery of tax has to be made. As is well known, arrangements of this kind date back no further than 110 BC under Hàn

		Wǔdì. Apart from the extensions of the *Jūn shū* principle to the fair sharing of labour tasks, other omissions include problems of pursuit and mutual approach, for instance by travellers at different speeds.
7	盈不足 *Yíng bù zú* 'Excess and deficit'	No major omissions.
8	方程 *Fāng chēng* 'The rectangular array'	Absent.
9	勾股 *Gōu gǔ* 'Base and altitude'	Absent.

There are no mathematical procedures used in the *Suàn shù shū* that are not discussed in the Nine Chapters. On the other hand, one particular problem type in the *Suàn shù shū* does not appear in the later work, and that is the "error correction" problem seen in Group 8. Likewise the frequent references to the *chéng* 程 "Norm" in the *Suàn shù shū* are not typical of the Nine Chapters. They do however strongly recall the flavour of the Qín administrative regulations recovered from the Shuìhǔdì 睡虎地 Qín tomb which are discussed by Hulsewé (1985), and indeed section 36 on the conversion rates between different types of grain is largely word for word identical to parts of the Qín text: see Shuìhǔdì (2001), 29-30, and Hulsewé (1985) 61 on 'norms'.

(b) The problem of the Nine Chapters in relation to the Suàn shù shū

Clearly the *Suàn shù shū* is sufficiently different from the Nine Chapters to cause us to hesitate about treating each book as simply a different recension of the other. In addition, there is a further aspect which does not emerge from a tabulation of content, and that is the aspect of form and style. From the points in my translation where examples from the Nine Chapters are cited as parallels to the *Suàn shù shū*, it becomes obvious that the Nine Chapters is a highly regularised text in which we meet the same small-scale patterns of text (such as the sequence problem/result/method) and even the same wording over and over again. In the *Suàn shù shū* on the other hand we may meet the same thing said in two different ways on the same strip of bamboo. And although the original order of the *Suàn shù shū* is irrecoverable, there is no conceivable shuffling of the strips that could produce anything like the larger-scale structure of the Nine Chapters, whether within individual chapters or at the level of the inter-relation of the chapters.

So what might the relations of the Nine Chapters and the *Suàn shù shū* be? Obviously the question of dating will be a crucial factor in any evidence-based answer to that question. What can be said about the dating of the *Suàn shù shū* has already been said: it was found in a tomb that was probably closed in the early second century BC, and contains language that in part resembles laws known to have been in force under the Qín dynasty a little before 200 BC. The only positive evidence which might help date the Nine Chapters is of two kinds. Firstly, we know that its earliest commentator, Liú Huī 劉徽 worked around AD 263 (*Suí shū* 16, 404). That fixes a date by which the Nine Chapters must have been in something like its present form. Secondly, we have a number of instances in ancient texts where scholars before Liú Huī are referred to in connection with the Nine Chapters. The earliest of these is a certain Mǎ Xù 馬續 the son of Mǎ Yán 馬嚴 (AD 17-98) and elder brother of the famous scholar and commentator on the classics Mǎ Róng 馬融 (AD 79-166). Hence he presumably flourished c. AD 110-120. According to the *Hòu Hàn shū* 後漢書 (History of the Later [=Eastern] Hàn dynasty, completed c. AD 450： 博觀群籍。善九章筭術 'He was widely acquainted with the mass of documentary sources, and excelled in the *Jiǔ zhāng suàn shù*' (*Hòu Hàn shū* 24, 862). As for negative evidence, the bibliographical monograph of the *Hàn shū* 漢書 (History of the [Western] Hàn dynasty, completed c. AD 92) draws on a listing of the contents of the imperial library made close to 5 BC. It contains references to various (now lost) books on mathematics and related topics, but has no title that suggests the presence of the Nine Chapters. In the face of this evidence, it seems safest to assume that the Nine Chapters was not in existence much before the beginning of the Christian Era. I discount here the story given by Liú Huī himself in the preface to his commentary, in which he tells of the creation of the Nine Chapters by a sage statesman of the early Zhōu dynasty a little before 1000 BC, and its alleged destruction by the Qín dynasty as part of an effort to wipe out political dissent by obliterating historical records - an event which if it did occur at all certainly spared all books on useful subjects (which would include the content of the present text) as well as those held by officials of state (which must have included material on administrative calculations of the type that constitutes most of the Nine Chapters). He continues the story by claiming that the Nine Chapters was then laboriously reconstituted from fragments by a number of figures famous for mathematical skills in the Western Hàn, but whose surviving biographical details make no mention of the Nine Chapters. As I shall argue in detail elsewhere, it is most likely that Liú Huī is simply attempting to reconstruct a likely history of the Nine Chapters to fill the void in the historical record before the first century AD. Taking the evidence overall it seems probable that the Nine Chapters was in fact put together shortly after the Christian Era, as part of a more general effort to edit and reorganise ancient materials.

(c) From the Suàn shù shū to the Nine Chapters?

It seems therefore that about two centuries separate the *Suàn shù shū* from the Nine Chapters. While the contents of these texts clearly represent parts of a single tradition of doing mathematics - the parallels are too close to make any other hypothesis plausible - the differences of form and style are also very striking. But any attempt to construct a historical narrative linking the two texts has to confront major difficulties, including the following:

(1) If we take the *Suàn shù shū* and the Nine Chapters as representative of the actual states of mathematical literature in the early Western Hàn and Eastern Hàn, we have to explain how the practice of mathematics and the modes of transmission of mathematical knowledge can have changed sufficiently to move us from a world in which the *Suàn shù shū* was seen as normal to one in which the Nine Chapters were seen as normal.

(2) However, we have rather little direct information on the activities of those who used mathematics in the intervening centuries, or on the social framework within which they operated. The only ancient attempt at writing something like the history of mathematics - Liú Huī's preface to his 3rd century AD commentary on the Nine Chapters - is, as we have seen, unlikely to give us much help in addition to what we can piece together from other more general sources.

(3) Further, although there is plenty of evidence that the Nine Chapters was widely seen as the central text of mathematics from the second century AD onwards, we have no idea how representative the *Suàn shù shū* might have been when it was laid in the tomb at Zhāngjiāshān in 186 BC. How do we know that the person who compiled this text was not an anomalous example of a mathematical scribbler with no sense of order or intellectual discipline? Of course the picture is not quite so stark as that: the magpie habits of the compiler of the *Suàn shù shū* give us a certain amount of evidence that there were a number of different mathematical collections available to him, and the nature of the material he draws from them is such as to suggest that they too were made up of relatively short units of text. But none the less a single surviving collection is not the strongest basis on which to rest the Western Han end of our narrative.

These problems are not trivial ones, and might well cause one to hesitate before trying to construct a history of Han mathematics capable of bridging the gap that confronts us. It is therefore very fortunate that we already have the main outlines of the history of another technical field that showed major changes in the writing down and transmitting of knowledge between Western and Eastern Hàn - changes similar to those that would take us from the *Suàn shù shū* to the Nine Chapters. The field in question is that of medicine. In medicine, as in mathematics, we find ourselves contemplating the impact of major finds of manuscript material from early Western Hàn tomb deposits, in the context of a canonical literature that cannot be reliably traced back further than the start of the Christian era.

For a description and evaluation of the major portions of the Western Hàn medical material, the reader may turn first to the work of Harper (1998), to which may be added the research results of Vivienne Lo embodied in her unpublished PhD thesis. A pioneering and original discussion appeared in Yamada (1979), and there is also the important study by Sivin (1995). But for our present purpose it is the work of Keegan (1988) that is most relevant. What Keegan's pioneering study achieved was to give us a new picture of what technical medical literature was like under the Western Hàn, and to elucidate some of the ways the Western Hàn heritage was transformed in succeeding centuries to produce the canonical literature of the received tradition.

In summary, Keegan's study of a group of Western Hàn medical manuscripts, mainly the so-called "vessel texts" from the Mǎwángduī 馬王堆 tomb, led him to the conclusion that the elementary unit of that literature was not to be seen as the "text" in its usual meaning of an extensive piece of writing equivalent to what we would call a book, but rather what we may call a "textlet", a shorter piece of writing capable of being transmitted on its own. Different extended texts might contain overlapping but not identical collections of textlets, and one text might separate textlets contiguous in another text, while bringing together textlets separated in other collections. In studying medicine, one increased one's knowledge base in part by receiving more material from a variety of teachers. who might sometimes pass on textlets only after an imposing ritual requiring a commitment not to transmit them to the unworthy - a process reconstructed and indeed evidenced from historical texts by Sivin. Clearly different doctors within a given tradition of medicine would tend to have overlapping but often differently ordered collections of material at their disposal.

By the Eastern Hàn, however, Keegan claims that this process had led to the formation of more than one large ossified collections of material that was no longer subject to "textlet" transmission in the old way. This is his explanation of the origin of the different recensions of the so-called Huángdì 黃帝 'Yellow Emperor' medical corpus of which signs appear for the first time in the *Hàn shū* bibliography. From the Sui dynasty onwards, editions of these recensions began to be edited by scholars and provided with commentaries. By the early Táng, we can be fairly sure that three of these recensions had reached something close to the form in which we have them today. These were the *Huángdì nèijīng sùwèn* 黃帝內經素問, the *Huángdì nèijīng língshū* 黃帝內精靈樞, and the *Huángdì nèijīng tàisù* 黃帝內經太素. An examination of these works shows that they do indeed embody much material common to one another, and that some of their contents closely resemble 'textlets' from the Mǎwángduī material - but in an order and arrangement different enough to witness to the ability of the 'textlet' to be transmitted independently.

These three extant representatives of the Huángdì corpus are by no means chaotic works, although any attempt to read them as deliberately composed and systematic treatises will lead rapidly to a sense of confusion on the part of the conscientious reader. However more systematised works do exist: one of them, the *Huángdì jiǎ yǐ jīng* 黃帝甲乙經 'Huángdì's ABC canon' was composed by Huángfǔ Mì 皇甫謐 around AD 256-282, so that he might have been a contemporary of Liú Huī. This uses material from the Huángdì corpus, edited and re-arranged to give a systematic account of acupuncture and moxibustion. We may also note the existence of a major text which is held by some to stand outside the Huángdì corpus, the *Nán jīng* 難經 'Canon of difficulties': this is a highly formalised and ordered book in which 81 sections each raise a question in the form of a 'difficulty' which is then answered. With the Nine Chapters in mind, we may note that 9 x 9 = 81. It is usually held that the *Nán jīng* dates from the second or first centuries AD, since it is quoted shortly thereafter. Another famous work that dates to just before the Eastern Hàn is also divided into 81 sections: this is the *Tàixuán jīng* 太玄經 of Yáng Xióng 楊雄 (53 BC - AD 18), a work of cosmology and divination intended to rival the *Yì jīng* 易經 'Book of Change': see Nylan (1993).

Turning back indeed to the Nine Chapters and its relation to the *Suàn shù shū*, it does seem that a pattern similar to the one sketched above for the case of medicine can be detected. Even without the work of the scholars mentioned here, an inspection of the *Suàn shù shū* suggests that in one technical field, that of mathematics, the independently circulating unit of knowledge in the early Western Hàn - the "molecule of written information", so to speak - was a textlet, often written on a single bamboo strip, rather than an extended and orderly treatise. In many cases the textlets of the *Suàn shù shū* take the form of a complete section beginning with a title. In other cases, a section with a title contains what is clearly a deliberately collected group of related textlets, which make it plain that an individual scribe has felt that gathering textlets in this way was just what his reader would expect him to have done. The frequent duplications and repetitions that result from this practice indicate how much the accumulation of transmitted textlets was valued for its own sake, even if each addition to the collection nowadays seems to add little or nothing to the sum of mathematical knowledge already gathered. Here of course we are speaking of what we can deduce from the activity of the final and apparently anonymous hand to work on this material in an editorial capacity, ignoring any intervening process of simple copying.

Unlike the case of the medical texts, however, the presence of the names of Wáng and Yáng on some strips gives us the chance to look back a little further than the manuscript itself, though our ability to draw any reliable conclusions is reduced by our inability to say just what sort of people they were, and what roles they played. But we can note that the way their names appear does not suggest any association with long passages of text: the longest continuous passages marked with their names are the two 'weaving problems', at the end of which we are told that they were 'checked' *chóu* 讎 by Wáng and Yáng respectively. This pattern is consistent with the notion that at this period a mathematically active individual was more likely to generate mathematics in the form of textlet sized packages that to write a discursive treatise. In other words, Western Hàn mathematicians managed their knowledge bases in ways similar in part to the practices of contemporary doctors.

The appearance of the Nine Chapters in the early Eastern Hàn may thus be seen as parallel to the emergence of such works as the Huángdì corpus, the *Nán jīng* and the *Huángdì jiă yĭ jīng* at the same period. To get from the *Suàn shù shū* to the Nine Chapters we need to apply the same kinds of transformations that take us from the Mǎwángduī material to the more systematic medical texts of the Eastern Hàn. Reversing the argument, we may say that given the existence of the Nine Chapters in the Eastern Hàn, a manuscript such as the *Suàn shù shū* is just the kind of mathematical text we would expect to find from the early Western Hàn.

This is not the time to attempt a systematic review of the opinions of Chinese scholars on the origins of the *Suàn shù shū* and its connections with the Nine Chapters. Briefly, however, we may say that two main currents of thought are clear. One of them, represented by the leading historian of mathematics Guō Shūchūn 郭书春 takes Liú Huī's account fairly literally (see Guō 2003). It is assumed that the Nine Chapters actually did exist before the Qín, and that it was damaged or scattered and later reconstituted as Liú Huī tells us. The question therefore arises whether the *Suàn shù shū* is in some way in the true line of descent that leads to the Nine Chapters, and the answer is negative. The polymath historian of science Lǐ Dí 李迪 on the other hand discounts Liú Huī as a reliable chronicler, and believes that mathematical knowledge in the Western Hàn circulated in the form of what he calls *guān jiǎn* 官簡 'official bamboo strips'. It was from such material that the Nine Chapters was assembled and edited (Lǐ 1997, 88-138). Although Lǐ Dí 's views were expressed before the text of *Suàn shù shū* was fully published and widely discussed, he has recently stated that they remain basically unchanged (private communication, 2004). It will be seen that my views are closer to those of Lǐ than to those of Guō; the comparison with the medical literature is one for which I must however bear responsibility.

It will no doubt take some time to unfold the full implications of the evidence considered here on the early history of Chinese mathematics. Even when we seem to be close to finding new answers to our old questions, the answers seem to lead on to new questions in turn. The contrast between the *Suàn shù shū* and the Nine Chapters certainly reveals aspects of the latter that would never have become evident from inspection of that text in isolation. Consider for instance the now increasingly obsolete *idée reçue* that the material of the Nine Chapters (as opposed to Liú Huī's commentary) reveals a mathematical style totally concerned with the practical and administrative obsessions of officials. Interest in mathematics for mathematics' sake, it is thus assumed, is first shown by Liú Huī himself in his commentary. Now however it is the *Suàn shù shū* that seems to be in large measure the practical book for the use of officials, while the Nine Chapters, even before Liú Huī, looks like a book for mathematicians. If we examine the material in the Nine Chapters that adds to the mathematics of the *Suàn shù shū*, this point becomes stronger. It is very hard indeed to see any practical point whatsoever in the multiple unknown problems that the *Fāng chéng* chapter solves by what are essentially matrix methods. Nor, somewhat to my own surprise, does a re-reading of the Pythagorean problems of the *Gōu gǔ* chapter succeed in suggesting any realistic situation in which an official would find the knowledge provided of any use at all. What bureaucrat would want to know about the dimensions of doors left ajar, broken bamboos in ponds or creepers winding up a tree? These are surely mathematical problems for people who are interested in mathematics for mathematics' sake. That is in general far from the spirit of the *Suàn shù shū*, which sticks to the relentlessly useful almost all the time. Almost, but not quite. Without having been a Qín official, it is difficult to be sure whether the more complex problems about amounts of grain, and elaborate arrangements about sharing and and mixtures requiring treatment by the Rule of False Position really do reflect any practical needs likely to be met in the course of one's work. However there is no doubt in my mind that in the pair of "weaving" problems found in sections 15 and 21 we are faced by texts where the main interest is in mathematical structures rather than any conceivable administrative reality. And since it is notable that these are the two cases where named persons are linked with problems in the clearest and most formal manner, it does seem evident that behind all the bureaucratic machinery with which much of this material is concerned, at least two persons near the beginnings of imperial China were for at least part of the time interested in displaying their ability to create (or at least to pass on) mathematics whose interest was more technical than practical.

The intriguing but unanswered question that then presents itself is who the audience for such mathematical virtuosity might have been, what criteria that audience operated in deciding what counted as good mathematics, and what rewards followed from a reputation for mathematical skill beyond the call of official duty. That we are currently far from being able to guess at. There is however one feature strongly marked in the case of medicine that does not appear to have been associated with mathematical learning. In Western Hàn medicine we have some evidence (both external to the texts and internal to them) that those who passed on these medical writings sometimes did so on the basis that the recipient was not to reveal them to others, or at least not to anyone 'unworthy'. The *Suàn shù shū* gives us no indication that such sanctions operated in the field of mathematical knowledge. We may perhaps see something a little similar in the dialogue of Chén Zǐ and Róng Fāng in the *Zhōu bì*, where Chén Zǐ goes through a ritual rejection of his student's approaches until he has extracted a confession of total ignorance and a humble request for instruction (see Cullen 1996, 176-178). But once instruction begins no promise of secrecy is imposed. One might perhaps suppose that a reputation for possessing secret knowledge was less likely to be important to a calculator than to a healer. For the healer, patient confidence and morale were often nine-tenths of the battle; and these might often depend crucially on a reputation as the trusted student of great predecessors. But for the calculator the publicly verifiable fact that one's calculations gave the right answers might be thought more important - though intellectual lineage no doubt counted for a good deal.

That is my first point on the historical problem of the transition from the Suàn shù shū to the Nine Chapters. My second relates to the aims of mathematical study in the Hàn dynasty. To characterise what these were, let us make an East-West comparison that will be useful, though perhaps a little crude. It was the fate of the Nine Chapters to be spoken of in later centuries as a paradigmatic work which summed up the essential spirit and content of mathematics in China. The same thing, more or less, happened to Euclid's *Elements* in the West, and with about as little justification, as can easily be seen by considering the contrast between Euclid and (for instance)

Diophantos or Heron. But even stereotypes may tell us something interesting, and there is an illuminating contrast here.
To make the contrast, let us start with Euclid. The *Elements*, as we know, treat mathematics with a well-defined programme in mind, which may in part be described as follows. We start from the smallest possible number of statements which the author has to ask us to accept as true. From these we attempt to derive logically the largest possible number of true propositions. So impressively is this programme executed that it is not surprising that some of Euclid's later readers were tempted to think that this was what all 'real' or 'true' mathematics should be like, and that anything else was in some sense a falling-short. Now judged in that way, the Nine Chapters is a lamentable failure to do real mathematics at all. But whenever we find ourselves thinking that some writing from another time and place is not up to our own exalted standards, we should ask ourselves whether we have understood what it is trying to do. It is after all rather unlikely that the compiler of the Nine Chapters worked with an eye on what Alexandrian geometers had been up to. There is in fact strong justification for thinking that the Nine Chapters had an aim that was in some sense orthogonal to that of Euclid: whereas the Elements sought to move from a few assumptions to a potentially unlimited number of true propositions by logical deduction, the Nine Chapters sought to move from the infinite variety of mathematical problems to the smallest number of general algorithms that could solve them all, grouped under the nine main headings of its chapters. Indeed the claim made on behalf of the Nine Chapters by its first commentator Liú Huī in the 3rd century AD was even more ambitious:

> 事類相推，各有攸歸，故枝條雖分，而同本幹者，知發其一端而已
> The categories under which the matters [treated herein fall] extend each other [when compared], so that each benefits [from the comparison]. So even though the branches are separate they come from the same root, and one may know that they each show a separate tip [of the same tree]. (Preface to Nine Chapters, in Guō 1990, 177)

The implication here is that, properly understood, all the problems in the book are in essence solved by the one and the same method.
But in the same way that you cannot get to the *Elements* in one step, you cannot get to the Nine Chapters in one step. I leave it to my Hellenist colleagues to sketch how we can see the project of the *Elements* in a process of inchoation before the work of Euclid. To conclude now, I shall point to the fortunate fact that we have a piece of meta-mathematical writing that probably dates from the first half of the Hàn dynasty, and which sums up the essentials of the work in which Chinese mathematicians were then engaged that led them to the creation of the Nine Chapters on the basis of collections such as the *Suàn shù shū*.
The material in question comes from the *Zhōu bì*, the early Chinese mathematical and astronomical collection of which I published a translation some years ago. In part of this book, a part which I argue is from the first century BC, a teacher, Chén Zǐ is represented as telling his student Róng Fāng how to learn mathematics. It is notable that as in the case of the dialogue between Socrates and his host's slave-boy in Plato's *Meno*, the first piece of Chinese writing about mathematics is in the form of a conversation between master and student:

> 陳子曰·然·此皆算術之所及·子之于算·足以知此矣·若誠累思之·[...] 子之於數·未能通類·是智有所不及·而神有所窮·夫道術、言約而用博者·智類之明·問一類而以萬事達者·謂之知道·今子所學·算數之術·是用智矣·而尚有所難·是子之智類單·夫道術所以難通者·既學矣·患其不博·既博矣·患其不習·既習矣·患其不能知·故同術相學·同事相觀·此列士之愚智·賢不肖之所分·是故能類以合類·此賢者業精習智之質也·
>
> Chén Zǐ said "Yes, these are all things to which calculation procedures can attain. In regard to mathematics, you have the ability to understand these matters, if only you give sincere and repeated thought to them [...] In relation to mathematics, you are not as yet able to generalise categories [i.e. categories of problems]. This shows there are things your knowledge does not extend to, and there are things that are beyond the capacity of your spirit. Now in the procedures of the Way [that I teach], illuminating knowledge of categories [is shown] when words are simple but their application is wide-ranging. When you ask about one category and are thus able to comprehend a myriad matters, I call that understanding [my] Way. Now what you are studying are the procedures of reckoning [算數 *suàn shù* as in the title of the collection we are discussing]. and this is what you are using your understanding for. But still you have problems, which shows that your understanding of the categories is too simple. The difficult part about understanding the Way, is that when one has studied it, one has to worry about broad application of it. Once it has been broadly applied, one has to worry about not [being able to] put it into practice. Once one has put it into practice, one worries about not being able to understand it. So similar procedures are studied comparatively, and similar problems are comparatively considered. This is what sorts the stupid scholar from the clever one, and the worthy from the worthless. So being able to categorise in order to unite categories - this is the substance of how the worthy will devote themselves to refining practice and understanding. (Cullen 1996, 175-178, from which this translation is slightly modified)

I have quoted this passage at length because I believe it sums up the spirit of early Hàn dynasty mathematical learning that led to the formation of such a systematic and powerful book as the Nine Chapters, in which 'words are simple but their application is wide-ranging [and] when you ask about one category [you] are thus able to comprehend a myriad matters'. That is what I believed when I first translated those words of Chén Zǐ, at a time when the *Suàn shù shū* was just being unearthed. Now I have seen the *Suàn shù shū*, I am all the more sure that Chén Zǐ's advice to his student embodies the authentic voice of the early Hàn mathematicians who assembled collections such as the *Suàn shù shū.* The aim of those who gathered the material in each section of that collection was precisely to bring together related problems and methods so that as Chén Zǐ says 'similar procedures are studied comparatively, and similar problems are comparatively considered.' Far from it being a puzzle that the Nine Chapters should have as a predecessor the *Suàn shù shū*, it turns out to have been what one would have expected. And that is a satisfying way to be able to resolve a problem in any sphere, including the history of mathematics.

BIBLIOGRAPHY.

Pre-modern works

[The listing of editions used is given here for convenience of reference. In the case of all Hàn and pre-Hàn texts, readers will find substantial discussions of content, text history and associated questions in Loewe (ed.) 1993.]

Jìn Shū 晉書 'History of the Jìn dynasty' (AD 265—419) by Fáng Xuánlíng 房玄齡, c. AD 648, Beijing 1974 edn.

Jiǔ zhāng suàn shù 九章算術 'Nine Chapters on mathematical procedures' compiled c. AD 100, edn. of Guō Shūchūn (1990); see also edition in Guō Shūchūn 郭书春 and Liú Dùn 刘钝 (2001) [translated in Shen (1999) and Chemla & Guō (forthcoming)].

Hàn shū 漢書 'History of [Western] Hàn dynasty' by Bān Gù 班固, largely complete on his death in AD 92, Beijing 1962 edn.

Hòu Hàn shū 後漢書 'History of Later [i.e. Eastern] Hàn dynasty' by Fàn Yè 范曄 , c. AD 450, Beijing 1963 edn.

Sān guó zhì 三國志 'History of the Three Kingdoms (AD 221-280)' by Chén Shòu 陳壽 c. AD 290, Beijing 1962 edn.

Shǐ jì 史記 'Records of the Historian' by Sīmǎ Qiān 司馬遷 c 90 BC, Beijing 1962 edn.

Suí shū 隋書 'History of the Suí dynasty' (AD 581-617) by Wéi Zhēng 魏徵 AD 636 (monographs added AD 656), Beijing 1973 edn.

Yì jīng 易經 'Book of change' originally a manual of divination (pre-Qin), with later accretions dating as late as W. Han, Sìbù cóngkān edn.

Modern works

Chace, Arnold Buffum, (1979 repr. of first publication 1927/9) The Rhind mathematical papyrus : free translation and commentary with selected photographs, transcriptions, transliterations and literal translations, Reston, Va, Nat. Council of Teachers of Mathematics.

Chemla, Karine (2003) 'Generality above Abstraction. The General Expressed in Terms of the Paradigmatic in Ancient China's Mathematics' Science in Context 16.3 413-458.

Cullen, Christopher (1996) *Astronomy and Mathematics in Ancient China: the Zhou bi suan jing.* Cambridge, University Press.

Duàn Yàoyǒng 段耀勇 and Zōu Dàhǎi 邹大海 (2003)《算数书》"以睘材方", "以方材睘"两问校正, 自然科学史研究 2003.2: 168-172 [Reviews interpretations of these two sections, and proposes new emendations].

Gillings, Richard J. (1972) *Mathematics in the Time of the Pharaohs,* Dover, New York.

Guō Shūchūn **郭书春** (1990) **九章算術** Shenyang [Critical edition of the Nine Chapters]

Guō Shūchūn **郭书春（2001）《筭数书》校勘，中国科技史料** 22 (3) 202-219. [Collation notes on the text of Wén wù transcription. "The author thinks that the ways for the expression of fraction in Suanshushu are not the same, which is an intrinsic phenomenon. It was only when Zhang Cang in the Hàn dynasty and other people sorted out Nine Chapters on Mathematical procedures that such ways were standardised. Therefore it is not possible for us to take the model of the latter to modify the former."]

Guō Shūchūn **郭书春**（2003）《**筭數書》初探, 國學研究** 11:307-349. [A general outline of the mathematical content of the text; comparison with the content of mathematical texts later than the Nine Chapters; distinctive features of the work; the composition of the work and its connection with the Nine Chapters. Conclusions: this work contains some features more advanced than some texts already known. The disorderly nature of the work, with repetitions and internal inconsistencies, suggest it was compiled from a number of sources without any systematic editing. In the cases where material in this text bears a resemblance to the Nine Chapters, differences of wording are quite large. So while this text bears some 'blood relationship' to the Nine Chapters, it cannot be an earlier form of that work.]

Guō Shūchūn **郭书春** and Liú Dùn **刘钝**（2001），**算经十书。九章出版社，台北** [Critical edition of the texts of ten mathematical classics, including the Nine Chapters]

Harper, Donald J. (1998*), Early Chinese Medical Literature: The Mawangdui Medical Manuscripts*. Sir Henry Wellcome Asian Series. Royal Asiatic Society, London.

Heath, Sir Thomas L. (1956*) The Thirteen Books of Euclid's Elements*, Translated from the text of Heiberg, with introduction and commentary. 2 vols. Dover, New York (repr.)

Horng Wann-sheng **洪萬生** (2000) **算數書》初探，師大學報。科學教育類** 45 (2): 77-91. [Preliminary survey on the basis of the Wenwu transcription. Suggests that the lack of proof in "method" sections of Nine Chapters points to "sole interest in practical calculation", whereas SSS "offered syntactical connectives ... in order to make a conclusive argument."

Horng Wann-sheng **洪萬生**（2001）**《算數書》的幾則論證，台灣歷史學會會訊** 11: 44-52. [Develops further the points on argument etc, in Horng 2000.]

Horng Wann-sheng **洪萬生**（2002）**再論 《算數書》** HPM **通訊** 5.2 & 5.3, 1-30. [Textual notes and annotations on the whole of the SSS, with other material, and a bibliography of SSS research up to 2001.]

Huang, H.T. (2000) *Science and Civilisation in China* vol 6 part 5, Cambridge

Hulsewé, A.F.P. (1985) *Remnants of Ch'in Law*. Brill, Leiden

Imhausen, Annette (2003) 'Egyptian Mathematical Texts and Their Contexts' *Science in Context* 16.3 367-389.

Jì Zhìgāng 纪志刚 (2004) 赞叹与遗憾: 读彭浩著《張家山漢簡〈筭數術〉註釋》, 自然科学史研究 2004.1: 91-96 [Review of Péng Hào 彭浩 (2001) " ... Although Peng's book is really praiseworthy in providing facilities for further study, the numerous defects in it make readers feel regrettable indeed"]

Jiānglíng 江陵 [anonymous collective work by team in charge of Jiānglíng documents] (2000): 文物 Wénwù 2000.9 78-84 江陵张家山汉简《算数书》释文 ' Jiānglíng Zhāngjiāshān Hànjiǎn 'Suàn shù shū' shìwén' 江陵张家山汉简《算数书》释文 [English title given in original publication as 'Transcription of Bamboo Suan shu shu or A Book of Arithmetic from Jiānglíng'. This preliminary transcription uses simplified characters and does not mark divisions between bamboo strips.]

Jochi Shigeru 城地茂 (2001):《算數書》日本語訳, 和算研究所紀要 4: 19-46. [A translation of the *Suàn shù shū* into Japanese]

Keegan, David (1988), ' "The Huang-ti nei-ching": The Structure of the Compilation; The Significance of the Structure', PhD thesis in History, University of California at Berkeley

Lam, Lay Yong and Ang, Tian Se (1992) *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China*, Singapore.

Lǐ Dí 李迪 (1997) 中国数学通史: 上古到五代卷 Nanjing. [History of Chinese mathematics from early times to the Five Dynasties]

Lǐ Dí 李迪 (1982)《九章算术》争鸣问题的概述 in 吴文俊 (1982), 28-59 [Review of controversies on the dating of the Nine Chapters]

Liú Dùn 刘钝 (1993) 大哉言数 Shenyang [On topics from the history of Chinese mathematics, with strong historical and cultural context provided]

Loewe, Michael (ed.) (1993), *Early Chinese texts: a bibliographical guide*, Cambridge

Netz, Reveil (2003) 'Introduction; The History of Early Mathematics - Ways of Re-writing' *Science in Context* 16.3 275-286.

Nylan, Michael (1993) *The Canon of supreme mystery by Yang Hsiung* [= *Yang Xiong*]*; a translation with commentary of the T'ai hsüan ching* [=*Tai xuan jing*] Albany, New York.

Péng Hào 彭浩 (2000) 中国最早的数学著作《算数书》, 文物 2000, 9, 85-90. [Preliminary discussion of the *Suàn shù shū*, to accompany the transcription in Jiānglíng 2000.]

Péng Hào 彭浩 (2001) Zhāngjiāshān Hànjiǎn «Suàn shù shū» zhùshì 張家山漢簡《筭數術》註釋 'The Hàn dynasty book on wooden strips *Suàn shù shū* found at Zhāngjiāshān with a commentary and explanation' Beijing, Science Press [Transcription of the text in full characters, with commentary and prefatory essay. Reviewed in Jì Zhìgāng 纪志刚 (2004)].

Péng Hào 彭浩 (2002), 张家山汉简《算数书》的"并祖"与"启丛(纵)", 考古 2002.5: 85-87. [Suggests that some *Suàn shù shū* procedures absent from the Nine Chapters are present in Liú Huī's work]

Shen, Kangshen (1999), with Crossley, John N. and Lun, Anthony W.-C. (trs). *The Nine Chapters on the Mathematical Art: Companion and Commentary*, Oxford, the University Press.

Shuìhǔdì 2001 [anonymous collective work by team in charge of Shuìhǔdì documents] (1990, repr. 2001) **睡虎地秦墓竹簡整理小組《睡虎地秦墓竹簡》文物出版社** Beijing [Photographs of all the Shuìhǔdì material, with transcription and textual notes.]

Sivin, Nathan (1995) 'Text and Experience in Classical Chinese Medicine' in *Knowledge and the Scholarly Medical Traditions*, ed. Don G. Bates, pp. 177-204. Cambridge University Press.

Struik, D.J. ed. (1986) *A Source Book in Mathematics* 1200-1800, Princeton

Unguru, Sabetai (1975) 'On the Need to Rewrite the History of Greek Mathematics' *Archive for History of Exact Sciences* 15, 67-114

Wú Wénjùn 吴文俊 (1982) 《九章算术》 与刘徽 Beijing [Discussion of Liú Huī's work on the Nine Chapters]

Wú Wénjùn **吳文俊** (1986(a)) **吳文俊文集** Jinan [Collected papers]

Wú Wénjùn **吳文俊** (1986(b)) 'Recent Studies of the History of Chinese Mathematics' *Proceedings of the International Congress of Mathematicians* Berkeley, 1657-1667.

Yamada Keiji (1979). 'The Formation of the Huang-ti Nei-ching'. *Acta Asiatica*, 36 : 67-89.

Zhāngjiāshān **張家山** [anonymous collective work by team in charge of Zhāngjiāshān documents] (2001) **張家山漢墓竹簡 【二四七號墓】文物出版社 北京** [Photographs of all the Zhāngjiāshān material, with transcription and textual notes.]

Zōu Dàhǎi 邹大海 (2001) 出土《算数书》初探，自然科学史研究 2001.3: 193-205. [Abstract: "...the Suanshu shu is a combined book with a least two origins; there are no direct influences between the texts of the Suanshushu and the Nine Chapters on Mathematical Procedures, which have the same origins in Pre-Qin Period; serving to the lower government administration, many parts of the Suanshu shu were written in the light of a certain mathematics book (or its ramification) in Pre-Qin Period which had changed into the primary origin of the Nine Chapters on Mathematical Procedures."]

Zōu Dàhǎi 邹大海 (2003) 从《算数书》和秦简看上古粮米的比率,自然科学史研究 2003.4: 318-328 [Considers the comparative values of grains in the Nine Chapters, *Shuō wén*, *Suàn shù shū* , together with Shuihudi Qin strips. "The kinds of rice husked from millet and paddy were belonged to two series, which cannot be confused. The documents on the ratios of different rice of the Shuowen and the Nine Chapters did not directly accept the influence of the Qin's bamboo strips and the Suanshushu."]

Translation and commentary

Principles and patterns of translation

This translation is based on my own critical edition of the *Suàn shù shū* text, which is presented and described later in this monograph. In view of the comments of one helpful colleague on a previous draft, I should like to make it clear that my use of the word 'text' in referring to the material I translate is purely a matter of convenience. I certainly do not mean to imply that the Chinese writings on which I have worked were intended as a single integrated body of writing by whoever assembled them before they were entombed. It is partly to avoid that implication that I have taken advantage of the possibility of translating the word *shū* 書 in the title as 'writings' rather than as 'book'.

It is not easy to translate ancient Chinese mathematics into modern English. I would have preferred avoid all unnatural English in my translation, but on the other hand I have felt impelled to attempt to represent something of the structure and flavour of the original language, including its specific mathematical vocabulary and syntax - and even the clearest mathematical writing in modern English is hardly natural in comparison with everyday speech. The tension between these aims cannot always be concealed in the version I offer here. It may therefore be helpful to begin with some remarks on the problems that such a text can present to a translator.

My initial aim in beginning my commented translation was to reconstruct an understanding of this material as near as possible to that likely to have been formed by a reader of the second century BC, the period during which it was entombed. Using the word 'aim' in such a context may however be over-optimistic, since it suggests that one is trying to hit a target that is clearly visible. The fact is that apart from the *Suàn shù shū* itself we have little direct and datable evidence of how Chinese people thought about mathematics at the period in question. It is not until the early second century AD, three centuries after the entombment of the *Suàn shù shū* that we can be sure of the existence of another mathematical text that is still extant today - the Nine Chapters. And it is not until the third century AD that we find in the commentary of Liú Huī 劉徽 the first firmly datable Chinese discursive writing on mathematical procedures by a historically identifiable individual. Perhaps the aim needs to be rephrased to something more modest: to use the act of translating the *Suàn shù shū* as an occasion for reconstructing at least a partial picture of the mathematical thinking of China in the second century BC.

What principles might one bear in mind to guide on in this task? A number of scholars have already reflected on the problems posed by the study of ancient mathematical texts. In China we have the example of the mathematician turned historian of mathematics, Wú Wénjùn 吴文俊, who argued that in attempting to reconstruct the way in which the results stated in an ancient text were arrived at, one should respect three basic principles:

> '1. The reconstruction should accord with the circumstances of mathematical development in the relevant place and time, and must not make use of results or methods that are modern, or originate elsewhere.
> 2. The reconstruction should be based on historical facts and historical material, and must not be concocted on the basis of mere imagination.
> 3. The reconstruction should lead naturally to the desired result or equation, and must not involve artificial elaborations designed to led to a predetermined conclusion.' (Wú 1986(a), 53-73, translated here from the original Chinese)

In a publication in English, Wú reduced these principles to two, which though conceived in a specifically Chinese context are capable of easy generalisation (Wú 1986(b) 1657):

> '1. All conclusions drawn should be based on original texts fortunately preserved up to the present time.
> 2. All conclusions drawn should be based on reasonings in the manner of our ancestors in making use of knowledge and in utilizing auxiliary tools and methods available only at that ancient time. [...] The use of algebraic symbolic manipulations or parallel-line drawings should be strictly forbidden in any deductions of algebra or geometry since they were seemingly non-existent in ancient Chinese classics.'

These points in themselves seem to me a good and sufficient guide to avoiding positive error in approaching material such as the *Suàn shù shū* . But it is also interesting to see what Western scholars have had to say on related questions. For that purpose, we may conveniently start with the recent special issue of *Science in Context* (vol 16, 2003) devoted to ancient mathematics. Particularly relevant to our concerns are the general introduction to the issue of translating and interpreting ancient mathematics by Reveil Netz (2003), and the article of Annette Imhausen (2003) on Egyptian mathematics. Both of them refer back to the often-cited study of Unguru (1975) as a paradigmatic text in the methodology of the history of mathematics, and it is to Unguru that I shall now turn. Unguru's paper develops its attack along two main axes:

> (1) A criticism of the specific claim that the work of certain ancient Greek geometers can and should be interpreted as concealing within its ostensibly geometric formulation a form of algebra - 'geometrical algebra' - and that as a result the geometrical propositions concerned can and should be translated into modern symbolic algebra.
>
> (2) A more general criticism of the historiography of mathematics practised by (in Unguru's view) modern mathematicians, who he believes are all too liable to assume that there is a supra-historical

> entity called 'mathematics' to which they have privileged access, and 'exactly because this content (like the inert gases) is essentially unaffected by its formal surroundings, the ability of the modern mathematician to uncover it and give it a 'palatable' (i.e. modern) form constitutes not only the best modern reading of ancient 'burdensome' and 'oppressive' mathematical texts but also the only correct reading, and at the same time, the proof that this is what the ancient mathematician had in mind when he put down (in an awkward fashion, to be sure) for posterity his mathematical thoughts.' (Unguru 1975, 73). For Unguru, a common way in which this false historiography is given effect in practice is by 'apply[ing] mechanically to [ancient] mathematics the manipulations and jugglings of modern mathematical symbolism' which amounts to 'betraying [ancient] mathematics, only by applying to it foreign categories of post-Renaissance mathematical thinking' (Unguru 1975, 111-112).

There is clearly a basic concord between Wú and Unguru, despite the fact that Wú is precisely the kind of historian of mathematics that Unguru might have expected to commit the faults exposed in his paper - an eminent professional mathematician who has turned to the history of mathematics in his later years. I find that the point of view expressed by these two historians in apparent ignorance of one another's work is consonant with my own experience in confronting ancient Chinese mathematical texts. I therefore intend to proceed as far as possible to interpret the *Suàn shù shū* in a way that avoids the pitfalls they indicate. In particular, I shall avoid the assumption that when I want to explain how the text solves its problems and describes its methods, all I have to do is to translate these into modern symbolic algebra. That does not mean that there will be no algebra at all in this study, for two reasons:

(1) Not all 'algebra' is modern symbolic algebra. The Arabic mathematicians such as Al-Khwārizmī (c. AD 825) who first used the Arabic original of this word mathematically were engaged in describing how to manipulate known quantities in order to find unknown quantities. They did this very effectively without writing down 'algebraic equations' in the modern sense of symbolic equations with unknowns designated by symbols that are manipulated by rule until the desired unknown is isolated. Thus where we would today write $x^2 + 10x = 39$, and proceed to solve for x by changing this equation into a number of different forms until we arrive at the form $x = 3$ (ignoring for the moment the other root $x = -13$), Al-Khwārizmī writes:

> '...what is the square which combined with ten of its roots will give the sum total 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this which is 8,subtract from it half the roots, 5, leaving 3. The number 3 therefore represents one root of this square ...' (Struik 1986, 58).

The *Suàn shù shū* certainly contains algebraic procedures in the sense that the word 'algebraic' can be applied to the work of Al-Khwārizmī, but not in the modern symbolic sense. The pleasant (but sometimes dangerous) thing about symbolic algebra is that as long as you observe the rules for manipulating the symbols on the page, you can often reach a result without having to think very much about what is actually happening to the numbers. But it is clear that to understand pre-modern non-symbolic algebra, it is essential to avoid assuming that it is enough to translate the problem into symbolic algebraic and find a solution by manipulation, since otherwise we shall not face up to the challenge of finding out how ancient mathematicians without symbolic algebra did their thinking.

2. The reader of this translation is not a Chinese person of the second century BC, not least in the respect that he or she does not have a second century BC Chinese teacher to introduce the material, demonstrate its application using counting rods, and so on. No doubt in the end any intelligent and diligent modern reader would be able to see what the text is doing without explanation. But most readers will need some kind of temporary scaffolding on which to construct such an understanding, and I have used modern symbolism for that limited purpose. Thus in the section on 'Excess and Deficit', I freely admit that when I first began to translate the Chinese into English I jotted down in the margin a translation into symbolic algebra in order to speed up my initial comprehension of the text and check that I was understanding it properly. But in my commentary I set out to construct an interpretation and justification of the methods underlying the text without any reference to symbolic algebra at all, and it seems to me that the results justify the additional labour for both author and reader alike.

Next, there is the question of numbers in general. I suggest that we should make a clear distinction between two questions:

> (a) How a reader of the *Suàn shù shū* would have set out numbers for the purposes of calculation.
> (b) How the scribes of the *Suàn shù shū* and their contemporaries recorded numbers when writing normal prose.

As for (a) the *Suàn shù shū* itself gives us no direct evidence. We do know from other texts that at the period from which the text comes practical calculation was carried out using bamboo rods arrayed on a surface which may perhaps have been marked with a grid. It is not until centuries later that we have clear descriptions of how these 'rod numbers' worked, but when evidence is available it shows us a system in which 'place value' operates - that is, the rod numeral for 5 may mean 5,000 in one position, but 500, 50 or 5 in others. In such as system, a gap in the array of numbers is clearly important, and needs to be preserved and represented by an appropriate symbol if the numerical array is to be recorded by writing on paper. In modern numerals, this is the purpose of the zero, '0'. When however numbers were written down in Chinese as part of normal prose, as in the *Suàn shù shū*, they were represented using a basic set of characters for the numbers 1 to 9, but without the use of a common marker for zero, so that simple place-value cannot be applied. Instead multiples of ten are specified in the number, so that the number 57,982 is written **五萬七千九百八十二** *wǔ wàn qī qiān jiǔ bǎi bā shí èr* literally 'five myriads, seven thousands, nine hundreds, eight tens, two' and 6003 would be 六千三 *liù qiàn sān* 'six thousands, three'. In pre-modern Chinese the distinction between words for numbers and figures to represent them therefore does not exist in normal writing. But faced with the choice posed by English usage, I have usually preferred for the sake of clarity to render number phrases in figures rather than using words - thus I write 153 rather than one hundred and fifty three for 一百五十三 *yī bǎi wǔ shí sān*. Apart from the digits 1 to 9 and the usual characters for 10, 100, 1,000 and 10,000, the *Suàn shù shū* also frequently uses the characters *niàn* 廿 for 20, *sà* 卅 for 30, and *xì* 卌 for 40, replacing the more regular forms *èr shí* 二十, *sān shí* 三十.and *sì shí* 四十 ('two tens', 'three tens', 'four tens'). These special forms are clearly simply groupings of multiples of the character for ten, 十 *shí*. There is also a special sign for 70 , 卄, of unknown pronunciation, but perhaps derived from the old form of the character for seven as used in the *Suàn shù shū* (a horizontal line with a short vertical bar through its mid-point) placed on top of the character for ten.

Let us turn to the basic operations of arithmetic. I suggest that we should not take the seemingly obvious step of asking asking how addition, subtraction, multiplication and division are represented, but rather begin by looking at the operative significance of common words used in specifying calculations. As will appear, there is not a simple one-to-one correspondence between these words and what a modern reader tends to assume must be the basic operations in all schemes of arithmetic in any culture.

We are frequently told that two quantities are to be 'combined' *bìng* 并, and from the results given it is clear that this process is effectively equivalent to the operation of addition. Thus for instance:

> Strip 33
> ... 并三人出錢數以為法 ... 'combine the number of cash paid out by the three men to make the divisor'

I have hesitated over whether this word should simply be translated as 'add', but have decided to leave it with the broader sense given here, since there are places where 'add' does not seems to work well (strip 36, strip 43, strip 117, strip 119)

We are also told from time to time that two quantities which are clearly being added must *xiāng cōng* 相從, which I have chosen to render literally as 'go with one another'.

> Strip 21
> 合分
> ... 母相類子相從 ... 'if denominators are of the same kind, numerators go with one another'

Once more, the results given show clearly that this is equivalent to addition.

The word *chú* 除 literally means 'lessen' 'remove' or 'take away': it is used when one quantity is to be removed from the other, for instance:

> Strip 17
> **約分**
> ... **以子除母** 'reduce the denominator by the numerator'
>
> Strip 31
> ... **以少除多** ... 'reduce the greater by the lesser'

In these and other cases, the operation of subtraction is taking place. In a few cases this word occurs in a context where it is clear that the subtraction is to be repeated as many times as possible, so that in effect we are dealing with division:

> Strip 78
> **絲練**
> ... **因而十二之除十六而得一** '... take and 12-fold it. Obtain 1 for each reduction by 16.'
>
> Strip 165
> ... **除積步如法得從一步** '... reduce the accumulated *bù* by accommodating the divisor to obtain 1 *bù* of length'; similarly on strip 167.

I have rendered *chú* **除** throughout as 'reduce', which seems to carry the ambiguity of the original quite well.The common expression for subtraction in later texts uses the word *jiǎn* **減**, but in the *Suàn shù shū* this occurs only once, and seems to bear the sense of 'make smaller' without the precise mathematical connotation of 'subtraction':

> Strip 13
> **增減分**
> ... **減分者增其母** 'to decrease a part [i.e. a fraction] increase its denominator'

Chéng **乘**, literally 'mount upon' occurs in such contexts as:

> Strip 12
> ... **十乘千萬也** ... 'ten mounting on a thousand is a myriad'

In all cases *chéng* appears, as here , to be equivalent to 'multiply', and I have so rendered it. The use of this word may come from the way that two rod numbers are arranged one above the other when multiplication takes place.

Faced with phrases such as *sān zhī* **三之**, literally '3 it', I have had to decide whether to be as literal as that, or whether to say 'triple it' instead. The former involves using a number as a transitive verb, which is not at all natural in English, while the latter would imply we had on occasion to go all the way up to 'nonuple' and 'decuple', which would be distinctly odd. Since the text specifically refrains from using the word *chéng* 乘 it seems illegitimate to use 'multiply'. For *sān zhī* and similar cases I have therefore decided to translate on the pattern '3-fold it', which seems to do the job as well as it ever can be done. At times the verbal use of numbers can become quite complex, as when (for instance) section 68 has 三之有三五之 '3-fold it, then 3 times 5-fold it', i.e. multiply by 3×5^3.

The final type of expression to be interpreted is exemplified in such phrases as:

> Strip 18
> … 各如法而成一 'as each [number] accords with the *fǎ*, form one'
>
> Strip 56
> … 實如法而一尺 … 'as the dividend accords with the *fǎ*, then one *chǐ*'
>
> Strip 33
> … 即以四錢各乘所出錢數如法得一錢 'then multiply each number of cash spent by four cash, and as [the results] accord with the *fǎ* get one cash'

The results given for these processes are in each case the number of times that the quantity referred to as the *fǎ* can be subtracted from the given number. This process will in general leave a remainder, which may be dealt with in the following fashion:

> Strip 58
> 如法一錢不盈以法命分 'as [the number] accords with the *fǎ*, then one cash. What does not suffice, denominate [those] parts by the *fǎ* [as denominator]'

Where possible I have preferred fairly literal renderings of such phrases as those given above, since to render X 如Y 而一 as 'divide X by Y' seems too much like a mere paraphrase rather than a translation. When division takes place, the two elements involved are usually referred to as the *shí* 實 literally 'full, solid' and the *fǎ* 法 'rule, measure, pattern'. The term *shí* is also used for the product of a multiplication. Effectively division consists of measuring the *shí* in terms of multiples of the *fǎ*. Since no translation (as opposed to paraphrase) of *shí* seems quite adequate, I have simply rendered *shí* and *fǎ* as 'dividend' and 'divisor' respectively.

The topic of division naturally leads into that of fractions. When faced with the standard form of expression *sān fēn zhī yī* 三分之一 , literally 'one of three parts' I have decided simply to write ⅓. It does seem reasonable to take the solidus line as relating 1 and 3 just as *fēn zhī* 分之 'of [ ] parts' relates *yī* 一 and *sān* 三 in the reverse order. The term *fēn* 分 has been translated as 'part' rather than 'fraction'; this seems closer to the usage of the text. As for the names for the numbers needed to specify a fraction, in the example just given 3 would be referred to as the *mǔ* 母, literally 'mother' and 1 would be the *zǐ* 子, literally 'child'. After consideration, a translation full of mothers and children seemed a little too bizarre, so I have simply opted for for 'denominator' and 'numerator' in every case. Where a particular fraction has its own special name, I have translated it in words rather than showing it as numbers and solidus. This applies to *bàn* 半 'half', *shǎo bàn* 少半 'diminished half', which is ⅓, and *dà bàn* 大半 'augmented half' which is ⅔.

Weights and measures

The units found in the *Suàn shù shū* are those commonly known from Hàn texts. For convenience they will be summarised here, in the groupings in which they are found in this text. In the present translation I have simply transliterated the names of these units rather than attempting to find modern English rough equivalents which might mislead the reader into thinking (for instance) that a *mǔ* was equivalent to an acre.

Length for general purposes other than land measurement

1 *zhāng* 丈 = 10 *chí* 尺
1 *chí* 尺 = 10 *cùn* 寸

From these arise units of area and volume, such as the square and cubic *chí*; these are not however distinguished from the length units by any special prefix.
In a few sections (36, 61 and 62) we find the special unit *wéi* 圍 used for measurements round the circumference of a circle: it appears that it is the same length as a *chí*. For comparison, a *cùn* (literally 'thumb', cf. French *pouce*) is about one modern inch or 23 mm., so that a *chí* is comparable with a foot.

Land measurement

For measuring the linear dimensions of a piece of land, the double-pace *bù* 步 is used throughout. Elsewhere in Hàn texts the *bù* is 6 *chí,* but at no time are *bù* and (for instance) *chí* interconverted in the present text. We frequently find land areas expressed in (square) *bù*. The *mǔ* 畝 (about 1/10 of an English acre) is also common, and one on occasion (section 68) we find the larger unit *qīng* 頃. The inter-relations of the three area units are:

1 *qīng* 頃 = 100 *mǔ* 畝
1 *mǔ* 畝 = 240 (square) *bù* 步

Section 68 also demonstrates the use of the largest-scale length unit, the *lǐ* 里for measuring large expanses of land, both as length and area. From the figures given there it is straightforward to deduce that in linear terms 1 *lǐ* is 300 *bù*.

Volumes of grains and liquids

1 *shí* 石 = 10 *dǒu* 斗
1 *dǒu* 斗 = 10 *shēng* 升

One *shēng* is about 200 cc, ⅕ litre, so a *dǒu* is 2 litres and a volume *shí* is 20 litres.

Mass

A complete set of mass units is set out in section 17, with the implication that:

1 *shí* 石 = 4 *jūn* 鈞
1 *jūn* 鈞 = 30 *jīn* 斤
1 *jīn* 斤 = 16 *liǎng* 兩
1 *liǎng* 兩 = 24 *zhū* 銖

Note that although 石 is nowadays read as *dàn* rather than *shí* when it is a weight, this reading has no ancient attestation. The Hàn dynasty *shí* was about 29.5 kg.in modern terms, which makes a *jīn* about a quarter of a kilogram.

Currency

In general all currency is reckoned in units of *qián* 錢, the standard copper coin, for which I use the conventional 'cash'. In section 29 however we also find the *suàn* 算, here used in the sense of a string of cash (a cord was threaded through the central square hole in each coin). There is no indication in this text of what was regarded as a standard number of coins per string.

Commodities and their names, principally grains

The problems and data in the *Suàn shù shū* relate closely to details of the social, economic and administrative life of early imperial China. They are therefore a fascinating resource for the wider history of China, well beyond the limits of the purely technical history of mathematics. I have tried to present translations which make it possible for the reader to think about the broader significance of the material before us and which make clear what manner of things are being made the subject of calculations, but without overloading the text with lexicographical scruples which would only be of significance to highly specialised scholars.

One aspect of this is my rendering of the various terms for grains, whose frequent occurrence underlines the importance of transactions and valuations in kind rather than in cash in the Western Hàn economy. Of all commodities, grain was paramount: official salaries were reckoned in *shí* 石 of grain rather than in money. Several different of kinds of food grain (in which I include both cereals and legumes) are mentioned in the *Suàn shù shū* . As noted above, I have decided to leave units of measurement in translated form since they are after all different in size from any modern western system. But since there are modern English names for all the grains cultivated in ancient China, I have tried to use them. For detailed discussions of the problems of identifying ancient Chinese grain names in modern terms, see for instance Huang (2000) 17-31.

A few issues of translation may conveniently be mentioned here:

粟 *sù* refers to grain that is in its unprocessed state, as removed from the ear by threshing and still bearing its husk (or hull). We may note, however, that in Western Hàn China it is very probable that the grain referred to by an isolated instance of this term was foxtail millet (*setaria italica*). The term for broomcorn or 'panicled' millet (*panicum mileaceum*), 黍 *shǔ* is found only on strips 88, 138 and 139. Grain as threshed cannot be used as human food since the husk is indigestible even when the grain is cooked. Hence at a minimum it must be subjected to a preliminary pounding, 舂 *chōng* which removes the outer husk or hull (see for example section 48). The result of this process is 米 *mǐ* which I render as 'hulled grain' in contrast to 粟 *sù* 'unhulled grain'.

Other common grains to which I have given translated names are:

麥 *mài* (Strips 43,44, 89, 90, 98, 99, 100, 102, 103, 109, 111): I have rendered as 'wheat' in all cases, although the term can also cover barley.

荅 *dá* (Strips 43, 44, 90, 109) I have rendered as 'beans'; it is probably the adzuki bean that is referred to. See SCC vol. 6 part 2, 515.

菽 *shú* (Strips 90, 109) 'soybean' (same reference, 511-514)

麻 *má* (Strips 90, 109) There is doubt whether this word in the Western Hàn refers to hemp or sesame seeds: see SCC vol. 6 part 5, 28-31. I have preferred to render as 'hempseed', on the grounds that hempseeds were identified as a food item in a Western Hàn burial (same reference).

禾 *hé* This is a problematic word in the *Suàn shù shū* . It occurs alone on strips 43 and 44, where it is evidently a crop in its own right, contrasted with wheat and beans. It is also found alone on strips 84 and 93. Strip 88 is headed 程禾 *chéng hé,* which apparently means 'The Norm for *hé*', but the strip itself has the combination 禾黍*hé shǔ* of which 1 *shí* is said to be equivalent to 16 ⅔ *dǒu* of unhulled grain. Strip 89 has 稻禾 *dào hé*, of which 1 *shí* is said to be equivalent to 20 *dǒu* of unhulled grain. On strips 109-110 we are told that 5 units of 禾粟*hé sù* are equivalent to 4 of 稻粟 *dào sù*.

Now *hé* on its own can simply mean 'cereals' in general, but if we need a specific meaning we can it seems choose between paddy rice and millet (see Huang 2000, 18-19 and 22). I am inclined to adopt a rendering of 'millet' for *hé* alone, since millet was the more common grain in the Western Hàn - although *hé* may of course mean different things in different places. As for the pairings, the last two make some potential sense in their context as 'unhulled *hé* grain' and 'unhulled rice grain'; we never see 稻 *dào* 'rice' on its own in the *Suàn shù shū* . In the case of the others, one can do little more than guess. In a literary context 禾黍*hé shǔ* has been attested as a general term for cereals, as 'rice and millet', but whatever it does mean here must be some specific grain with a particular exchange value. Thus one might speculate that 禾黍 *hé shǔ* may involve panicled millet, while 稻禾 *dào hé* involves rice again. For discussion of the significance of the quantitative statements made about all these grains as an aid to their identification, see the sections in question.

While many grains can be cooked and consumed once the outer husk has been removed, the grain may be rendered more palatable (though less nutritious) if the process of milling by pounding is continued until some or all of the outer coating of the grain itself has been removed. At each stage of processing the weight of grain remaining is somewhat reduced, and Hàn administrators were naturally anxious to know what reduction it was reasonable to expect, partly no doubt in order to know whether the milling was up to standard, and partly also in order to ensure that the wastage during milling was not used as a cover for petty theft. The Nine Chapters begins its second chapter with a complete list of equivalences of various types of grain and grain products, including millet grain at various stages of processing, for which the names and figures are as follows (I include the translations used in Shen (1999) 141:

粟	*sù*	50	[unhulled] millet
糲米	*lì mǐ*	30	hulled millet
粺米	*bài mǐ*	27	milled millet
糳米	*zuò mǐ*	24	highly milled millet
御米	*yù mǐ*	21	imperial millet

Similar sequences are found in the *Suàn shù shū* . Thus in section 36 (strip 88) we have the sequence

粟	*sù*	16 ⅔
糲米	*lì mǐ*	10
糳米	*zuò mǐ*	9
毀米	*huǐ mǐ*	8

Since the figures given for relative amounts are exactly one third of those in the first four positions of the Nine Chapters list, it is clear that the same four stages of processing are designated. But Nine Chapters' *bài mǐ* becomes *zuò mǐ*, and Nine Chapters' *zuò mǐ* becomes *huǐ mǐ*. However in section 40 (strips 98, 99, 100) we have a series of statements which imply a different sequence:

粟	*sù*
米	*mǐ*
粺米	*bài mǐ*
毀米	*huǐ mǐ*

The proportions implied by the calculations linking these four types are however identical to those between the first four types of the preceding two lists. It appears therefore that although between the two sections of the *Suàn shù shū* noted here and the Nine Chapters there was a consistent view of what quantitative changes should be seen at each stage of grain processing, there might be significant variations in what those stages were to be called. Given that state of affairs, it is clearly not possible to avoid simply transliterating the Chinese terms for at least the third and fourth stages of processing, since if they were simply rendered as 'milled millet' and 'highly milled millet' the difference in terminology would be obliterated. It also seems worth while to write '*lì* hulled grain' explicitly whenever this term is used as distinct from the simple *mǐ* for hulled grain.

Divisions of the text.

The topic of the divisions to be made in the text of this collection is discussed elsewhere. For the moment I shall simply explain what the reader can expect to find in the translation below. If we start at the beginning of the text, after the title there comes the heading '[Group 1: Elementary operations]'. This is placed in square brackets to show that like all such 'Group' headings it is an editorial insertion by myself, based on no more than the observation that the following material deals with a common theme. The fact that such groupings suggest themselves is of course simply a consequence of the fact that the modern Chinese editors decided to group similar material together when sorting the jumbled bamboo strips found in the tomb.

Next is the heading 'S1 (Yáng), S2 (long gap)'. This indicates that the following material is to be found on strips 1 and 2, and that the text in the following translation runs from one strip to the other without any obvious discontinuity, so that is is clear that the strips belonged together in the original arrangement when they were tied together with string. The name 'Yáng' is written below the lower node of strip 1, and there is a noticeable gap between the end of the writing on strip 2 and the lower node, suggesting that the end of a unit of text has been reached.

Next comes the heading '(1) Multiplying together'. The words here are the title given to this section of text in the original, and were originally written above the upper node of strip 1. The section number is an editorial insertion by myself, for convenience of reference. Within each section, I have identified and signalled with small letters in brackets, (a), (b), (c) etc., subdivisions that in my opinion are significant. In some cases I have done this to draw attention to obvious discontinuities of topic or diction between parts of a section that was physically continuous. Examples of this are to be found In section (7), where subsections (a), (b) and (c) are explicitly presented as three separate statements of the method for simplifying fractions, clearly from different sources though deliberately copied into the same section by the scribe. In a case such as section (10) however, the division into subsections (a) and (b) is based on the fact that the first subsection is written on strips 28 and 29, which ends with a gap, and the second begins at the top of strip 30 and ends with a gap at the end of strip 30, and ends with a gap at the end of strip 31. In such cases as this, I mark the numbers of the relevant strips before each subsection in turn, rather than together at the start of the section as I do when the text runs continuously.

I have had to make a decision about the rendering of another kind of division in the text, relating to to the conventional structural markers that recur in various versions throughout. One of the most common is the word 問 *wèn*, literally 'ask'. When this occurs I have rendered it as 'Question', followed by a colon, rather than as 'It is asked' or a similar phrase. In the same spirit, when confronted by 术曰 *shù* *yuē* literally 'the method says', and **得日** *dé yuē* literally 'what one gets says' I have translated as 'Method:' and 'Result:' on the grounds that *yuē* 'say' often has more of the nature of a punctuation mark indicating that another statement is following than a verb. Faced however with *yuē* on its own, rather than simply inserting a semi-colon I have wherever possible marked its presence by translating 'Reply:', which fits the context in almost every case, and is mindful of the common role of this word as marking the alternation of speakers in a dialogue.

Annotations

Where appropriate each section can be followed by annotations of three different kinds. 'Content' annotations bear on general points about the type of subject with which the section deals, and its relation to the subjects of other sections. 'Parallels' annotations discuss related material from other texts, principally the *Jiǔ zhāng suàn shù* **九章算術** 'Nine Chapters on the Mathematical Art'. 'Mathematical notes' focus on technical points relating to the calculations in the section in question. For the convenience of the reader, when quoting from the Nine Chapters I have given references to the critical text of Guō Shūchūn (1990) and to the translation and study of Shen Kangshen and other (1999). The latter has very rich illustrative citations giving parallels from other mathematical texts from China and also from the rest of the world.

筭數書
Writings on Reckoning

[Group 1: Elementary operations]

S1(Yáng),S2(long gap)

(1) Multiplying together

(a) a *cùn* multiplying a *cùn* is a *cùn*; multiplying a *chí*, [it] is one tenth of a *chí*; multiplying ten *chí*, [it] is one *chí*; multiplying a hundred *chí*, [it] is ten *chí*; multiplying a thousand *chí*, [it] is a hundred *chí*;

(b) a half *cùn* multiplying a *chí* is one twentieth of a *chí*; one third of a *cùn* multiplying one *chí* is one thirtieth of a *chí*; one eighth of a *cùn* multiplying one *chí* is one eightieth of a *chí*;

S3(Yáng),S4,S5,S6

(c) one half multiplying one is a half; multiplying a half, [it] is one quarter; a third multiplying one is one third; multiplying a half, [it] is one sixth; multiplying a third, [it] is one ninth;

(d) a quarter multiplying one is one quarter; multiplying a half, [it] is one eighth of a *chí*;

(e) a quarter *cùn* multiplying a *chí* is one fortieth of a *chí*; a fifth of a *cùn* multiplying a *chí* is one fiftieth of a *chí*; a sixth of a *cùn* multiplying a *chí* is one sixtieth of a *chí*; a seventh of a *cùn* multiplying a *chí* is one seventieth of a *chí*;

(f) multiplying a third, [that] is one twelfth; multiplying a quarter, [that] is one sixteenth;

(g) a fifth multiplying one is one fifth; multiplying a half, [it] is one tenth; multiplying a third, [it] is one fifteenth; multiplying a quarter, [it] is one twentieth; multiplying a fifth, [it] is one twenty-fifth;

(h) The method for multiplying parts: Denominator multiplies denominator to make the divisor; numerators multiply together to make the dividend.

Content

This section is a miscellany of notes on multiplying fractions ('parts' 分 *fēn*), consisting of several sections with common patterns. In some cases, such as (a) length units are multiplied, so that the product represents (in modern terms) area units.

S7(long gap)

(2) Parts multiplying

The method for a part multiplying a part [is] always: The denominators multiply together to make the divisor; the numerators multiply together to make the dividend.

Content

This is another version of the rule given earlier. The repetition is consistent with the view that we are not dealing with a text intended as a systematic exposition, but rather with a compilation of material grouped roughly under broad categories.

Parallel:

The rule from the Nine Chapters (1: *Fāng tián* 方田, Guō 1990, 187; Shen 1999, 82) is as follows:

乘分術曰母相乘為法子相乘為實實如法而一

The method for multiplying parts:

The denominators multiply together to make the divisor; the numerators multiply together to make the dividend; [count] one for [each time] the dividend accommodates the divisor.

S8,S9,S10(long gap)

(3) Multiplying

(a) Diminished half multiplying diminished half is one ninth; half a *bù* multiplying half a *bù* is one quarter; half a *bù* multiplying diminished half of a *bù* is one sixth; diminished half multiplying augmented half is two ninths.

(b) A fifth multiplying a fifth is one twenty-fifth; a quarter multiplying a quarter is one sixteenth; a quarter multiplying a fifth is one twentieth; a fifth multiplying a sixth is one thirtieth; a seventh multiplying a seventh is one forty-ninth; a sixth multiplying a sixth is one thirty-sixth; a sixth multiplying a seventh is one forty-second; a seventh multiplying an eighth is one fifty-sixth.

S11, S12 (2 character gap)

(c) One multiplying ten is ten; ten multiplying a myriad is ten myriad; a thousand multiplying a myriad is a thousand myriad; one multiplying ten myriad is ten myriad; ten multiplying ten myriad is a hundred myriad; half multiplying a thousand is five hundred; one multiplying a hundred myriad is a hundred myriad; ten multiplying a hundred myriad is a thousand myriad; half multiplying a myriad is five thousand; ten multiplying a thousand is a myriad; a hundred multiplying a myriad is a hundred myriad; half multiplying a hundred is fifty.

Content
Subsection (a) deals with the products of the diminished and augmented half - i.e. with $\frac{1}{3}$ and $\frac{2}{3}$. Subsection (b) covers various products of fractions from $\frac{1}{4}$ to $^{1}/_{7}$, and appears incomplete. Subsection (c) deals with multiples of 10 up to a hundred myriad (1,000,000), with the addition of three cases of halving.

S13(long gap)

(4) Increasing or decreasing parts

When increasing a part one increases its numerator; when decreasing a part one increases its denominator.

Content
The second clause of this general rule, but not the first, is applied in the section that follows.

S14,S15(long gap)

(5) When parts should be halved

For all parts that should be halved, double the denominator; for those that should be [made] a diminished half, 3-fold the denominator; for those that should be quartered, 4-fold the denominator; for those that should be fifthed, 5-fold the denominator; for those that should be tenthed or hundredthed, then 10-fold or 100-fold the denominator, according to the part you wish for.

Text and Content
See under section 6.

S16(long gap)

(6) Parts that are halved

Even if there are a hundred [further] parts proceed in this manner.

Text note
It seems possible that this fragment is simply a continuation of the last line of the preceding section. But the presence of the long gap at the end of S15 and the title at the head of S16 count against that possibility.

Content
In sections 5 and 6 taken together, examples are given of applying the rule stated in the second clause of section 4. These sections are not however bound up with 4, since they conclude with a general statement of their own.

S17,S18(long gap)

(7) Simplifying parts

(a) The method for simplifying parts: Take the numerator from the denominator; in turn take the denominator from the numerator. When the numbers [on the sides of] the numerator and denominator are equal to one another, then you can go on to simplify.

(b) Further, the method for simplifying parts:
What can be halved, halve it. Where one [can be counted for each multiple] of some amount, [count] one for [each multiple of] that amount.

(c)・One method:
Take the numerator of the part from the denominator. [If that is] the lesser take the denominator from the numerator. When [the numbers on the sides of] the numerator and denominator are equal, take that [number] as the divisor. For numerator and denominator complete one for [each time] they accommodate the divisor.

S19(long gap)

(d) Where there is not enough to take away, that can be halved. When halving the denominator, go on to halve the numerator.

S20(long gap)

(e) 162/2016; Simplify it [to] 9/112

Content
Subsections (a), (b), (c) and (d) all give rules for simplifying fractions, while (e) is a numerical example given without working.

Parallel:
The rule given in the Nine Chapters is as follows (1: *Fāng tián* 方田, Guō 1990, 183; Shen 1999, 64). While the principle is the same, there is no obvious textual parallel.

約分術曰
可半者半之不可半者副置分母子之數以少減多更相減損
求其等也以等數約之

The method for simplifying parts:
What can be halved, halve them. As for what cannot be halved, separately set out the numbers for the denominator and numerator. Then alternately reduce them by subtraction. This is seeking for the equality. Simplify using this equal number.

Mathematical note
We may compare this with the procedure in Euclid, Book 7 proposition 2 (paralleled for magnitudes rather than numbers by Book 10 proposition 3);

> *Given two numbers not prime to one another, to find their greatest common measure.*
>
> Let AB, CD be the two given numbers not prime to one another. Thus it is required to find the greatest common measure of AB, CD.
> If now CD measures AB--and it also measures itself--CD is a common measure of CD, AB. And it is manifest that it is also the greatest; for no greater number than CD will measure CD.
> But, if CD does not measure AB, then, the less of the numbers AB, CD being continually subtracted from the greater, some number will be left which will measure the one before it... (Heath 1956, vol. 2, 298-300)

And Euclid goes on to show that this process of alternate subtraction of the lesser from the greater will produce 'the greatest common measure' (or highest common factor in more modern English usage). The point of the alternating subtractions in the present case is of course to find a number that is a factor of both the numerator and denominator. The principle is easily understood if one imagines applying this process, as in Euclid, to two lines of different lengths each made up of multiples of some given line segment. Clearly the process of subtraction will end when the basic 'building block' segment is reached. If the length of the basic segment is not unity, then the lengths of the lines have a common factor other than unity. Applied to the example given above, it would work like this:

2016 and 162 can be halved to give 1008 and 81

1008 - 81 = 927
927 - 81 = 846
... and so on until
117 - 81 = 36
81- 36 = 45
45 - 36 = 9
36 - 9 = 27
27 - 9 = 18
18 - 9 = 9

So 9 is the so-called *děng shù* **等數** 'equality number', the largest common divisor of the two numbers. Dividing both by nine we arrive at the simplified fraction 9/112. The example given is of course poorly chosen to illustrate the rule: anyone familiar with the multiplication table would immediately notice that 9 is a factor of 81, and try dividing 1008 by 9 without following through the process given above.

S21,S22,S23,S24,S25

(8) Joining parts

(a) The method for joining parts: [when] denominators are of the same kind as one another, [then] numerators go with one another [in addition]; [when] denominators are not of the same kind as one another, double what it is fitting to double, 3-fold what it is fitting to 3-fold, 4-fold what it is fitting to 4-fold, 5-fold what it is fitting to 5-fold, and 6-fold what it is fitting to 6-fold; as for the numerators, just double or 3-fold, 4-fold or 5-fold them like the denominators.

(b) In a case where denominators are of the same kind as one another, numerators go with one another [in addition]; [For] those not of the same kind as each other, multiply the denominators together to make the divisor. The numerators multiply the opposite denominators and combine to make the dividend. Complete one for [each time the dividend] accommodates the divisor.

(c) Now we have 2/5, 3/6, 8/10, 7/12, 2/3. How much does this make? 2 cash and 57/60 cash; the method is like the recipe above.

(d) Five men divide 7 cash and a diminished half, and a half cash. A man gets 1 cash and 17/30 cash. The method: in the lowest [place there is] a third, [so] make 6 from 1; then go on to six-fold the [number of] men to make the divisor; likewise six-fold the [number of] cash to make the dividend.

(e) Again: denominators multiply denominators to make the divisor; numerators multiply denominators crosswise to make the dividend; obtain one for [each time the] dividend accommodates the divisor.

(f) One [account]: 10-fold what it is fitting to 10-fold, 9-fold what it is fitting to 9-fold, 8-fold what it is fitting to 8-fold, 7-fold what it is fitting to 7-fold, 6-fold what it is fitting to 6-fold, 5-fold what it is fitting to 5-fold, 4-fold what it is fitting to 4-fold, 3-fold what it is fitting to 3-fold, double what it is fitting to double; [when] denominators are of the same kind as one another, stop; [when] denominators are of the same kind as one another, [then] numerators go with one another [in addition].

Content

This section contains four separate recipes for adding fractions - (a), (b), (e) and (f). In (c) we have an example of the process,. On the other hand (d) seems to be more of a 'sharing' problem similar to the one in the next section (9) 'Direct sharing' *jìng fēn* **徑分**, although it does involve the addition of units, halves and thirds. It seems that the core of the method is to be found in the repeated aphorism **母相類子相從** '[when] denominators are of the same kind as one another, [then] numerators go with one another [in addition]'. This appears in essentially identical form in (a) (b) and (f). Note however that the Nine Chapters do not use this wording.

Parallel:
The Nine Chapters gives the following rule (1: *Fāng tián* **方田**, Guō 1990, 183-4; Shen 1999, 70)

> **合分術曰**
> **母互乘子并以為實母相乘為法實如法而一不滿法者以法命之其母同者直相從之·**
>
> The method for joining parts:
>
> Denominators multiply numerators reciprocally; add to make the dividend; denominators multiply one another to make the divisor; [count] one for [each time the] dividend accommodates the divisor; [as for that part of the dividend] which does not fill the divisor, count it off against the divisor; as for those cases where the denominators are the same, set them out to go with one another [in addition].

Although the wording of the rule in the final line does not closely parallel that given in this text, other parallels are close enough to be suggestive.

The process described may be illustrated with the example written in modern terms as ¾+⅔. We write the numerators and denominators as follows:

numerators:	**3**	**2**	Diagonal multiplication gives 3x3+2x4 = 9 + 8 = 17
denominators:	**4**	**3**	Multiplication gives 4 x 3 = 12

So the result is 17/12

Liú Huī's commentary (same reference) gives the following rationale for the procedures described in the main text:

> 母互乘子；
> 約而言之者，其分麄；繁而言之者，其分細。雖則麄細有殊，然其實一也。眾分錯雜，非細不會。乘而散之。所以通之。通之則可並也。凡母互乘子謂之齊。羣母相乘謂之同。同者，相與通同共一母也；齊者，子與母齊，勢不可失本數也。
>
> 'Denominators multiply numerators reciprocally':
> That which is called 'simple'. its parts are coarse; that which is called complex, its parts are fine. But even though they are different in being coarse or fine, all the same they are identical in substance. The multitude of fractions is manifold and varied, and if one does not divide them finely they will not fit together. The way to make them interchangeable is to divide them more finely by multiplication. When

> they have been made interchangeable, then they can be combined. When denominators multiply numerators reciprocally, we call it 'adjusting'. When a flock of denominators multiply one another, we call it 'sharing'. 'Sharing' means that [everything] is interchangeable and shares in common a single denominator. 'Adjusting' means that numerators are adjusted to the [resulting] denominator, so that in relation to one another they cannot lose the original number.

The descriptive mathematical terms used by Liú Huī in this passage are not found in the Nine Chapters itself, nor in the *Suàn shù shū* . My English renderings attempt to reflect the fact that these terms have common meanings in Chinese outside the mathematical sphere, while also suggesting something of their specific mathematical meaning in this context. I do not of course claim that these are the only useful renderings: all translation is a process of makeshift approximation. But here is a sketch of the reasoning behind my choices:

> 通 *tōng* : my 'interchangeable' plays on the fact that the word basically means refers to things interpenetrating, interrelating or intercommunicating; in the inscriptions on coins it means 'current'. The mathematical significance is that (for instance) two thirds and three quarters cannot be added as they stand to make five units, any more than two pounds and three euros make five currency units. The fractions, like the money, need to be put into common terms (twelfths, dollars) to make them interchangeable before they can be used together in any arithmetical process.
>
> 同 *tóng* : 'sharing' is a basic sense of this word, and what it shared here is simply a common denominator.
>
> 齊 *qí* : Things which are or have been adjusted so as to be level, even, or in proper order generally are *qí*. If one takes a fraction such as ¾ and changes the denominator to 12, the numerator 3 has to be adjusted to a new value of 9 in order to keep its previous relation to the numerator. The word 勢 *shì* here rendered as 'relation' refers to the power or potential that one thing has in virtue of its position relative to other elements in its setting. Thus 3 in relation to 4 has the same *shì* that 9 has in relation to 12.

We shall see the use of these terms extended somewhat in the discussion of the 'Excess and Deficit' method later on (Group 11).

[Group 2: Sharing; sharing in proportion; progressions]

S26,S27

(9) Direct sharing

(a) Direct sharing counts off the dividend by [each] single person. Thus: 5 men share 3 and one half and a diminished half. Each receives 23/30. The method: in the lowest [place] there is a diminished half [so] make 6 from 1; make 3 from a half; make 2 from a diminished half; combine them to make 23; then set out the number of men, and then six-fold it for counting off from the dividend.

(b) Further: The method: when in the lowest [place] there is a half then double it; when in the lowest [place] there is a third then 3-fold it; when in the lowest [place] there is a quarter then 4-fold it.

Content

Subsection (a) gives a brief characterisation of the problem followed by an illustration; (b) is clearly linked in theme, but may come from a different source since it refers to ⅓ as 三分 rather than 少半 as in (b).

Parallel:

In the Nine Chapters the method is given as follows (1: *Fāng tián* 方田, Guō 1990, 187; Shen 1999, 80):

> 經 分 術 日
> 以 人 數 為 法 錢 數 為 實 實 如 法 而 一 有 分 者 通 之 重 有 分 者 同 而 通 之
>
> The method for direct sharing:
>
> Take the number of men as the divisor [and] the number of cash as the dividend; [count] one for [each time the] dividend accommodates the divisor. In cases where there are parts, make them interchangeable; Where there are repeated parts [of the same kind] combine them and make them interchangeable [with those of other kinds].

Here the method is named *jīng fēn* 經分 rather than the *jìng fēn* 徑分 used in the

present text. It is striking that Liú Huī 劉徽 and other commentators are careful to explain that here *jīng* 經 does in fact mean *jìng* 徑, and their explanations of *jìng fēn* 徑分 are based on this.
What the Nine Chapters means by *tōng zhī* 通之 'make them interchangeable' is of course the series of steps in (a) by which the units and assorted fractions are made capable of being rolled up into a single total through multiplication by 6.

S28,S29(7 character gap)

(10) Paying out gold

(a) There are 3 *zhū* and 5/9 *zhū* of gold. Now it is desired to pay out 6/7 *zhū* of it. Question: how much is the remaining gold? The remaining gold is 2 *zhū* and 44/63 *zhū*. The method: The denominators are multiplied together to make the divisor; the numerators multiply the denominators reciprocally, each making a dividend of its own; diminish it by the [amount] paid out; then the remainder is the remaining [gold]. Multiply 3 *zhū* by the 9 parts of a *zhū* and combine it with the smaller 5.

S30,S31(long gap)

(b) Now there are 3/7 *zhū* of gold. How much should one increase it to make 7/9? Increase it [by] 22/63 *zhū*. Method: The denominators are multiplied together to make the divisor; the numerators multiply the denominators reciprocally, each making a dividend of its own; reduce the greater by the lesser; then the remainder is the increase.

Content
This section contains two separate examples, both involving the subtraction of one fraction from another. The final sentence at the end of (a) looks like a scholium added to the original, explaining how to turn 3 and 5/9 into a number of ninth parts; there is no corresponding addition to (b). It is preceded by the 'blob' mark. 'The smaller 5' is evidently a usage referring to the number of parts; compare 'the smaller 10' in section 67.

Parallel:

Both of the examples given here perform the same function as the procedure for subtraction of fractions given in the Nine Chapters (1: *Fāng tián* 方田, Guō 1990, 184-185; Shen 1999, 76), but there are no parallels specific to this operation rather than to manipulations of fractions in general.

S32,S33

(11) Buying timber in common

Three men buy timber in common. In accordance with the price one man pays out 5 cash, one man pays out 3, and one man pays out 2 cash. Now there is a surplus of 4 cash. It is desired to divide it differentially according to the number of cash. He who paid out 5 gets 2 cash; he who paid out 3 gets 1 cash and ⅕ cash; he who paid out 2 gets ⅘ cash. Method: combine the number of cash paid out by the three men to make the divisor; then for each multiply the number of cash paid out by 4 cash; you obtain one cash for [each time these amounts] accommodate the divisor.

Content:
This is the first problem in the collection relating to rates of sharing a total in varying proportions amongst a number of recipients. The fourth sentence contains the phrase **衰分** *Cuī fēn* 'differential division' as a name for this process, and this is the title of the third of the Nine Chapters, in which such problems are treated at length. There are however no precise parallels to this example.

S34,S35(6 character gap)

(12) The fox goes through a customs-post
A fox, a wild-cat and a dog go through a customs-post; they are taxed 111 cash. The dog says to the wild-cat, and the wild-cat says to the fox 'Your skin is worth twice mine; you should pay twice as much tax!' Question : how much is paid out in each case? Result: the dog pays out 15 cash and 6/7 cash; the wild-cat pays out 31 cash and 5 parts; the fox pays out 63 cash and 3 parts. Method: let them be double one another, and combine them [into] 7 to make the divisor; multiply each by the tax to make the dividends; obtain one for [each time] the dividend accommodates the divisor.

Content:
This is another problem of the 'differential division' type.

Parallel:
The *Cuī fēn* **衰分** section of the Nine Chapters has its own 'animal story' example with the same structure of successive doubling, but this time it is not the animals that speak but the owners of an ox, a horse and a sheep who have to share liability for some fodder eaten by their animals - and of course the ox is said to have eaten twice as much as the horse, who has eaten twice as much as the sheep (3: *Cuī fēn* **衰分**, Guō 1990, 236-237; Shen 1999, 162)

今有牛馬羊食人苗苗主責之粟五斗羊主曰我羊食半馬馬主曰我馬食半牛今欲衰償之問各出幾何

荅曰
牛主出二斗八升七分升之四馬主出一斗四升七分升之二羊主出七升七分升之一

術曰
置牛四馬二羊一各自為列衰副并為法以五斗乘未并者各

自為實 實如法得一斗

Now there are an ox, a horse and a sheep that eat someone's sprouting grain; the owner of the sprouting grain sets the value at 5 *dǒu* of unhulled grain; the owner of the sheep says : 'My sheep ate half what the horse did'; the owner of the horse says: 'My horse ate half what the ox did'. Now it is desired to penalise them differentially. Question: how much does each one pay out?

Answer:
The owner of the ox pays out 2 *dǒu* 8 *shēng* and 4/7 *shēng*; the owner of the horse pays out 1 *dǒu* 4 *shēng* and 2/7 *shēng*; the owner of the sheep pays out 7 *shēng* and 1/7 *shēng*.

Method:
Set out 4 for the ox, 2 for the horse and 1 for the sheep; let each of these be the successive differentials; make an auxiliary combination of these to make the divisor; Multiply the uncombined [differentials] by 5 dǒu to make the dividend for each one; obtain 1 *dǒu* for each time the dividend accommodates the divisor.

As ever, the Nine Chapters version of the problem is considerably more formal and regular than in the the *Suàn shù shū* . Immediately after its 'animals' problem, the Nine Chapters has a problem in which tax paid by three persons going through a customs post is differentially distributed - so both elements in the *Suàn shù shū* example are found in the Nine Chapters, separately but in a way that suggests some real connection.

今有甲持錢五百六十乙持錢三百五十丙持錢一百八十凡三人俱出關關税百錢
欲以錢數多少衰出之問各幾何

荅曰
甲出五十一錢一百九分錢之四十一·乙出三十二錢一百九分錢之一十二·丙出一十六錢一百九分錢之五十六·

術曰
各置錢數為列衰副并為法以百錢乘未并者各自為實實如法得一錢

Now we have: A brings 560 cash, B brings 350 cash, and C brings 180 cash. All three go through a customs post together; the customs post [sets] duty of 100 cash; it is desired to pay differentially in accordance with the size of the number of cash. Question: how much for each?

Answer:
A pays 51 cash and 41/109 cash; B pays 32 cash and 12/109 cash; C pays 16 cash and 56/109 cash;

Method:
set out the number of cash for each to be the successive differentials; make an auxiliary combination of these to make the divisor; Multiply the uncombined [differentials] by 100 cash to make the dividend for each one; obtain 1 cash for each time the dividend accommodates the divisor.

Mathematical note
The problem is solved here, as are other examples, by constructing a sequence of numbers (in this case 1,2 and 4) having the required ratios to one another, and sharing out in proportion to the ratio each number bears to the total of all three. As in the following problem, once the denominator of the fraction in the answer has been stated once, it is omitted in the remaining two answers, which just refer to numbers of 分 *fēn* 'parts'. This is not a usual practice elsewhere, and marks out these two problems from the rest.

S36,S37(long gap)

(13) The fox's skin
The fox's skin is 35 *cái*; The wild-cat's skin is 25 *cái*; The dog's skin is 12 *cái*. They all pass through a customs-post; the customs-post takes a combined tax of 25 cash. Question: how much does each pay out? Result: The fox pays out 12 and 11/72; the wild-cat pays out 8 and 49 parts; the dog pays out 4 and 12 parts. Method: combine the prices to make the divisor; multiply each by the tax to make the dividends.

Content:
The setting and wording of this problem parallel 13 closely enough to suggest that they come from the same source. What exactly 裁 *cái* represents in this context is unclear, but it obviously serves as a measure of quantity of skin and hence of value.

S38,S39(long gap)

(14) Carrying hulled grain
A man is carrying hulled grain - we do not know how much - as he passes through three customs posts. [Each] post takes a duty of 1 in 3. After leaving he has one *dǒu* of hulled grain left. Question: when he started going, how much hulled grain did he bring? Result: The hulled grain he brought was 3 *dǒu* 3 *shēng* and ¾. Method: Set out one, and thrice double it to make the divisor. Again set out one *dǒu* of hulled grain and 3-fold it Again three-fold it and [multiply by] the number of passes to make the dividend for it.

Content:
This problem falls under one of the types discussed in the 均輸 *Jūn shū* 'equitable transport' section of the Nine Chapters where some of the problems show how to

work back to an original amount that has been reduced by some proportional deduction. Within the present text it resembles other sections such as 18 and 19 in which similar principles operate.

Mathematical note
The statement of the working seem a little confused. The mathematics involve three successive multiplications of the remaining hulled grain by (3/2) to restore the original amount. The text can be read as saying that the divisor should be 2x2x2 (see the parallel in 68 below), but what follows seems an awkward way to refer to a multiplication of 1 *dǒu* by 3x3x3, although the answer given is accurate. There is however no obvious and simple emendation that can restore order.

Parallel:
The Nine Chapters have a very close parallel to this problem (6: *Jūn shū* **均輸**, Guō 1990, 339; Shen 1999, 345) although the wording (as so often) is clearer and more regular compared to the present text.

> **今有人持米出三關外關三而取一中關五而取一內關七而取一餘米五斗問本持米幾何**
>
> **荅曰**
> **十斗九升八分升之三術曰置米五斗以所稅者三之五之七之為實以餘不稅者二四六相乘為法實如法得一斗**
>
> Now there is a man carrying hulled grain, who passes through three customs posts:
> the outer post takes 1 in 3;
> the middle post takes 1 in 5;
> the inner post takes 1 in 7;
> the remaining hulled grain is 5 *dǒu*.
> Question: how much was he originally carrying?
>
> Answer: 10 *dǒu* 9 *shēng* and ⅜ *shēng*.
>
> Method:
> set out the hulled grain, 5 *dǒu*;
> using the amounts taxable, 3-fold it, 5-fold it, 7-fold it to make the dividend;
> using the tax-free remainders of 2, 4, and 6 multiply them together to make the divisor;
> [count] one *dǒu* for [each time] the dividend accommodates the divisor.

S40,S41,S42(5 character gap; 'checked by Wáng')

(15) The woman weaving

In a neighbouring village there is a woman good at weaving, who doubles her [production each] day. In weaving, [she] says: 'On the fifth day I [had] woven five *chí*.' Question: on the day she began weaving and the subsequent ones, how much [was produced] in each case? Reply: At the start she wove 1 *cùn* and 38/62 *cùn*; next 3 *cùn* and 14/62 *cùn*; next 6 *cùn* and 28/62 *cùn*; next 1 *chí* 2 *cùn* and 56/62 *cùn*; next 2 *chí* 5 *cùn* and 50/62 *cùn*. Method: set out 2; set out 4; set out 8; set out 16; set out 32; combine them to make the divisor; multiply them by 5 *chí* on one side, each to make its own dividend; obtain a *chí* for [each time] the dividend accommodates the divisor; what does not fill a *chí*, 10-fold it; [count] 1 *cùn* for [each time the result] accommodates the divisor; for what does not fill a *cùn*, designate the part by the divisor.

Content:
This is one of two 'weaving rates' problems in this collection. This one is linked with the name of Wáng, while 21 below (in which there is a mistake) mentions Yáng.

Mathematical note:
The solution starts from the notion that the production of successive days can be said to be in the ratio 2:4:8:16:32. The total of 5 *cùn* is thus divided into (2+4+8+16+32) = 62 parts, and these are allocated to days in accordance with the ratios listed. It is not clear why the ratios start from 2 rather than 1, nor why the fractions in the result are not reduced to their lowest terms.

Parallel:
The *Cuī fēn* 衰分 section of the Nine Chapters has a very close parallel (3:*Cuī fēn* 衰分, Guō 1990, 237-238; Shen 1999, 162-163). Note however that unlike the present text, the Nine Chapters begins its doubling with 1 rather than 2; also the fractions are reduced to their simplest terms.

今有女子善織日自倍五日織五尺問日織幾何

荅曰
初日織一寸三十一分寸之十九次日織三寸三十一分寸之七次日織六寸三十一分寸之十四次日織一尺二寸三十一分寸之二十八次日織二尺五寸三十一分寸之二十五

術曰
置一二四八十六為列衰副并為法 以五尺乘未并者各自為實實如法得一尺 ·

Now there is a girl good at weaving who doubles her [production] every day; in 5 days she weaves five *chí*. Question: in [each] day how much does she weave?

Answer:
On the first day she weaves 1 *cùn* 19/31 *cùn*; on the next day she weaves 3 *cùn* 7/31 *cùn*; On the next day she weaves 6 *cùn* 14/31 *cùn*; on the next day she weaves 1 *chí* 2 *cùn* 28/31 *cùn*; On the next day she weaves 2 *chí* 5 *cùn* 25/31 *cùn*.

Method:
set out 1, 2, 4, 8, 16 as separate differentials; adjointly combine them to make the divisor; multiply the uncombined [differentials] by 5 *chí*, each to make its own dividend; obtain 1 *chí* for [each time] the dividend accommodates the divisor.

S43,S44,S45(long gap)

(16) Combined tax
3 *bù* [planted with] millet: [produce] 1 *dǒu*; 4 *bù* [planted with] wheat [produce] 1 *dǒu*; 5 *bù* [planted with] beans [produce] 1 *dǒu*. Now combining them, one taxes 1 *shí*. Question: how much is the tax? Result; millet: the tax is 4 *dǒu* and 12/47; wheat: the tax is 3 *dǒu* and 9 parts; beans: the tax is 2 *dǒu* and 26 parts. Method: S et out 3 *bù of* millet; 4 *bù of* wheat; 5 *bù of* beans. Let millet multiply wheat to make the dividend for beans; beans multiply millet to make the dividend for wheat; wheat multiply beans to make the dividend for millet. Set out each [dividend] separately, multiply each by one *shí* and 10-fold it to make the dividend. One *dǒu* [comes from taking] 47 as the divisor.

Parallel:
I do not know of a parallel with this type of problem in the Nine Chapters.

Mathematical note
In order to produce the results given, the underlying assumption must be that equal amounts of land are devoted to each type of crop, and that they are then taxed at the same fractional rate of product per area. producing a total of 1 *shí* of the three grains combined. Clearly then the contributions of each type of grain to the total will be in inverse proportion to the land area required to produce 1 *dǒu* of each type of grain. So if we designate the amount of each type of grain in the 1 *shí* tax by the initial letters of each type of grain, we find that (in modern terms) the contributions will be in the ratios:

millet:wheat:beans :: $(\frac{1}{3}):(\frac{1}{4}):(\frac{1}{5})$

And so in *Suàn shù shū* terms we may use these three fractions as the cuī 衰 'differentials' for the three grains as in previous examples. Thus for example the share of the 1 *shí* allocated to millet will be:

1 *shí* x $(\frac{1}{3})/\{(\frac{1}{3})+(\frac{1}{4})+(\frac{1}{5})\}$
= 1 *shí* x 4 x 5/{4x5+3x5 + 3x4}
= 1 *shí* x 4 x 5/47
= 10 *dǒu* x 4 x 5/47
= 10 *dǒu* x 4 x 5/47
= 4 *dǒu* and 12/47 as stated above.

S46

(17) The price of gold
(a) The price of gold: 1 *liǎng* is 315 cash. Now there is 1 *zhū* [of gold]. Question: how many cash does one get? Reply: One gets 13 cash and ⅛. Method: Set out 1 *liǎng*'s number of *zhū* to make the divisor; take the number of cash as the dividend; obtain one cash for [each time] the dividend accommodates the divisor.

S47(long gap)

(b) 24 *zhū* are 1 *liǎng*; 384 *zhū* are 1 *jīn*;. 11520 *zhū* are 1 *jūn*; 46080 *zhū* are 1 *shí*.

Content:
This is a simple price calculation, followed by a list of measures with their equivalents. The calculation finishes just at the end of strip 46, so there is no evidence from continuity of text that the list on strip 47 originally belonged at this point, despite its general relevance to the calculation. I have therefore marked it as a separate subsection.

Mathematical note:
From the table of measures in terms of *zhū* at the end of this section, we may deduce that :

24 *zhū* are 1 *liǎng*
16 *liǎng* are 1 *jīn*
30 *jīn* are 1 *jūn*
4 *jūn* are 1 *shí*

These are the usual ratios; the Hàn dynasty *shí* was about 29.5 kg.in modern terms

[Group 3: Wastage]

S48,S49(long gap)

(18) Hulling unhulled grain
There is conferred unhulled grain, 1 *shí*: hulling it makes 8 *dǒu* 8 *shēng*. How much wastage unhulled grain should be added? Reply: 1 *dǒu* 3 *shēng* 7/11 *shēng*. Method: Set out the number of *shēng* in the hulled grain obtained as the divisor; further set out the number of *shēng* in one *shí* of unhulled grain, and multiply it by the number of *shēng* of wasted hulled grain; obtain 1 *shēng* for [each time this result] accommodates the divisor.

Content
The next section, 19 also deal with the question of wastage occurring during processing of commodities. The aim here is to ensure that 1 *shí* remains after hulling has been performed.

Mathematical note
1 *shí* is 10 *dǒu*. So if the product from 1 *shí* after hulling is 8 *dǒu* 8 *shēng*, we have lost 1 *dǒu* 2 *shēng*. This extra amount of hulled grain corresponds to the following amount of unhulled grain:
(1 *dǒu* 2 *shēng*) x (10 *dǒu*)/(8 *dǒu* 8 *shēng*) = 12 x 100/88 *shēng*
= 13 + 7/11 *shēng*
= 1 *dǒu* 3 *shēng* 7/11 *shēng.*

We may note that the ratio of unhulled to hulled grain produced used here is 100:88 = 50:44. This is different from the ratio 50:30 found in the Nine Chapters and also in section 36.

Parallel:
The mentions of *hào* 耗 'wastage' in the Nine Chapters occur in problems relating to drying of commodities (rather than the hulling of grain, or the melting of bronze as in section 19. On drying grain in the *Suàn shù shū* see 34 and 35; drying of textile fibres is dealt with in 37. 41 has a parallel instance of wastage in hulling, although the rate of loss is slightly different from this. It does however seem worthwhile to cite the Nine Chapters (3: *Cuī fēn*, Guō 1990, 243-244; Shen 1999, 170-171) here, since the underlying mathematical principles parallel this section

今有絲一斤耗七兩今有絲二十三斤五兩問耗幾何？

荅曰
一百六十三兩四銖半·

術曰
以一斤展十六兩為法以七兩乘今有絲兩數為實實如法得耗數

今有生絲三十斤乾之耗三斤十二兩今有乾絲一十二斤問生絲幾何

荅曰：
一十三斤一十一兩十銖七分銖之二

術曰
置生絲兩數除耗數餘以為法三十斤乘乾絲兩數為實實如法得生絲數

Now there is 1 *jīn* of undressed silk; the wastage is 7 *liǎng*. Now there is 23 *jīn* 5 *liǎng* of undressed silk. Question: how much is the wastage?

Answer: 163 *liǎng* 4 ½ *zhū*.

Method:
Take 1 *jīn* unfolded into 16 *liǎng* to be the divisor; multiply the number of *liǎng* in the present amount of undressed silk by 7 *liǎng* to make the dividend; by accommodating the dividend to the divisor obtain the number of the wastage.

Now there is 30 *jīn* of fresh undressed silk; in drying it the wastage is 3 *jīn* and 12 *liǎng*. Now there is 12 *jīn* of dried undressed silk; Question: how much fresh undressed silk [was there]?

Answer: 13 *jīn* 11 *liǎng* 10 *zhū* $^{2}/_{7}$ *zhū*.

Method:
Set out the number of *liǎng* of fresh undressed silk; reduce by the number of wasted [*liǎng*]; take the remainder as the divisor; 30 *jīn* multiplies the number of *liǎng* of dried undressed silk to make the dividend; by accommodating the dividend to the divisor obtain the number [of *jīn*] of fresh undressed silk.

S50.S51(long gap)

(19) Bronze wastage
In casting bronze the wastage on 1 *shí* is 7 *jīn* 8 *liǎng*. Now there is 1 *jīn* 8 *liǎng* 8 *zhū* of bronze. Question: how much wastage is there? Result: 1 *liǎng* 12 *zhū* 72/144 *zhū*. Method: Set out the number of *zhū* in one *shí* as the divisor; then set out the number of *zhū* in the 7 *jīn* 8 *liǎng*; multiply it by the number of *zhū* in the 1 *jīn* 8 *liǎng* 8 *zhū*; [count] 1 *zhū* [for each time the dividend] accommodates the divisor.

Content:
This problem is one of the 'wastage' category referred to in the notes to 18.

Mathematical note:
This problem depends on the conversion of all larger units into *zhū* in the manner set out in 17: Thus we are told that the wastage on 46080 *zhū* is

(7x384+8x24) *zhū* = 2880 *zhū*.

So the wastage on (1x384 + 8x24 + 8) *zhū* = 584 *zhū* is:

584 x 2880/46080 *zhū* = 36 *zhū* 23040/46080 *zhū*
= 1 *liǎng* 12 *zhū* 72/144 *zhū*

(leaving the fraction over the denominator used in the text, rather than simplifying further).

[Group 4: Sharing, contributions and pricing]

S52,S53(long gap)

(20) Post horses

Post horses: for 1 day, 2 [horses] share 2 *shí* of hay and stalks. Let the hay be 3 and the stalks 2. Now one [further horse] arrives early. Question: how much hay and stalks must one provide? Reply: Provide 4 *dǒu* of hay and 2 *dǒu* ⅔ *dǒu* of stalks. Method: Set out the hay's 3 and the stalks' 2 and combine them; multiply [the result] by the 3 horses to make the divisor; multiply what has been set out [i.e. the proportions of hay and stalks] by 2 *shí*, each to make its own dividend.

Content:
The problem involves a situation where a post-station has sufficient rations for two horses, but finds itself unexpectedly having to cope with three, since one horse arrives before either of the two original horses leaves. The answer gives the amounts of hay and stalks for each horse per day, assuming the given proportion between the two types of fodder. Contemporary regulations for the feeding of post-horses and other official beasts of burden can be found in Zhāngjiāshān (2001), 189-190. There are however no exact parallels to the amounts given here.

Mathematical note:
The calculation as follows.

In the text, the result is obtained by calculating:

hay: (2 *shí*) x 3/(5 x 3)   stalks: (2 *shí*) x 2/(5 x 3)

The thinking is evidently that since the balance of types of fodder is expressed out of a total of five parts, and there are three horses, the shares of each horse will be made up of a number of 15th parts of the total fodder, distributed proportionately between the two fodder types.

S54,S55,S56(5 character gap; checked by Yáng)

(21) Women weaving

There are 3 women; The eldest one weaves 50 *chí* in 1 day; the middle one weaves 50 *chí* in 2 days; the youngest one weaves 50 *chí* in 3 days. Now their weaving produces 50 *chí*. Question: how many *chí* does each deliver? The result: The eldest delivers 25 *chí*; the middle one delivers 16 *chí* and 12/18 *chí*; the youngest delivers 8 *chí* and 6/18 *chí*. Method: set out 1; set out 2; set out 3;

then let each [contribute] as many to make the divisor. Then 10 and 5 -fold them to make the dividends; one *chí* results from [each time the dividends] accommodate the divisor; for what does not fill a *chí*, designate the parts according to the divisor. 3 is the dividend for the eldest one; 2 is [the dividend] for the middle one; 1 is [the dividend] for the youngest one.

Contents:
This is another weaving problem, related but not identical to 15, which is annotated with the name of Wáng. If these names are those of the authors of problems, and if these two authors were aware of one another's existence, it seems possible that Yáng may have wanted to contribute a distinctive weaving problem of his own involving a series of numbers based on rates of work, leading to a division of a total length of cloth - 50 *chí* in both cases. Unfortunately there is a flaw in the reasoning which has led to the wrong answer - see below.

Parallel:
The Nine Chapters (6: Jūn shū 均輸, Guō 1990, 338; Shen 1999, 343-345) has a problem involving the same basic principle of several sources contributing to a known total at different rates:

今有池五渠注之其一渠開之少半日一滿次一日一滿次二日半一滿次三日一滿次五日一滿今皆決之問幾何日滿池

荅曰
七十四分日之十五

術曰
各置渠一日滿池之數并以為法以一日為實實如法得一日
其一術
列置日數及滿數令日互相乘滿并以為法日數相乘為實實如法得一日

Now there is a pool with five channels running into it. When one channel is opened it fills the pool once in a diminished half of a day; when the next is opened it fills the pool once in 1 day; when the next is opened it fills the pool once in 2 ½ days; when the next one is opened it fills the pool once in 3 days; when the next one is opened it fills the pool once in 5 days. Now they all pour into it. Question: how many days will it take to fill the pool?

Answer:
15/74 day.

Method:
set out the number of times each channel will fill the pool in one day; combine these to make the divisor; take one day as the dividend; obtain one day for each time the dividend accommodates the divisor.

One method:

> set out in order the numbers of days and the number of fillings; let the days multiply the fillings reciprocally; combine them to make the divisor; the day numbers multiply each other to make the dividend; obtain one day for each time the dividend accommodates the divisor.

Mathematical note:
In the Nine Chapters example, a pool is filled by five separate streams, and in each case we are told how many days each would take to fill the pool on its own. The correct step is then to take the reciprocals of these rates to find how many times each day the pool would be filled by each stream alone,. These are then amalgamated and divided into one day to find how long all the streams together take to fill the pool. Clearly Yáng's calculation would have worked in the same way if he had taken the number of times each woman wove 50 *chí* in one day as his basic rates, rather than the days each took to produce 50 *chí*. Thus, for all the women weaving together to produce 50 *chí* would take:

1 day/ (1 + ½ + ⅓) = 6/11 day
From this it follows easily that the actual productions of each of the three women during this time will be:

50 x 6/11 *chí* = 27 3/11 *chí*
50/2 x 6/11 *chí* = 13 7/11 *chí*
50/3 x 6/11 *chí* = 9 1/11 *chí*

It seems that the problem author's check of his answer must have been limited to seeing whether his total was 50 *chí* - which it certainly is. He cannot have asked himself whether the ratios of the contributions made sense, since otherwise he would have been warned by the fact that the production of the middle woman is not half of the production of the eldest, as it should have been.

S57,S58(long gap)

> (22) Feathering arrows
> 2 quills of feathers are 5 cash. Now there are 37/57 of a quill. Question: how much does one get [for it]? Reply: One gets 1 cash and 71/114 cash. Method: 2 multiplies 57 to make the divisor; multiply 37 by 5 to make the dividend; [count] 1 cash [for each time the dividend] accommodates the divisor; [for what is] not filled, denominate the part by the divisor.

Content:
This is a very simple problem on the application of a known per unit price to a non-unit quantity. The content of 23 is very similar, and 24 is only a little more elaborate. The complex fraction of a feather involved here is of course purely for the sake of creating an arithmetical exercise.

Mathematical note:
The calculation is
(37 x 5)/(2 x 57) cash = 1 and 71/114 cash

Parallel:
The purchase of quills of feathers is mentioned in one problem in the Nine Chapters (2: *Sù mǐ* 粟米, Guō 1990, 227; Shen 1999, 154-155), but the principle behind the problem is more different and more complex (the quills have two unknown prices in integral numbers of cash), and there is no obvious parallel.

S59,S60(damaged; three visible characters at the top, then seems to have a long gap before the break)

(23) Lacquer money
A *dǒu* of lacquer is 35 cash. Now there is 5/40 *dǒu*. Question: how many cash does one get? Reply: one gets 4 cash and ⅜ cash. Method: Take 40 as the divisor; multiply 35 by 5 to make the dividend; obtain 1 cash for each time the dividend accommodates the divisor.

Content:
The problem (which involves calculating 35 x 5/40) seems to be completely stated and solved in the text we see, despite the damage to the second strip.

S61,S62,S63(long gap)

(24) Silken strip
A silken strip is 22 *cùn* in breadth and 10 *cùn* in length. Its price is 23 cash. Now it is desired to buy a cut along the length 3 *cùn* in breadth and 60 *cùn* in length. Question: the total *cùn*, and the price in cash - how much is each one? Reply: 8 *cùn* and 2/11 *cùn*; the price is 18 cash and 9/11 cash. Method: take 22 *cùn* as the divisor; take the breadth and length multiplied together as the dividend; obtain 1 *cùn* [for each time the dividend] accommodates the divisor. Go on to take the number of *cùn* in a *chí* as the divisor; multiply the number of cash in the price of 1 *chí* by the number of *cùn* obtained [above] to make the dividend. Obtain 1 cash [for each time] the dividend accommodates the divisor.

Content:
This odd little problem has no parallels that I know of in the Nine Chapters or elsewhere. The object is to cut an amount from a strip of a certain width, 1 *chí*, that will be of an area equivalent to a piece of given dimensions, and to calculate its price. We may note that according to excavated Qín regulations the breadth of a regulation bolt of cloth was 25 *cùn*: see Shuìhǔdì (2001) p. 36, whereas in Hàn times the breadth was 22 *cùn* as in this example: see Zhāngjiāshān (2001) p. 168.

Mathematical note:
We calculate the length required to be cut from the standard 22 *cùn* strip from
3 x 60/22 *cùn* = 8 and 2/11 *cùn*
Since a 10 *cùn* length costs 23 cash, the price of this is
23 cash x (8 and 2/11 *cùn*)/10 *cùn*
= 18 and 9/11 cash

S64,S65(long gap)

(25) Interest money

[If] the capital is 100 cash, the interest is 3 a month. Now the capital is 60 cash; it is returned when the month has not yet filled 16 days. Calculate how much the interest is. Result: 24/25 cash. Method: calculate the amount of accumulated cash for 100 cash over one month and make that the divisor; set out the cash of the capital and multiply by the interest on 100 cash for one month; then multiply by the number of days and make it the dividend; obtain 1 cash of interest for [each time] the dividend accommodates the divisor.

Content:

This problem is about simple interest: it is the only reference to money-lending in the present text.

Mathematical note:

In modern terms, the method finds the integral of capital lent over the lending period; this quantity is labelled *jī qián* 積錢 'accumulated cash', following a similar usage in astronomical writing: see also section 67 in the present text.

Parallels:

The Nine Chapters has a close parallel to this problem (3: *Cuī fēn* 衰分, Guō 1990, 243; Shen 1999, 172-174)

今有貸人千錢月息三十今有貸人七百五十錢九日歸之問息幾何

荅曰
六錢四分錢之三

術曰
以月三十日乘千錢為法以息三十乘今所貸錢數又以九日乘之為實實如法得一錢・

Now we have a case of lending someone 1000 cash; The monthly interest is 30. Now we have a case of lending someone 750 cash; in nine days it is returned. Question: how much is the interest?

Answer:
6 cash and ¾ cash.

Method:
multiply the 1000 cash by the 30 days of a month to make the divisor; multiply the number of cash now being lent by the interest of 30; then multiply it by 9 days and make it the dividend; obtain 1 cash for [each time] the dividend accommodates the divisor.

It is notable that the interest rate is the same (3% a month) although the amount of capital and length of lending period are different.

S66,S67(long gap)

(26) Pouring [water] into lacquer

[Into] 1 *dǒu* of lacquer one pours 3 *dǒu* of water, while for a pan into which water is poured, 2 *dǒu* and 7 *shēng* fill the pan. Question: the remaining lacquer and the remaining water - how much is each? Reply: the remaining lacquer is 30/37 *shēng*; the remaining water is 2 *shēng* 7/37 *shēng*. Method: putting together the 2 *dǒu* and 7 *shēng* and the 1 *dǒu*, make 37 the divisor; further set out the 27 and the 10 *shēng*, and 3-fold each to make a dividend; [count] one for [each time] the dividend accommodates the divisor.

Content:
On the face of if, this is a straightforward problem about the proportions of components in a mixture, and the resulting amount of each component in some given volume of mixture. The administrative practice underlying the problem is undoubtedly the Qín regulations cited by Péng Hào (2001) 9-10, which deal with the testing of batches of lacquer to reveal the loss of water by evaporation, with progressive penalties for those responsible for delivering lacquer with high water loss. The testing process seems to involve adding water to the lacquer sample until the mark showing the level of the original fresh lacquer is reached. However the way the problem is phrased is by no means simple and clear. Compare Hulsewé (1985) 97-98.

Mathematical note:
Looking at the mathematical structure of this problem, we note that the total of the 'remaining' water and lacquer is 3 *shēng*, which is what would overflow if 3 *dǒu* (= 30 *shēng*) of liquid was poured into the pan, which is said to have a capacity of 2 *dǒu* and 7 *shēng*. That much makes obvious sense. But turning to the proportions of the water and lacquer in this 3 *shēng* of overflow, they are;

water: lacquer = 27/37 : 10/37 = 2.7:1

which suggests that we started off with a mixture totalling 37 units (let us assume they were *shēng*) made up of 27 units of water and 10 of lacquer. These could be assumed to have been contained in the pan full of water (27 *shēng*) and a full *dǒu* of lacquer (10 *shēng*).
On this basis, if we look at the amount of non-remaining ingredients, we would have:

water: 27 - 2 7/37 = 24 30/37 = 918/37
lacquer: 10 - 30/37 = 9 7/37 = 340/37

These are also in the ratio 2.7: 1, and total up to 34 *shēng*

The assumption seems to be therefore that 10 *shēng* of lacquer were mixed with 27 *shēng* of water - but that the resultant volume was only 30 *shēng.* This was then poured into the 27 *shēng* pan, and an overflow of 3 *shēng* occurred, containing ingredients in the proportions calculated here. There is clearly some flaw in the way this problem is conceived.

[Group 5: changes in rates]

S68,S69(long gap)

(27) Duty on a field
There is duty on a field of 24 *bù*. 8 *bù* [produces] 1 *dǒu*. the tax is 3 *dǒu*. Now in error it is ticketed at 3 *dǒu* 1 *shēng*. Question: how many *bù* give a *dǒu* [on that basis]? Result: 7 *bù* 23/31 *bù* [produce] 1 *dǒu*. Method: make the 3 *dǒu* 1 *shēng* the divisor; 10-fold the tax [and the] field; Make it accommodate the divisor [to obtain] 1 *bù*.

Content:
This problem seems to be concerned with the consequences of bureaucratic error: a field that should have been rated (literally 'ticketed' 券 *quàn*) at a given rate is assessed at a higher total rate, and the object is to decide what the implied rate per unit area would be. The initial statement of the total area of the field is not used in the calculation. Sections 38 and 39 are also concerned with the consequences of mistakes in tax assessment.

Mathematical note:
The calculation is
10 x 3 x 8/(31) *bù* = 7 and 23/31 *bù*
Clearly the 3 *dǒu* 1 *shēng* are converted to 31 *shēng* for the calculation.

S70 (4 character gap)

(26) Norms for bamboo

(a) The Norm: A bamboo of 8 *cùn* [circumference] makes 183 slips of 3 *chí*. Now slips are made from a 9 *cùn* bamboo. How many slips should there be? Reply: it makes 205 slips and ⅞ of a slip. Method: Take 8 *cùn* as the divisor

S71

(b) The Norm: One piece of 8 *cùn* bamboo makes 366 slips of 1 *chí* 5 *cùn*. Now it is desired to use this bamboo to make slips of 1 *chí* 6 *cùn*. How many slips should there be? Reply: It makes 343 and ⅛ slip. Method: Take 16 *cùn* as the divisor.

Content:
The heading of this section contains the first occurrence so far in this text of the

term *chéng* 程, familiar from Qín administrative documents with the sense of ‘regulation’, ‘standard’, or (as suggested here) ‘norm’ as a standard for productivity; compare Hulsewé (1985) 61. For examples, see 45-46 in Shuìhǔdì (1990) where there are three items labelled *gōng rén chéng* 工人程 ‘*chéng* for workmen’ detailing standards for production by workers. Note 1 of Shuìhǔdì (1990) to strip 108 interprets 程 in this context as the amount of work done per day. This word is found twenty times in the present text, usually in circumstances that suggest that the writer is quoting some official standard. It is therefore fitting that the first instance of its use should refer to the production of bamboo writing slips, the standard office stationery of the early imperial bureaucrat.

Mathematical note:
The two problems here, obviously from the same source, simply look at the consequences of changing the circumference of the bamboo to be used, and the length of the strips to be cut. The width of strips and the length of bamboo to be used do not enter into the calculation. Unusually, the ‘method’ section gives only a brief gesture towards the calculations that is to be made, which are of course:

$9 \times 183 / 8 = 1647 / 8 = 205 \frac{7}{8}$

$366 \times 15 / 16 = 5490 / 16 = 343 \frac{1}{8}$

S72(damaged and only partly decipherable),S73(perhaps 5 character gap)

> (29) The doctor
> The Norm: A doctor treating the sick gets 60 strings of cash ... 20 strings of cash ... the Norm ... not .... How much does he owe if he gets 60? Reply: he owes 17 strings and 11/269 strings. Method: take the strings he gets now as the divisor; let 60 multiply the strings he owes as the dividend.

Content:
This reference to a doctor is fascinating in a number of ways. Firstly, there is the mere fact that an official interest is taken in his income. Is he being taxed, or is he being paid in some way out of official funds? It seems clear that here *fù* 負 should be taken in the sense of ‘owe’ rather than ‘carry’ as on strip 38: cf. Hulsewé (1985) 98. If he is being paid. is he a state employee? Then there is the size of the amount involved, 60 strings of cash (this is, by the way, the only time the word *suàn* 筭 occurs in the present text, apart from in the title). While in the title it clearly refers to calculation, here it can hardly mean anything other than a string of cash, and 60 strings is quite a large amount. Unfortunately the indecipherable state of much of the strips greatly reduces our ability to draw useful conclusions from this material. Furthermore, it is not certain that strip 73 really belongs after strip 72 (which ends with ‘not ...’). The only reasons for the link are the fact that both have the word *suàn* 筭, and the figure 60 is common to both.

Mathematical note:
The calculation here seems to amount to:

17 11/269 = 60 x o/s

where o is the number of strings he 'owes' when the number of strings he 'gets' is s

Thus

4584/269 = 60 x o/s

therefore:

768/269 = 10 x o/s

But from this no reasonable value with o and s as small whole numbers emerges. Guō Shūchūn (2001, 209) deals with this situation by inserting a large number of characters to give figures that work, but there is clearly no room for any of this on the strips themselves.

[Group 6: Rating by unit]

S74,S75(long gap)

(30) Rating by the *shí*

The method for rating by the *shí* says: take what is bought or sold as the divisor; multiply the number [of units of the relevant measure in] 1 *shí* by the cash obtained. If there is a half in the lowest [place], double it. If [there is] a diminished half, 3-fold it; in cases where there are *dǒu* and *shēng*, or *jīn*, *liǎng*, and *zhū*, then in such cases throughout break the higher [units into lower ones], and let the lower ones follow them [in addition] to make the divisor. As for what the cash multiplies, likewise break it like this.

Content:
This seems to be the general statement for which 31 serves as a specific example.

S76,S77(long gap)

(31) Pricing salt

Now there is salt: 1 *shí* 4 *dǒu* 5 *shēng* and a diminished half *shēng*. As a price one obtains 150 cash. It is desired to rate it by the *shí*. How many cash does that make? Reply: 103 cash and 92/436 cash. Method: 3-fold the amount of salt to make the divisor; likewise 3-fold the number of *shēng* in 1 *shí*; multiply it by the cash to make the dividend.

Mathematical note
Following the method previously given we 'break' the higher units in 1 *shí* 4 *dǒu* 5 *shēng* to obtain 145 *shēng*. To deal with the extra ⅓ ('diminished half') *shēng* we multiply 145 by three and add one to obtain 436. This is 'what is bought or sold', and gives us the divisor. There are 100 *shēng* in 1 *shí*, so this is 'the number [of units of the relevant measure in] 1 *shí*'. This too must be multiplied by 3 because of the way we dealt with the 'diminished half'. Finally we multiply by the number of cash and the calculation is:

300 x 150/436 = 103 and 92/436

Parallels:
The Nine chapters has a group of problems in the its chapter (2: *Sù mǐ* 粟米, Guō 1990, 233-234; Shen 1999, 151), of which the last is representative and similar to the ensemble of 30 and 31 given in the present text:

> **今有出錢一萬三千六百七十買絲一石二鈞一十七斤欲石率之問石幾何**
>
> **荅曰**
> **一石八千三百二十六錢一百九十七分錢之一百七十八**
>
> **經率術曰**
> **以所求率乘錢數為實，以所買率為法，實如法得一**
>
> Now one pays out 13670 cash, to buy 1 *shí* 2 *jūn* 17 *jīn* of silk. It is desired to rate it by the *shí*, Question how much is a *shí*?
>
> Answer:
> 1 *shí* is 8326 cash and 178/197 cash.
>
> The method for direct rating says:
> multiply the number of cash by that according to which you seek to rate it, to make the dividend; make the rate at which you buy the divisor; obtain 1 for [each time] the dividend accommodates the divisor.

S78(long gap)

(32) Undressed and dressed silk fibre
Seeking dressed silk from undressed fibre: take and 12-fold it. Obtain 1 for each reduction by 16.

Content:
This is a simple statement of a conversion ratio between the weights of silk fibre at successive stages of processing. This is one of the cases where modern 'division' is

effected by repeated 'reduction', i.e. subtraction.

Parallel:
Liú Huī's commentary to a problem in the Nine Chapters about the conversion of types of fibre (6: *Jūn shū* 均輸, Guō 1990, 327; Shen 1999, 326) points to the same conversion rate as specified here:

> ... 此絡率一六練率十二也
>
> ...So here the rate for unreeled fibre is 16 and the rate for dressed fibre is 12.

S79,S80,S81,S82(long gap)

(33) Worked fat

(a) There are 3 *dǒu* of hulled grain. Question: how much each of fat and water will be used to make how much worked [product]? Reply: Use 6 *jīn* of fat and 4 *shēng* and half a *shēng* of water, to make worked fat, 10 *jīn* 12 *liǎng* 19 *zhū* ⅕ *zhū*.

(b) To make worked [product]: 1 *dǒu* of hulled grain, one *dǒu* and half a *dǒu* of water, 20 *jīn* of X fat, make 36 *jīn* of worked fat. Now there are 5 *jīn* of Y fat. Question: using hulled grain and water to make worked [product], how much is it of each? Result: use 2 *shēng* and half a *shēng* of hulled grain, and 3 *shēng* and ¾ *shēng* of water, to make 9 *jīn* of worked [product]. Method: Take 20 as the divisor; set out 15 of water, 10 of hulled grain and 36 of worked [product], and multiply by 5 to make the dividends; [for each time] the dividends accommodate the divisor, obtain 1 *shēng* of water or hulled grain, and 1 *jīn* of worked [product]. For what does not fill [the divisor], designate the parts according to the divisor.

(c) From each one *liǎng* of worked [product], hulled grain, and X, one gets 5/9 of Y.

Content:
The nature and purpose of this (to modern ears) rather unpleasant sounding product (fat, hulled grain and water, somehow amalgamated) are not clear. In (b) and (c), X and Y represent two unknown characters 峑 and 峕 evidently designating types of fat. But mathematically, the problem is a simple one of the proportion of components in a mixture. The text runs continuously from one strip to the next. This section divides fairly clearly into (a) in which an answer to a problem is given without working, and (b) in which a statement of the proportion of ingredients for a given amount of product leads on naturally to a stated problem, a solution and a method statement. In (c) we may be seeing a garbled statement of the fact that each unit of product corresponds to 5/9 of a unit of fat.

Mathematical note:
On the basis of the emended text, the figures in (b) work out in obvious proportions:

water (dou):hulled grain (dou): fat (jin): product (jin) :: 15: 10: 20: 36

Now in (a) the amounts of water, hulled grain and fat are 3/10 those given in (b). Since

36 *jīn* = 36 x 16 x 24 *zhū* = 13824 *zhū*

the expected amount of product in (a) is

3/10 x 13824 *zhū* = 4147 *zhū* and ⅕ *zhū* = 10 *jīn* 12 *liǎng* 19 *zhū* ⅕ *zhū*

So the figures in (a) and (b) are consistent.

[Group 7: wastage, and equivalents]

S83

(34) The Norm for collecting
(a) The Norm for collecting is 10 *bù* for 1 *dǒu*. Now when we dry it, [we get] 8 *shēng*. Question: how many *bù* [for] one *dǒu*? Result: 12 *bù* and a half [gives] 1 *dǒu*. Method: The 8 *shēng* makes the divisor; set out the number of *bù* for one *dǒu* and 10-fold it; [for each time it] accommodates the divisor, [count] 1 *bù*.

S84(8 character gap)

(b) The Norm [for collecting]: [from] 37 *bù* one gets 19 *dǒu* and 7 *shēng* of millet. Question: how many *bù* for 1 *dǒu*? Result: Reducing the field, 1 *bù* and 173/197 *bù* for 1 *dǒu*

S85(2 character gap)

(c) The Norm for collection: 5 *bù* for 1 *dǒu*. Now when one dries it, [one gets] 1 *dǒu* 1 *shēng*. It is desired to reduce the field to make it 1 *dǒu*. Result: Reduce the field by 5/11 *bù*. Method: Multiply 5 *bù* by the number of *shēng* in 1 *dǒu*. Let there be 1 for 11.

Content:
This section consists of three subsections all apparently relating to the same basic

theme: there is a norm for the taxable productivity of a certain field, but when the produce is dried after harvest it is found to depart from this. Since there is no continuity of text from one strip to another, the only evidence that they belong together is that they all clearly fall under the topic named in the title of strip 83. In (a) the field is under-producing and the problem is to find its true rate of production. In (c) a piece of land that is over-producing has land removed from it to bring its total product back to the norm. In (b) the problem seems to be to find the amount of land to produce one unit of grain, though the figures only work as here on the basis of an emended text. This section is clearly related to 35.

S86,S87(long gap)

(35) Wastage on tax

In wastage on tax, the production is more and the dried [product] is less. The Norm for collection is 7 *bù* ¼ *bù* [yielding] 1 *dǒu*. Now one dries it to 7 *shēng* and a diminished half *shēng*. It is desired to take the number of *bù* for 1 *dǒu*. Method: Set out 10 *shēng* to multiply the 7 *bù* ¼ *bù*. Every one accommodation of the dried [norm] is one count. Reply: 9 *bù* 39/44 *bù* for 1 *dǒu*. The Norm for other things is similar to this.

Content:
This is another 'wastage' problem similar to 33, though seemingly from a different source.

Mathematical note:
The calculation works as follows
$10 \times (29/4)/(22/3)$
$= (3 \times 29)/(88)$
$= 9 + 78/88$
$= 9 + 39/44$

S88(2 character gap; Wáng)

(36) The norm for millet

(a) The Norm: 1 *shí* of *hé shǔ* [panicled millet?] makes 16 *dǒu* and augmented half a *dǒu* of unhulled grain. Hull it to make 1 *shí* of *lì* hulled grain. 1 *shí* of *lì* hulled grain [is] 9 *dǒu* of *zuò* hulled grain. 9 *dǒu* of *zuò* hulled grain makes 8 *dǒu* of *huǐ* hulled grain.

S89(3 character gap)

(b) The Norm: 1 *shí* of *dào hé* [rice?] makes 20 *dǒu* of unhulled grain. Hull it to make 10 *dǒu* of hulled grain. It makes *huǐ càn* hulled grain 6 *dǒu* and ⅔ *dǒu*. 10 *dǒu* of wheat [is] 3 *dǒu* of [fine] crushed wheat.

S90(long gap to end)

(c) The Norm: wheat, soybeans, beans and hempseed are 15 *dǒu* for 1 *shí*. In conferring *huǐ* and *zuò* there are 10 *dǒu* to 1 *shí*.

Content:
This is section deals with the topic of 'grain equivalents', to which a whole chapter of the Nine Chapters is devoted (2: *Sù mǐ* 粟米, Guō 1990, 213-234; Shen 1999, 134-156). In problems related to this topic, we are typically concerned to convert one type of grain into an equivalent amount of some other type, either by exchange for equal value, or by finding the amount yielded when the original grain is subjected to processing, ranging from simple hulling to various degrees of polishing. No problems are stated in this section: we are simply given the ratios for various conversions. Related sections are the run of seven sections from 39-45 inclusive, and section 52. The reader may wish to refer to the discussion of translating grain names in the introduction.

Parallels:
Péng Hào (2001) 5 notes that the text of this section is very close to the Qín regulations from Shuìhǔdì; see Shuìhǔdì (1990) 29-30:

> [top of strip is broken - possibly 5 missing characters] 石六斗大半斗。舂之為糲米一石。糲米一石為鑿米九斗。九【斗】為毀米八斗。稻禾一石 (19 irrelevant characters intervene here , apparently through a scribal error) 為粟廿斗。舂為米十斗。十斗。粲毀米六斗大半斗。麥十斗。為 麴三斗。叔荅麻十五斗一石稟毀粺者以十斗為一石
>
> [...] *shí* 6 *dǒu* and augmented half a *dǒu* of unhulled grain. Hull it to make 1 *shí* of *lì* hulled grain. 1 *shí* of *lì* hulled grain makes 9 *dǒu* of *zuò* hulled grain. 9 [*dǒu* of *zuò* hulled grain] makes 8 *dǒu* of *huǐ* hulled grain.
>
> 1 *shí* of *dào hé* [...] makes 20 *dǒu* of unhulled grain. Hull it to make 10 *dǒu* of hulled grain. 10 *dǒu* [of hulled grain makes] *càn huǐ* hulled grain 6 *dǒu* and ⅔ *dǒu*. 10 *dǒu* of wheat [is] 3 *dǒu* of [fine] crushed wheat.
>
> Soybeans, beans and hempseed are 15 *dǒu* for 1 *shí*; In conferring *huǐ* and *bài* there are 10 *dǒu* to 1 *shí*.

Note that this material does not use the expression 'Norm' *chéng* 程 which begins each strip in the *Suàn shù shū* . The reason is, perhaps, that these regulations (or more probably some other version of them) *are* the 'Norm' from which the *Suàn shù shū* is quoting at this point. Compare Hulsewé (1985) 40-43.

Mathematical note:
For comparison with later information, we may note the equivalents given in 35 (a) in tabular form, together with the relevant equivalences from the start of the Nine Chapters *Sù mǐ* chapter :

SSS 35(a)	*hé shǔ*	*sù*	*lì mǐ*	*zuò mǐ*	*huǐ mǐ*
	10	16 ⅔	10	9	8
9 chapters	(absent)	*sù*	*lì mǐ*	*bài mǐ*	*zuò mǐ*
		50	30	27	24

It seems clear that *hé shǔ* (panicled millet?) is exchanged for a greater quantity of *sù (*unhulled setaria millet), which is then processed to three successive degrees of fineness. The terms *sù*, and *lì mǐ* are found in both texts for grain at the first two stages of processing, but whereas the Nine Chapters continues with *bài mǐ* and *zuò mǐ*, the last two stages in the present text are *zuò mǐ* and *huǐ mǐ.* Nevertheless the figures for the successive stages correspond perfectly, since:

16 ⅔:10:9:8 :: 50:30:27:24

As for the identity of *hé shǔ*, we may note that it exchanges with *sù* (unhulled millet) in the ratio 30 : 50, just as does *lì mǐ.* However in section 53 (b) (Strips 138-140) prices for *shǔ* and *mǐ* (which one would expect to be the same as *lì mǐ*) are given which suggest that these exchange in the ration 9 :10. So if the sources of these strips are all consistent (which is a large assumption) we might be in error in rendering *hé shǔ* as 'panicled millet' in this case.

Turning to (b) and its Shuìhǔdì parallel, we may note that whatever *dào hé* may be, it exchanges for unhulled millet in the ratio 10:20. Since the 'equivalence number' for unhulled millet in the Nine Chapters is 50, we should expect that the equivalence number for *dào hé* would be 25. But no product has that value in the Nine Chapters. It is also off to see that unhulled and unhulled grain interconvert at the ratio 20:10, which is again not in correspondence with the Nine Chapters. However (b) then goes on to tell us that hulled grain and *huǐ càn* hulled grain (or *huǐ càn* hulled grain as in the Shuìhǔdì material) interconvert at the ratio 10 : 6 ⅔ = 30 :20. That suggests that the type of hulled grain referred to here was a little finer than the **御米** 'imperial millet' of the Nine Chapters, for which the conversion ration would be 30 : 21. The final conversion in (b), of wheat to crushed wheat, is in the same ration as that given in the Nine Chapters for fine crushed wheat.

In (c) the initial statement implies that wheat, soybeans, beans and hempseed convert with something else at the ratio 15 :10. All these have the equivalence number 45 in the Nine Chapters, so that they would be interconverting with a commodity of equivalence number 30 - which turns out to be hulled millet. The significance of the final statement about *huǐ* and *zuò* hulled grain (*huǐ* and *bài in* Shuìhǔdì) is unclear.

The discussion of this material in the notes of Shuìhǔdì (1990) 29-30 brings to bear on these problems similar statements about the identities and relative values of types of grains in the *Shuō wén jiě zì* **說文解字** dictionary of AD 121. But for the present it does not seem necessary to argue at length on the more lexicographical aspects of the material now under discussion.

S91,S92(long gap)

(37) The Norm for collecting hemp

In collecting hemp, 10 *bù* give 1 bundle [of circumference] 3 *wéi* Now one dries it [and it becomes] 28 *cùn*. Question: how many *bù* yield one [standard] bundle? Method: Let the dried multiply itself to make the divisor. Let the fresh multiply itself, and further multiply by the number of *bù* [through] the dividend accommodating the divisor, one gets 11 *bù* and 47/98 *bù* for 1 bundle.

Content:
Another 'wastage' problem, this time with the interesting feature of the special circumferential measurement 韋 *wéi*, which is apparently 10 *cùn*.

Mathematical note:
The wastage is measured directly by noting that the circumference of a bundle shrinks on drying from 3 *wéi* (30 *cùn*) to 28 *cùn*. But, as the author of this problem is evidently aware, the quantity in the bundle depends on its cross-section area, which varies as the square of the circumference. Thus we have the final calculation to find the number of *bù* to produce a 3-*wéi* circumference dry bundle:

(30 x 30 x 10) / (28 x 28) = 9000/784

= 11 376/784

= 11 47/98

[Group 8: allowing for mistakes]

S93 (2 character gap), S94, S95 (long gap)

(38) Error in ticketing

(a) In taxing millet, when there has been mistaken ticketing: Method: when there are no *shēng*, take the quantity of dutiable field as the dividend and take the ticketed *dǒu* as 1, and let the number of *shí* be made tenfold. Combine to make the divisor. Accommodating the divisor, obtain 1 *bù*.

(b) As for the cases where the ticket has *dǒu*, set out the number of *bù* in the given field as the dividend, then take the ticketed *dǒu* as 1; let the *shí* be made tenfold; combine to make the divisor; accommodating the divisor, obtain 1 *bù*.

As for the cases where the ticket has *shēng,* set out the number of *bù* in the given field as the dividend, then take the *shēng* of the ticket as 1; let the *dǒu* be made tenfold; combine to make the divisor; accommodating the divisor, obtain 1 *bù*.

Content*:*
This section, like the next, is concerned with dealing with incorrect tax assessment, when the object is to find the area of the field in *bù* which yields a unit amount of product. The text seems to run continuously despite the short gap at the end of strip 93. The two procedures in (b) are clearly in the same pattern, and explain how to deal with cases where the smallest units are respectively *dǒu* and *shēng* while (a) seems to state the same method as the first part of (b) in different words - evidently it is from a different source.

S96,S97(long gap)

(39) Taxation: error in ticketing
A field of 1 *mǔ*: tax it at 1 *dǒu* for 10 *bù*. The overall tax is 2 *shí* 4 *dǒu*. Now it is wrongly ticketed at 2 *shí* 5 *dǒu*; it is desired to increase or cut down the number of *bù*. Question: how much should the increase or decrease be? Reply: 9 *bù* and ⅗ *bù* for 1 *dǒu*. Method: take the mistaken ticketing as the divisor; take the given field as the dividend.

Mathematical note:
Clearly the appropriate calculation here is:

area for 1 *dǒu* = (10 *bù*) x (2.4 *shí* / 2.5 *shí*)

which gives 9 *bù* and ⅗ *bù* as stated. To make sense of the text, it seems necessary to take 'the given field' as meaning 10 *bù* x 2.4 *shí*, which seems somewhat confused.

[Group 9: converting grains]

S98 (Yáng), S99, S100(long gap)

(40) *Bài* and *huǐ.*

(a) Hulled grain, a diminished half *shēng*, makes *bài* [hulled grain], 3/10 *shēng*;

9-fold it and take 1 for 10. Hulled grain, a diminished half *shēng*, makes *huǐ* hulled grain 4/15 *shēng*. 8-fold it and take 1 for 10. Hulled grain, a diminished half *shēng*, makes wheat, half a *shēng*. 3-fold it and take 1 for 2.

(b) Wheat, a diminished half *shēng*, makes unhulled grain, 10/27 *shēng*. 9-fold the denominator and 10-fold the numerator, [that is,] 10-fold it and take 1 for 9. Wheat, a diminished half *shēng*, makes hulled grain, 2/9 *shēng*. 3-fold the denominator and double the numerator, [that is,] 2-fold it and take 1 for 3. Wheat, a diminished half *shēng*, makes *bài* [hulled grain], ⅕ *shēng*. 15-fold the denominator and 9-fold the numerator, [that is,] 9-fold it and take 1 for 15. Wheat, a diminished half *shēng*, makes *huǐ* [hulled grain], 8/45 *shēng*. 15-fold the denominator and 8-fold the numerator.

S101(Yáng), S102, S103, S104 (long gap)

(c) *Bài* hulled grain, ¼ *shēng* makes unhulled grain, 25/54 *shēng*. 27-fold the denominator and 50-fold the numerator.
Bài hulled grain, ¼ *shēng* makes hulled grain, 5/18 *shēng*. 9-fold the denominator and 10-fold the numerator.
Bài hulled grain, ¼ *shēng* makes *huǐ* hulled grain, 2/9 *shēng*. 9-fold the denominator and 8-fold the numerator.
Bài [hulled grain], ¼ *shēng* makes wheat, 5/12 *shēng*. 9-fold the denominator and 15-fold the numerator.
Huǐ hulled grain, ¼ *shēng* makes hulled grain, 5/16 *shēng*. 8-fold the denominator and 10-fold the numerator.
Huǐ [hulled grain], ¼ *shēng* makes *bài* [hulled grain], 9/32 *shēng*. 8-fold the denominator and 9-fold the numerator.
Huǐ hulled grain, ¼ *shēng* makes wheat, 15/32 *shēng*. 8-fold the denominator and 15-fold the numerator.
Huǐ hulled grain, ¼ *shēng* makes unhulled grain, 25/48 *shēng*. 24-fold the denominator and 50-fold the numerator.

Content:
The aim of this lengthy passage is evidently to enable *sù* (unhulled grain, presumably setaria millet) and *mǐ* (hulled grain) to be interconverted with the successive further stages of processed grain *bài* and *huǐ*, and also with wheat. Although the text of (a) and (b) runs continuously, there does seem to be a clear change of style with (b). Subsection (c) begins at the head of a strip after a long gap at the end of a previous strip, and has its first strip marked with the name of Yáng, which suggests it is a separate unit of material. Unlike the first three clauses of (b) it does not add explanatory glosses giving the effect of the changes in numerator and denominator prescribed.

Mathematical note
The rules for conversion given here are clumsy compared with the neat tabulation given in the Nine Chapters for a similar purpose (2: *Sù mǐ* 粟米, Guō 1990, 213-214; Shen 1999, 141), in which each type of grain is given an 'equivalent' with unhulled millet setting the standard at 50. Comparing the figures given here with those of the Nine Chapters, it soon becomes clear that the 'equivalents' given in the Nine Chapters work perfectly with the stated relations between commodities in the present text, so long as names are slightly revised. This is best set out in tabular

form;

Nine Chapters	wheat	*sù*	*lì*	*bài*	*zuò*
Equivalents:	45	50	30	27	24
SSS 40	wheat	*sù*	*mǐ*	*bài*	*huǐ*

Thus for instance, we are told at the start of (a) that ⅓ of a unit of hulled grain *mǐ* is equivalent to 3/10 units of *bài*. Now :

(⅓) : (3/10) = 10 : 9 = 30 : 27

so the proportions are the same as in the Nine Chapters. The reader may easily check that this holds for all the other equivalents in this section.

Note that the sequence of grain processing terms here is also different from that seen in section 35 (a), which had *sù*, *lì*, *zuò*, *huǐ*. Presumably these discrepancies of terminology indicate that the relevant sections of text came from different sources, both of which represent terminologies slightly different from that of the Nine Sections. It is therefore noteworthy that this section contains the name Yáng, while section 35 contains the name Wáng.

S105(Yáng),S106(2 character gap)

(41) Wastage

(a) 1 *shí* of unhulled grain has a wastage of 1 *dǒu* 2 *shēng* and a diminished half *shēng*. In a case of conferring a diminished half *shēng* of hulled grain, one gets 500/789 *shēng* of unhulled grain. In the case of conferring 1 *shēng*, one gets 1 *shēng* 237 /263 *shēng* of unhulled grain. In the case of conferring 1 *dǒu*, one gets 1 *dǒu* 9 *shēng* and 3/263 *shēng* of unhulled grain. In the case of conferring 1 *shí*, one gets 19 *dǒu* 30/263 *shēng* of unhulled grain.

S107(Yáng)S108(5 character gap)

(b) A *shí* of unhulled grain wastes 5 *shēng.* In a case of conferring a diminished half *shēng* of hulled grain, one gets 100/171 *shēng* of unhulled grain. In the case of conferring 1 *shēng*, one gets 1 *shēng* 215 /285 *shēng* of unhulled grain. In the case of conferring 1 *dǒu*, one gets 17 *shēng* and 155/285 *shēng* of unhulled grain. In the case of conferring 1 *shí*, one gets 17 *dǒu* 5 *shēng* 125/285 *shēng* of unhulled grain.

Content:
This is a pair of procedures involving wastage at two different rates, but both involving the conversion of unhulled to hulled grain at the nominal rate of 50:30, as elsewhere. The material is not stated in the usual form of problem, answer and procedure, but simply in the form of final conversions. As in 39, the name Yáng occurs in the text.

Mathematical note:
The figures here seem puzzling at first sight. Evidently, the principle behind these calculations is that the unhulled grain is first converted to hulled grain at the usual 50:30 ratio. Wastage is then allowed for by subtraction. Choosing easy multiples, if one started with 500 units of unhulled grain, that would yield 300 units of hulled. At the rate of wastage given in (a), since one deducts 12 and ⅓ *shēng* from 100 *shēng*, that implies that from the 300 units of hulled grain expected there would in fact result:

$300 - 3 \times (12 + ⅓) = 300 - 37 = 263$ units,

which is what results from the initial 500 units of unhulled grain.

Thus, to solve the first example, if the result is to be ⅓ units one must start not with 500 units, but with $500/(3 \times 263) = 500/789$ units, as stated here. All the other calculations follow the same pattern.

S109(Yáng), S110(long gap)

(42) Unhulled becomes hulled
Hempseed, wheat, soybeans and beans - 3 match 2 of hulled grain; 9 match 10 of unhulled grain; 5 of unhulled make 3 of hulled; 10 of hulled make 9 of *bài*, and make 8 of *huǐ*. 3 of wheat match 4 of unhulled *dào*; 5 of unhulled *hé* make 4 of unhulled *dào*;

S111(Yáng),S112(long gap)

(43) Seeking hulled from unhulled grain
Seeking hulled from unhulled, 3-fold it and take 1 for 5; Seeking wheat from unhulled, 9-fold it and take 1 for 10; Seeking *bài* from unhulled, 27-fold it and take 1 for 50; Seeking *huǐ* from unhulled, 24-fold it and take 1 for 50; Seeking unhulled from hulled, 5-fold it and take 1 for 3.

Content:
More text from Yáng continues with the theme of conversions of grains. The rules given in (42) are consistent with preceding material by Yáng in section 40, so far as they refer to the relations of members of the sequence unhulled grain, hulled grain, *bài* hulled grain and *huǐ* hulled grain. The statement that for hempseed, wheat, soybeans and beans 3 match 2 of hulled grain is consistent with the Nine Chapters use of an equivalence number of 45 for the first four and 30 for the last. The ration 3 of wheat to 4 of unhulled *dào* suggests that we need to find an equivalence number of $3 \times 45/4 = 33\ ¾$ for an equivalent of 'unhulled *dào*' in the Nine Chapters. But no number of this value is to be found there. On similar reasoning, unhulled *hé* would have an equivalence of $3 \times 45/5 = 27$, the same as that for the stage after the initial hulling of millet.

Turning to 43, we see that it contains rules that in effect repeat the same ratios as given in section 40 as well as in 42 for wheat, unhulled grain, hulled grain, *bài* hulled grain and *huǐ* hulled grain. So material with the name of Yáng on it effectively repeats the same information no less than three times in different formulations. The suggestion is clear that Yáng (whoever he was and whatever was his precise role) was involved in the collection of material on the same topic from varied sources, just as was the overall compiler of the *Suàn shù shū* .

S113,S114(long gap)

(44) Seeking hulled from unhulled grain

Seeking hulled from unhulled grain, take and 3-fold it, then for 5 complete 1. Now we have unhulled grain, 1 *shēng* and 3/7; How much hulled grain does that match? Reply: It makes 6/7 *shēng* of hulled grain. Method: the denominators are multiplied together to make the divisor; multiply 10 by 3 to make the dividend.

Content:

This section clearly forms a pair with the next one.

S115,S116(long gap)

(45) Seeking unhulled from hulled grain

From hulled grain seeking unhulled, take and 5-fold it, then for 3 complete 1. Now we have hulled grain, 6/7 *shēng*; How much unhulled grain does that make? Reply: It makes 1 *shēng* 3/7 *shēng* of unhulled grain. Method: the denominators are multiplied together to make the divisor; multiply 6 by 5 to make the dividend.

Content:

These two subsections are inverse problems, both involving the simple hulled/unhulled conversion seen several times previously, but stating it with the full apparatus of problem, result and method, which is not a feature found with such problems before this point. Neither Yáng nor Wáng are identified here.

S117,S118(9 character gap)

(46) Hulled and unhulled grain combined.

There is 1 *shí* of hulled grain and 1 *shí* of unhulled grain. They are combined. Question: [the owners of] hulled and unhulled should each take how much? Reply: The owner of the hulled grain takes 1 *shí* 2 *dǒu* 8/16 *dǒu*; The owner of the unhulled grain takes 7 *dǒu* 8/16 *dǒu*. Method; Set out hulled grain 10 *dǒu* and 6 *dǒu*, and combine to make the divisor; separately multiply what has been set out by 2 *shí* so that each makes a dividend by itself. The 6 *dǒu* is the number of the unhulled grain in hulled [terms]

Content:

This is another formally stated problem, with result and method, this time based on the sharing of a pool of commodities in proportion to the amount contributed.

The usual conversion ratio is used. The final sentence appears to be an explanatory note rather than part of the main 'method' statement.

Parallel:
The Nine Chapters discusses a problem of pooling grains embodying the principle used here (3: *Cuī fēn* **衰分**, Guō 1990, 241; Shen 1999, 168-169). It is clear that the Nine Chapters performs in a highly systematised way the process that in the *Suàn shù shū* is still somewhat *ad hoc*, with the conversion of one grain into another being explained as an afterthought:

> **今有甲持粟三升乙持糲米三升丙持糲飯三升欲令合而分之問各幾何**
>
> **荅曰**
> **甲二升一十分升之七·乙四升一十分升之五·丙一升一十分升之八·**
>
> **術曰**
> **以粟率五十糲米率三十糲飯率七十五為衰而返衰之副并為法**
> **以九升乘未并者各自為實實如法得一升**
>
> Now we have: A brings 3 *shēng* of unhulled grain; B brings 3 *shēng* of *lì* hulled grain; C brings 3 *shēng* of *lì* cooked hulled millet; it is desired to combine and share them. Question: how much for each?
>
> Answer:
> A: 2 *shēng* 7/10 *shēng;* B: 4 *shēng* 5/10 *shēng;* C: 1 *shēng* 8/10 *shēng*
>
> Inversely differentiate them, using the rate for unhulled grain, 50, the rate for *lì* hulled grain 30, and the rate for *lì* cooked hulled millet 75 as the differentials, Do an auxiliary addition to make the divisor; Make each dividend by multiplying the as yet uncombined [factors] by 9 *shēng*; Obtain 1 *shēng* for each time the dividend accommodates the divisor.

S119(Wáng),

(47) Unhulled and hulled combined

(a) 1 hulled to 2 unhulled: in all 10 *dǒu*. One refines it to make 7 *dǒu* ⅓ *shēng*. Method: In all [cases] 5-fold the hulled and unhulled and combine to make the divisor; 5-fold the hulled and 3-fold the unhulled [and add them together]; multiply it by 10 *dǒu* to make the dividend.

S120(only ⅓ of a damaged strip)

...gets how much? Reply: unhulled grain ... 30 ... hulled grain ...

S121(poor condition, especially at start; Yáng),S122(6 character gap)
(b) Combining unhulled and hulled: 5 *shēng* each of unhulled and hulled grain are put together. How much does [the owner of the hulled grain] get? Result: 6 *shēng* and ¼ *shēng* of hulled grain. Method: Set out the 5 *shēng* of hulled grain. The 5 *shēng* of unhulled grain makes 3 *shēng* of hulled grain; combine [with] the 5 *shēng* of hulled grain [so] 8 is made the divisor; thereupon further set out the 5 *shēng* and 10-fold it making that accommodate the divisor gives 1 *shēng* for both unhulled and hulled grain.

S123(bottom third only;Yáng),S124(broken at start, then again after 4 characters, characters resume, long gap at end),
(c) ....*2 dǒu* 5 *shēng*. Method: Set out hulled and unhulled; 5-fold the hulled and 3-fold the unhulled .... combine to make the divisor .... combine hulled and unhulled and in each case multiply to make the dividend; form 1 for [each time] the dividend accommodates the divisor;

S125(nearly full length strip, but mostly obliterated)
.... *shí* 50 and ....

Content:
Although the lacunae are lengthy, there is enough evidence by way of internal structure and external parallels to enable us to attempt a rational restoration of the content of these strips in several places. There are clearly three different problems here, all dealing with mixtures of hulled and unhulled grains. Since (a) bears the name of Wáng and (b) and (c) are both labelled Yáng, it does seem very likely that a new section title appeared somewhere in the lacuna with which strip 121 begins.

In (a) the problem is to find the hulled equivalent of a mixture of hulled and unhulled in stated proportions - 'refining' 精 *jīng* evidently refers to the process of converting the whole mixture to unhulled grain, which uses the usual 5:3 ratio. The calculation specified is, in effect:

$(5 \times 1 + 3 \times 2) \times 10/(5 \times 3)$
$= 110/15$
$= 7 \frac{1}{3}$

In (b) the problem is clearly how much each contributor gets if 5 *shēng* each of hulled and unhulled grain are pooled. Despite the last clause, only one share is calculated, being that for the owner of the hulled grain. The calculation specified is:

$(5 \times 10)/(5+3)$
$=50/8$
$= 6 \frac{1}{4}$

As for (c), while the general topic is still clearly the mixture of hulled and unhulled grain there is not enough information here to reconstruct the original question reliably.

[Group 10: Rationalising and checking tasks]

S126,S127,S128(long gap)

(48) Carrying charcoal

[A man] carries charcoal in the mountains; in a day he makes finished charcoal, 7 *dǒu*, and gets it to the cart; the next day, he carries the charcoal and leading the cart, gets to the office, [with] 1 *shí*. Now it is desired that carrying charcoal through the journey to the office, he should take the charcoal all the way to the office. Question: how much charcoal arrives each day? Reply: In a day, the charcoal obtained is 4 *dǒu* 2/17 *dǒu*. Method: take the 7 *dǒu* and 10-fold it to get 7 *shí*; In 7 days that is what he carries to the office; then take 10 days and 7 days, and combine to make the divisor; obtain 1 *dǒu* for each time [the dividend of 7 *shí*] accommodates the divisor.

Content:
This is the first of a small group of three problems in which the object is to set a new overall norm for productivity after amalgamating tasks previously done separately. Here some unfortunate (probably a convicted criminal) has been set to make charcoal and deliver it to an official receiving post some distance away. Here *fù* 負 might mean either 'carry' or 'owe' (compare strips 38 and 73); I have chosen the former, since it seems to fit a little better.
The worker cannot simply be allowed to alternate days making and delivering charcoal, since the amount of charcoal he can make in each day is only 7/10 of the amount he could carry to the post the next day - so he would not be fully occupied, which can by no means be permitted. The official imagined here is therefore trying to find that amount of charcoal produced in one day that can just be transported to the office in the remaining working time. Of course the calculation is made completely without reference to the practicalities of making the charcoal, a process which it is assumed can be stopped and started at will. If we regard this as a practical problem, it seems that the desire to be able to maintain control over the labourer on a daily basis is seen as paramount. Otherwise we might see this as an instance where a problem stated in particular terms is in fact intended to illustrate a general mathematical principle.

Mathematical Note:
The solution proceeds by noting that in 10 days he could produce ten times 7 *dǒu* , which is 7 *shí*. This can then be transported to the office in 7 days at the rate of 1

shí per day. So his daily rate of charcoal production is:

7 *shí*/17 = 70 *dǒu*/17 = 4 + 2/17 *dǒu*

Parallel:
See below, section 50.

S129,S130(long gap)

(49) Bamboo tubes

The Norm: In one day cutting bamboos: 60 pieces; in one day making bamboo tubes: 15; one bamboo makes 3 bamboo tubes. Now it is desired to order the same person to cut bamboos himself and use them to make bamboo tubes. In one day, how many does he make? Reply: he makes 13 ¾ bamboo tubes. Method: Take 60 as the divisor; Multiply 55 by 15 to make the dividend.

Text note:
The meaning of the term *lú táng* 盧唐 translated here as 'bamboo tubes' is something of a mystery. No such term seems to be known from ancient literature. However in the late Warring States *Zhōu lǐ* 周禮 we find a possible link in the section known as the 'Artificers' Record' *Kǎo gōng jì* 考工記. According to that text, the Lú rén 廬人 make *lú qì* 廬器, which are shafted weapons such as spears and halberds. Zhāngjiāshān (2001)identifies the *lú táng* 盧唐 of this text with the same two characters found with the addition of the 'bamboo' radicall 籚篖 in the lists of grave goods from Mǎwángduì tomb 1 and Fènghuángshān tomb 168, which appears to refer to bamboo tubes (used for storage vessels?).

Content:
This is another 'task rationalisation' text, in which the aim is to ensure that a norm for steady production is set.

Mathematical note:
Something seems to be wrong with the calculation here. Counting each bamboo providing 3 unworked pieces of bamboo of the right size to make tubes, we have:

Piece production: 180 per day
Tube production; 15 per day

Following the method in the next section, we find:

production per day = 180x15/(180+15) = 13 165/195 = 13 33/39 = 13 11/13

Here the calculation specified is:

55x15/60 which does indeed come out at 13 ¾ as stated here.

Note that 55x15/60 = 165 x 15/180 = (180-15)x15/180

So it looks as if someone has in effect wrongly assumed that subtracting 15 from the 180 in the dividend has the same effect as adding it to the divisor.

Péng Hào's explanation, followed by Anon. (2001) ignores the fact that each bamboo makes three tubes, and works on the basis of 60 bamboos/ day, 15 *lutang*/day, so joint production would be:

(60 x 15)/(60+15) = 12

but this requires rewriting the figures on the strips, so that (e.g.) 13 ¾ becomes 12, which seems hard to justify.

Parallel:
See below, section 50.

S131(10 character gap)

(50) Feathering arrows

The Norm: 1 man in 1 day makes 30 arrows; he feathers 20 arrows. Now it is desired to instruct the same man to make arrows and feather them. In one day, how many does he make? Reply: he makes 12. Method: Combine the arrows and feathering to make the divisor; take the arrows and the feathering multiplied together to make the dividend.

Content:
This is the last of the three 'combined norm' sections, 48-50. It seems quite clear and correct.

Parallel:
The Nine Chapters also has three 'combined norm' problems (6: *Jūn shū* **均輸**, Guō 1990, 335-7; Shen 1999, 340-341), with one of another type between the second and third examples, of which the following is most similar to the ones given here:

今有一人一日為牡瓦三十八枚一人一日為牝瓦七十六枚
今令一人一日作瓦牝牡相半問成瓦幾何

荅曰：
二十五枚、少半枚

術曰：
并牝、牡為法 牝牡相乘為實實如法得一枚

Now there is a man who in one day makes 38 male roof-tiles; another man makes 76 female roof-tiles in one day. Now one instructs the same man to make half each of female and male tiles in one day. Question: how many tiles does he complete?

Answer:
25 units and a dimished half unit.

Method:
Combine the female and males tiles to make the divisor; Multiply the male and female together to make the dividend; obtain one unit for [each time] the dividend accommodates the divisor.

Mathematical note:
In modern notation, it is easy enough to represent such problems. Taking the example from the Nine Chapters, it is clear that a male tile takes 1/38 day to make, and a female tile takes 1/76 day. Thus to make a pair takes (1/38 + 1/ 76) day, and hence the number made in one day is:

1/(1/38 + 1/ 76)

Multiplying top and bottom by 38x76, one obtains:

38x76/(76+38) for the number of pairs of tiles he can make in a day.

But surely we need to think of a way that this result could be obtained directly, by a single conceptual leap, without manipulation in the modern style? The route seems to be as follows.

Suppose the man makes 38x76 pairs of tiles. The time it takes to do this will be 76 days to make the 38x76 male tiles (since he makes 38 daily) and 38 days to make the 38x76 female tiles (since he makes 76 daily). Thus the total number of days to make 38x76 pairs will be (76+38) days. Hence the rate of tile-pairs per day is as stated:

38x76/(76+38) pairs per day, which is the result required.

S132

(51) Travelling
X travels for 50 days. Now today is [sexagenary day] renshen [the 9th of the 60-day cycle] Question: on what day did he begin his journey? Method: Enquire in what decade renshen [9] falls. Reply: It is the decade [commencing] jiazi [1] Given that starting at jia [1] and counting to ren [9] is nine days set out 9, and let it be increased.

Content:
Although S132 is undamaged, the text seems incomplete. A strip may be missing at this point. Once again the motivation of this problem seems to be the wish to maintain tight control of all state-managed activity. Here we have someone who arrives on day 9 of the sixty- day cycle designated by the systematic pairing of the two sets of ten and twelve cyclical characters, beginning with jiazi as the first pair.

If he claims to have been travelling for 50 days, on what day did he set out? Presumably the aim is to reconcile the present date and his authorised journey time with the day marked on his official 'move order'.

Mathematical note:
The procedure seems to break off before it is complete. In 5-7 we set out to establish in which of the six *xún* 旬 decades of the sixty days the arrival day falls, and it is established that the arrival day is the ninth of the first decade. Since he has travelled for five decades, his departure must have taken place in the second decade of the preceding 60-day cycle, on day 19, with the cyclical name renwu. But the final clause does not quite seem to get us as far as that point.

[Group 11: Excess and Deficit ('Rule of false position')]

S133,S134(long gap)

(52) Sharing cash
In sharing cash, if [each] person [gets] 2 then the surplus is 3. If [each] person [gets] 3 then the deficit is 2. Question: how many persons, and how many cash? Result: 5 persons and 13 cash. Let the excess and the deficit multiply numerators reciprocally to make the dividend. Let the denominators go with one another [in addition] to make the divisor. In each case let the excess or deficit numerator reciprocally multiply the denominator and set each one out separately; reduce the numerator which is greater by the numerator which is lesser; let the remainder be made the divisor; (let the deficit be made the dividend).

Content:
This is the first of four sections involving the solution of problems by what is called in the West 'The Rule of False Position', but was known earlier in China under the name of 盈不足 *Yíng bù zú* 'excess and deficit'. This section appears to assume that the reader is already aware of the algorithm required to solve the problem, and will therefore recognise what is referred to by the 'numerators' and 'denominators' involved. The solution of the problem is not made completely explicit, and the seven final words do not seem to relate to what precedes them. As will become clear (see Mathematical note), to make the method complete we would need to be told to simplify both the dividend (excess + deficit) and the divisor (sum resulting from cross-multiplication) by the second divisor (difference of numerators). The result would be a fraction with the total cash as the numerator and the number of

persons as the denominator.

Parallels:
The general method used here and in the other problems of this group is illustrated throughout the seventh of the Nine Chapters Yíng bù zú **盈不足**. The content of the problems given in the *Suàn shù shū* is not however closely paralleled: thus instead of discussing the sharing out of money between a number of persons, the Nine Chapters takes its elementary examples from the common purchase of commodities by a group of purchasers.

For illustration the first and fourth of these examples are given here, followed by the general statement of method that follows the fourth example. (7: Ying bù zu **盈不足**, Guō 1990, 357-359; Shen 1999, 358-359).

今有共買物，人出八，盈三；人出七，不足四．問人數、物價各幾何？

荅曰
七人物價五十三．

Now there is a case of common purchase of something. If each person pays 8, the excess is 3. If each person pays 7, the deficiency is 4. Question: how much are the number of people and the price of the thing?

Answer:
7 people, and the price of the thing is 53.
[.....]

今有共買牛七家共出一百九十不足三百三十九家共出二百七十盈三十問家數牛價各幾何

荅曰
一百二十六家牛價三千七百五十

Now there is a case of common purchase of an ox. If [each group of] 7 families contribute 190, the deficiency is 330; if [each group of] 9 families contribute 270, the excess is 30. Question: how much are the number of families and the price of the ox?

Answer: 126 families; the ox price is 3750

盈不足術日

置所出率盈不足各居其下令維乘所出率并以為實并盈不足為法實如法而一

The method of excess and deficiency says:
Set out the rates of payment; the excess and deficiency are placed below each one; let them diagonally multiply the rates of payment; combine the rates produced to make the dividend; combine the excess and deficiency to make the divisor; [count] 1 for [each time] the dividend accommodates the divisor.

[Passage A of Liú Huī's commentary appears here; see below]

有分者通之

In cases where there are parts, make them interchangeable.

[Passage B of Liú Huī's commentary appears here; see below]

盈 不 足 相 與 同 其 買 物 者 置 所 出 率 以 少 減 多 餘 以 約 法 實 實 為 物 價 法 為 人 數

In cases where the excess or deficiency relates to sharing in the purchase of something: set out the rates of payment; reduce the greater by the lesser; use the surplus to simplify the divisor and the dividend; [then] the dividend is the price of the thing; the divisor is the number of persons.

[Passage C of Liú Huī's commentary appears here; see below]

Mathematical note:

It is clear that the Nine Chapters solves its problems by a method identical to that found in the *Suàn shù shū* . But neither text gives us any indication of how the method was discovered or explained to students. For most modern readers unfamiliar with such problems the procedure followed is certainly not intuitively obvious. But rather than resorting to modern algebra (which can tell us little or nothing about how an ancient Chinese mathematician would have thought), let us follow the lucid explanations of Liú Huī (same reference)

A: **按盈者謂之朓不足者謂之朒。所出率謂之假令。盈朒維乘兩設者。欲為同齊之意。按《共買物，人出八，盈三；人出七，不足四》齊其假令，同其盈朒，盈朒俱十二。通計齊則不盈不朒之正數。故可併之為實，併盈不足為法。齊之三十二者。是四假令。有盈十二。齊之二十一者。是三假令。亦朒十二。並七假令合為一實。故並三四為法**。

Note that 'excess' is called 'overstepping' and 'deficit' is called 'falling short'. The rates of spending are called 'assumptions'.
The diagonal multiplication of these suppositions [is similar to] the

> concept of wishing to make [numerators of fractions] share [a common denominator] and to adjust [these numerators to that new denominator].
> Note that [in the example]
> 'In the common purchase of something, if each person pays 8, the excess is 3. If each person pays 7, the deficiency is 4.'
> if one 'adjusts' the assumptions, and makes the excess and falling short 'shared', then the excess and falling short are [both] 12. If one makes an overall reckoning of [what has been] adjusted, then this is the correct number without excess or falling short. Thus one may combine them to make the dividend, and combine the excess and deficit to make the divisor. The 32 [that results from] adjusting is the assumption being multiplied by 4; [giving] an excess of 12. The 21 [that results from] adjusting is the assumption being multiplied by 3; [giving] in turn a falling short of 12. One combines the 7 assumptions to make a single dividend, so one combines the 3 and 4 to make the divisor.

It is clear here that Liú Huī I referring back to his earlier discussion of how fractions may be brought to a common denominator and added (see section 8). This involved writing out the numerators and denominators is a fashion similar to the following for the given example ¾+⅔.

numerators: **3** **2** Diagonal multiplication gives 3x3+2x4 = 9 + 8 = 17

denominators: **4** **3** Multiplication gives 4 x 3 = 12

So the result is 17/12

The Nine Chapters text asks us to lay out and process the different data of its problem in similar manner:

rates of expenditure: **8** **7** Diagonal multiplication gives 8x4+7x3= 32 + 21 = 53

excess or deficit: **3** **4** Multiplication gives 3 x 4 = 12

But why does the 'adjusting' and 'sharing' procedure previously applied to fractions work in the different context of 'excess and deficit' problems? The key point made by Liú Huī is that one can balance out excess and deficit if one takes a certain number of each of the two different cases and adds them. In the example, we take four cases of expenditure 8, each of which produces an excess of 3: the total excess will be 4 x 3 = 12, and the total cost will be 4 x 8 = 32. We also take three cases of expenditure 7, each of which produces a deficit of 4, so the total deficit is 3 x 4 = 12, and the total cost will be 7 x 3 = 21. Now if we add these results together, we get a total expenditure of 53, and the excess and deficit cancel out exactly. Clearly the rates of expenditure have been 'adjusted' and added just as in the case of fractions, and 12 is the 'shared' magnitude for excess and deficit. The additional final step - which is not analogous to the treatment of fractions - is the realisation that the situation of spending at a rate of 53 and getting no excess or deficit comes from adding the results of 4 + 3 = 7 cases. Thus the rate of expenditure in one case is 53/7.

B: **若兩設有分者，齊其子，同其母。令下維乘上，訖，以同約之。**

If both suppositions have parts, adjust the numerators and make their denominators shared. Let the lower diagonally multiply the upper, and when that is done, simplify them by what is identical.

Here we are simply being reminded of what needs to be done if the rates of expenditure have fractional parts.

C: **所出率以少減多者。餘謂之設差。以為少設，則並盈朒。是為定實。故以少設約定實。則法為人數。適足之實故為物價。**

[As for] subtracting the lesser of the rates of payment from the greater: the surplus is called 'the supposition difference', and we take that as a 'lesser supposition'. So when we combine the excess and falling short, this is making the fixed dividend. Therefore if we simplify the fixed dividend by the lesser supposition, then [we see that] the divisor becomes the number of people. The dividend corresponding to the right amount thus stands for the price of the thing.

The previous explanations by Liú Huī took us to the point where we could see why we had to 'combine [the results of cross-multiplication] to make the dividend, and combine the excess and deficit to make the divisor.' which in the example considered gave 53/7 - evidently the rate of expenditure per person when there is neither excess nor deficit. But how are we to find the actual total expenditure and the number of people? In the commentary quoted here, Liú Huī is clearly thinking of the fact that the change in rates of payment causes the result to swing by an amount equal to (excess + deficit). Dividing this sum ('the fixed dividend') by the change in rates ('the lesser supposition') will clearly transform the divisor in our fraction 53/7 into 'the number of people' who are involved in payment.

Now the 'lesser supposition' in this case is (8-7) = 1. So the simplification of dividend and divisor required by the Nine Chapters amounts in this case to a trivial division by 1. Thus when we see the fraction 53/7, which represents the rate of expenditure per person when there is neither excess nor deficit, we know that 7 actually *is* the number of persons. Thus, as Liú Huī points out, the numerator of this fraction must itself be the actual expenditure. The explanation of the method is now complete.

S135,S136,S137(long gap)

(53) Grain costs

(a) *Lì* hulled grain is 3 cash for 2 *dǒu*; *bài* is 2 cash for 3 *dǒu*; Now we have 10 *dǒu* of *lì* and *bài*. Selling it, we get 13 cash. Question: how much each of *lì* and *bài*. Reply: 7 *dǒu* and ⅗ of *bài*, 2 *dǒu* and ⅖ of *lì*. Method: Making it all *lì*, then the cash are 2 in excess Making it all *bài*, then the cash are 6 ⅓ in deficit Put the excess and deficit together to make the divisor; Multiply the 10 *dǒu* by the excess for the *bài*; Multiply the 10 *dǒu* by the deficit for the *lì*. In each case accommodate the divisor to obtain 1 *dǒu*.

S138,S139,S140(7 character gap)

(b) Setaria millet: a *dǒu* is 1 cash and ⅔ cash; Panicled millet: a *dǒu* is 1 cash and a half cash; Now let it be that for 16 cash one buys 10 *dǒu* of setaria millet and panicled millet. Question: how much is each, and how much money goes on each? Result: Setaria millet, 6 *dǒu*; panicled millet 4 *dǒu*; the setaria millet cash are 10 and the panicled millet cash are 6. Method: Use excess and deficit. Making it all setaria millet there is ⅔ cash too much; [if it is] all panicled millet, there is a cash too little. In the lowest place there is ⅓, so take 1 as 3, calling the 3 and the deficit 2. Combine the too much and too little to make the divisor; Further set out 2 and 3, and multiply each by 10 *dǒu.* Then swap over the results, and accommodate the divisor [to get] 1 *dǒu*.

Content:

In these two problems the aim is to find the proportions of ingredients of known unit price in a given total volume of known total cost. They differ enough in style to suggest that they are from two distinct sources.

Mathematical note:

Like the later problems in the *Yíng bù zú* 盈不足 chapter of the Nine Chapters, the two problems in this section are not initially stated in terms of excess and deficit, but are solved by application of that method as a result of applying two trial solutions. As the text says, we are to 'Use excess and deficit'.

The main calculations for (a) are:

lì hulled grain = (10 *dǒu* x 6 ⅓)/(2+ 6 ⅓)
= (10 *dǒu* x 19)/(6 + 19)
= 7 ⅗ *dǒu*

bài hulled grain = (10 *dǒu* x 2)/(2+ 6 ⅓)
= (10 *dǒu* x 6)/(6 + 19)
= 2 ⅖ *dǒu*

The calculations in (b) follow the same lines. *Mào* 貿 'swap over' clearly refers to the fact that the result for each grain is obtained by taking the result of multiplying by the excess or deficit for the other type.

Once again it seems wise to avoid explaining the algorithms used here by modern algebra. Perhaps the thinking behind them might have been something like this.

We have a total amount of grain which has to be shared between *lì* and *bài.*
Each unit of *bài* will make a contribution to putting the result in deficit.
Each unit of *lì* will make a contribution to putting the result in excess.
These contributions per unit will be in proportion to the excess or deficit caused when all the grain is of one sort.
Clearly we want to find the proportions of grain where the total contributions to excess and deficit just cancel out, and the actual amounts add up to the known total.
So the amounts of each sort of grain will be in inverse proportion to the excess or deficit caused when all the grain is of the given sort, i.e. the amounts of each sort of grain will be in direct proportion to the excess or deficit caused when all the grain is of the other sort.
Hence (given the need for the amounts to add up to 10 *dǒu*), the algorithms are obvious.

Thinking of the more general problem of grain exchanges, we may note the relative values of the commodities given here. Evidently the prices for one *dǒu* in (a) are in the ratio

lì price *: bài* price = 3/2 : 2/3
=9:4

If these prices were to be consistent with the fact that 30 units of *lì* exchange for 27 units of *bài*, then this ratio should have been 27:30 = 9:10. The discrepancy suggests that no attempt is being made to represent actual prices of commodities in the question.

In (b) the prices per unit are in the ratio:

setaria price : panicled price = 5/3 : 3/2

= 10: 9

which would suggest that the commodities would be exchanged at equivalent volumes in the ration 9 of panicled to 10 of setaria. This is, we may note, inconsistent with the statement in section 36 that

> '1 *shí* of *hé shǔ* [panicled millet?] makes 16 *dǒu* and augmented half a *dǒu* of unhulled grain'

This discrepancy casts some doubt on the possible identification of *hé shǔ* with panicled millet already made, given that the 'unhulled grain' in (36) is almost certainly setaria.

(The following section appeared here in the original *Wénwù* transcription, but both Péng Hào and Anon. (2001) shift it to a later position as reflected in the numbering of the two relevant strips. I prefer to retain it here to facilitate comparison with other applications of this method. All this underlines the fact that the various publications of this material do not give us sufficient information to decide how far the present ordering of the strips reflects the circumstances of their excavation, and how far it is a result of editorial decisions)

S185,S186(long gap)

(54) Squaring a field

There is a field of one *mǔ*: how many *bù* is it square? Reply: it is square 15 *bù* and 15/31 *bù*. Method: If it is square 15 *bù* it is in deficit by 15 *bù*; if it is square 16 *bù* there is a remainder of 16 *bù*. Reply: combine the excess and the deficit to make the divisor; Let the numerator of the deficit multiply the denominator of the excess, and the numerator of the excess multiply the denominator of the deficit; combine to make the dividend. Reverse this like the method for revealing the width.

Mathematical note:

It is intriguing to find the 'false position' method used here to find a good approximation to a square root, thereby perhaps suggesting the possibility that at the time of writing the algorithm for finding square roots had not yet been discovered. The algorithm for square roots is given explicitly in the Nine Chapters (4: Shao guang 少廣 258-259).

The main calculation is:

length of side ('numerator')	15	16
excess/deficit ('denominator')	15 (deficit)	16 (excess)

(15x16 + 16x15)/(15+16)
= 15 15/31

Of course the 'false position' method only works exactly when linear functions are involved, but in fact

$(15 + 15/31)^2 = 239 + 761/931$,

which is very close to the desired value of 240, the number of (square) *bù* in 1 *mǔ*..

The method works well for finding square roots if the two trial values are close enough to the solution so that a straight line is a good approximation to the second-degree curve on which the solution lies. In the Nine Chapters, the 'false position' method is applied to what are in effect non-linear equations in the 11th, 12th and 19th problems of the *Yíng bù zú* chapter.

The reference to 'reversing' *fù zhī* 复之 is also found in 65 (which does in fact bear the title 啟從 *qǐ cōng* 'revealing the width' rather than 啟廣) and in 66. The principle is that having obtained a length of an unknown side one multiplies with the known side to check whether the desired area has in fact been produced. Here of course one would have to square the result that is to be checked.

[Group 12: Shapes and volumes]

S141,S142(long gap)

(55) A cutting

A *yán chú* : let it be fixed at a *zhāng* square. The height is 1 *zhāng* 2 *chí*. Its cutting is a *zhāng* broad; the length is 5 *zhāng* 6 *chí*. One of its edges has no height. The volume is 3360 *chí*. Method: The breadths accumulate to 30 *chí*, [which] multiplies the height; multiply it by the breadths and the length to fix it. Form 1 for six.

Content:
The rendering 'cutting' is an attempt to find an English translation (as opposed to a paraphrase) for the way that *chú* 除 is apparently being used to refer to the removal of material in the process of forming the given excavation. A *yán chú* is commonly explained by commentators and dictionaries as the entrance ramp to a subterranean tomb. This is the first of a sequence of seven problems (55-60) involving finding the volume of a solid figure. There are regularities of phrasing that suggest a common source.

Parallel:
In the Nine Chapters we find (5: *Shāng gōng* 商功, Guō 1990, 288; Shen 1999, 279-280):

今有羨除下廣六尺上廣一丈深三尺
末廣八尺無深袤七尺
問積幾何？

荅曰：
八十四尺·

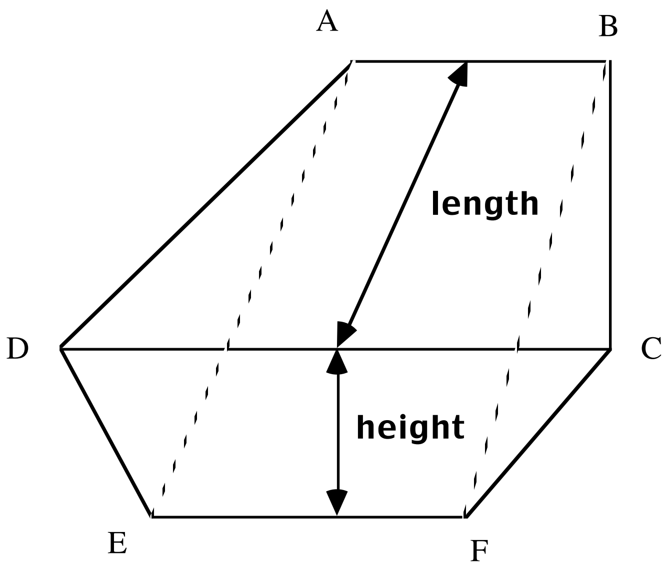
術曰
并三廣以深乘之又以袤乘之六而一

Now there is a *yán chú;* Its lower width is 6 *chí*; Its upper width is 1 *zhāng*; Its depth is 3 *chí*; its end width is 8 *chí* with no depth; its length is 7 *chí*. Question: what is the volume?

Answer:
84 *chí*.

Method:
combine the three breadths; multiply them by the depth; further multiply by the length; [take] 1 for 6.

Mathematical note:
Comparing the Nine Chapters *yán chú* to the case given here, an interesting point emerges: Although the cutting described here is of uniform width, unlike the general case described in the Nine Chapters, it is not treated in the most straightforward manner, which would have been simply to take it as a half - cuboid, and take (½) (length x breadth x height). Instead we are rather pointlessly made to take the three breadths added together - and then divide by 6 to deal with this. That surely makes it clear that the concept of the general *yán chú* of variable width seen in the Nine Chapters was known at the time the example here was composed, even though the case described here does not require use of the general formula.



The general *yán chú* ABCDEF is shown above. The faces ABCD and CDEF are perpendicular. The height and length are as marked, and AB, DC and EF are the 'three breadths'. Calling the height and length h and l, the algorithm given in the Nine Chapters may be rendered as a formula in modern notation thus:

Volume, $V = hl(AB+DC+EF)/6$

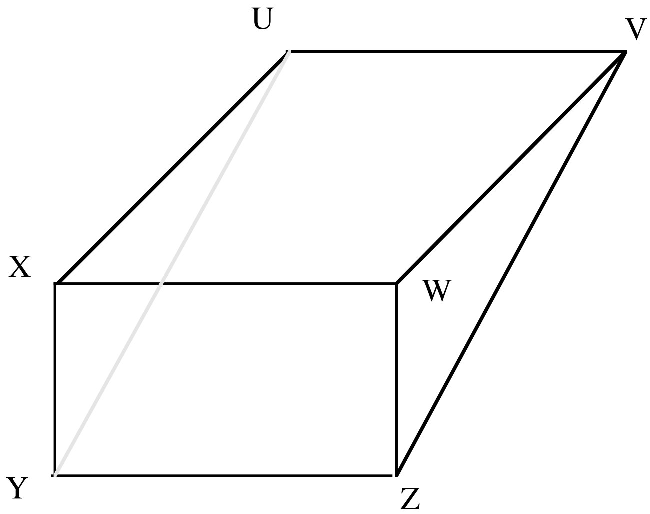
If AB, CD and EF were all equal (as here) to some width w, then we would have:

$V= hl.3w/6 = hlw/2$, half of the corresponding cuboid as already mentioned

The question arises as to how this (correct) formula was arrived at. In the Nine Chapters no explanation is given in the main text, but in his commentary (5: *Shāng gōng* **商功**, Guō 1990, 288; Shen 1999,280-281) Liú Huī describes how the *yán chú* may be dissected into a number of standard elements whose volume is already known, and from consideration of which the formula given here follows. It is noteworthy that two of these standard elements (the *yáng mǎ* 陽馬 and *biē nào* 鱉臑 and their volumes appear in the main Nine Chapters text (5: *Shāng gōng* 商功, Guō 1990, 286 and 287; Shen 1999, 269 and 278), although they serve no directly practical purpose in themselves, and function only as shapes useful for Liu Hui's dissections. Their presence suggests two (non-exclusive) possibilities:

(a) The dissection method involving such elements was already well-known before Liú Huī;
(b) Liú Huī wrote these elements into the Nine Chapters text at the time he composed his commentary.

However neither *yáng mǎ* not *biē nào* are mentioned in the *Suàn shù shū* . While this does not mean that a rigorous dissection method (four centuries before Liú Huī) did not lie behind formulae such as the one given here, this fact does suggest that we ought to consider the possibility of some other non-rigorous process having been involved. One route is suggested by the fact that we find a simple half-cuboid such as UVWXYZ treated as if it was the general shape ABCDE



The fact that we find the sum (UV + XW + YZ) formed here as if the three lengths might be different might suggest that someone may have started from the obvious half-cuboid of constant width and then made the leap of generalisation to construct a formula that applies when the width varies.

S143

(56) The wall of Yùn
A wall of Yùn has a lower thickness of 4 *chí*; its upper thickness is 2 *chí*; its height is 5 *chí*; Its length is 2 *zhāng* The volume is 133 *chí* and a diminished half *chí*. Method: Double the upper thickness; increase it by the lower thickness; multiply it by the height and the length; [take] 1 for 6.

Parallel:
The rendering of *yùn dū* 鄆都 as 'Wall of Yùn' is not intended to bear much weight. Yùn 鄆 is an ancient place-name in modern Shandong, while 都 is a possible loan for *dǔ* 堵. In the Nine Chapters the term graphically nearest to 鄆都 is perhaps *qiàn dǔ* 塹堵 (5: *Shāng gōng* 商功, Guō 1990, 286; Shen 1999, 267-268), also written 塹堵 - but that term refers to a simple prismatic length of wall or wedge of triangular vertical cross-section, and in any case Liú Huī tells us that he has never heard any explanation of where that name comes from.. The object described here is what the Nine Chapters (5: *Shāng gōng* 商功, Guō 1990, 289; Shen 1999, 287-288) calls a *chú méng* 芻甍 'straw thatch'. From what Liú Huī says, this has the shape of a hipped roof, whose ridge length is shorter than the side of the rectangular base to which it is parallel.

今有芻甍下廣三丈袤四丈上袤二丈無廣高一丈問積幾何

荅曰
五千尺

術曰
倍下袤上袤從之以廣乘之又以高乘之
六而一

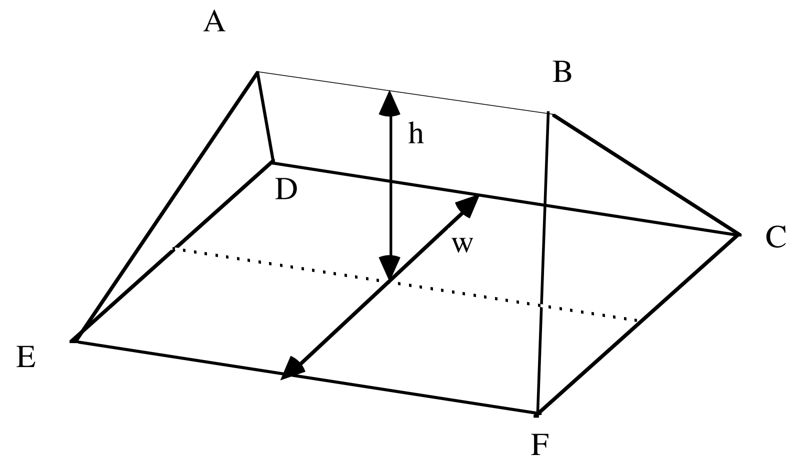
Now there is a straw thatch. Below, its width is 3 *zhāng* and its length is 4 *zhāng*; above, its length is 2 *zhāng* and it has no width; its height is 1 *zhāng*. Question: how much is its volume?

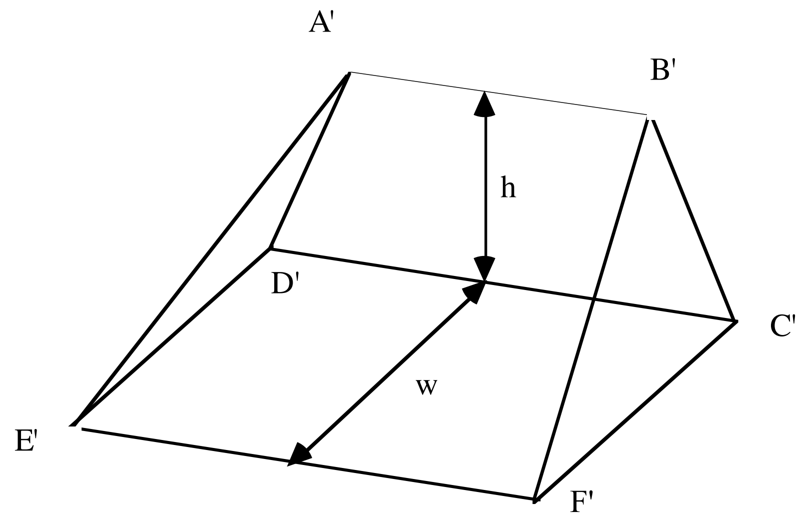
Answer:
5,000 *chí*.

Method:
double the lower length; the upper length follows it [in addition]; multiply it by the width; further multiply it by the height; [take] 1 for 6.

Mathematical note:
There is an obvious resemblance between the algorithm for the volume of the object described in both 56 and the Nine Chapters, and the algorithm for the *yán chú* volume, the only difference being that in this case we have twice one thickness (or length) plus another thickness, rather than the sum of three different lengths.

The reasons for this are obvious from inspection of the two figures below:





The first figure shows the roof-shape which (for Liú Huī at least) was a *chú méng*, where h is what both this text and the Nine Chapters call the height, and w is what the Nine chapters call the width and this text calls the length. It can however be transformed by horizontal deformation without change of volume into the second shape. in which A'B'C'D' is perpendicular to C'D'E'F'. (we may note that although Liú Huī limits the term *chú méng* to a symmetrical hipped roof there is nothing in the main text of the Nine Chapters to suggest this). But this is clearly a *yán chú* with two of its widths identical, just as the formula implies.

S144,S145(long gap)

(57) Straw

A 'straw-boy' or square watchtower has a lower width 1 *zhāng* 5 *chí*; the length is 3 *zhāng*; the upper width is 2 *zhāng*. the length is 4 *zhāng*; the height is a *zhāng* 5 *chí*. the volume is 9250 *chí*. Method: Let the upper width and

> length and the lower width and length each multiply themselves; further let the upper length follow the lower length [in addition] and multiply the upper width, while the lower length follows the upper length in addition to multiply the lower width; combine them all; multiply by the height; [take] 1 for 6.

Content:
The object described here is a hopper-shaped container for fodder; if it was inverted, it could be thought of as a tapering platform or 'watchtower' as its alternative name implies.

Parallel:
The Nine Chapters deals with the *chú tóng* shape as part of a related group (5: *Shāng gōng* **商功**, Guō 1990, 289; Shen 1999, 289-290):

> **芻童曲池盤池冥谷皆同術·**
>
> **術曰**
> **倍上袤下袤從之亦倍下袤上袤從之各以其廣乘之并以高若深乘之皆六而一**
> The *chú tóng*, *qū chí*, *pán chí*, and *míng gǔ* all share a common procedure.
>
> Method:
> double the upper length; the lower length follows it [in addition]; likewise double the lower length; the upper length follows it [in addition]; let each be multiplied by its width; [having] combined [them], multiply by the height or as it may be the depth; in all cases take 1 for 6.

Mathematical note:

The procedure in the *Suàn shù shū* amounts to calculating the volume as:

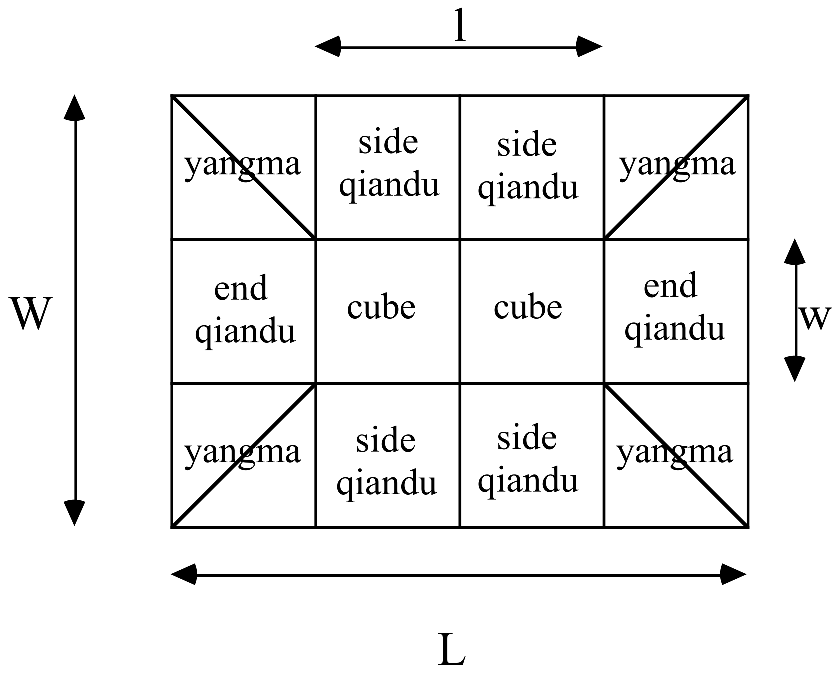
$V = h/6 \{(WL + wl) + (l + L) W + (L + l) w \}$

Whereas the Nine Chapters specifies, in effect:

$V = h/6 \{(2L + l) W + (2l + L)w\}$

A few moments' inspection of these algebraic expressions shows the modern reader that the two methods are equivalent to one another. It is however important to realise that this does not get us any closer to the thought processes of the ancient writers behind these texts. In what follows, expressions in symbolic algebra will serve only to save writing out expressions in prose, and we shall attempt to understand the methods given in ancient Chinese terms. In the case of the Nine Chapters method, Liú Huī supplies a justification through his usual dissection procedure. The principal of this is simple: If one has a shape that is not a rectangular parallelepiped (i.e. a cuboid), whose volume can be easily be calculated as length x breadth x height, then take sufficient multiples of the shape in question until one can form one or more rectangular parallelepipeds from the various components of the original shape. The volume of the desired shape can then be found as a fraction of the total volume which results from this process.

If we consider the following plan view of a *chú tóng*, we shall see all the elementary shapes, *qí* 棊 that Liu Hui uses in his explanation. In this and the following diagrams it is assumed (with Liu Hui) that the dimensions of the *chú tóng* are such as to allow it to fit into a simple cubic grid. Thus the height h of the object seen in plan here is equal to w. It will become clear that the way the problem is treated allows us to generalise the result to a *chú tóng* in which the horizontal and vertical scales are modified to produce a more general shape.



Two shapes are already familiar: the cube, and the *qiàn dǔ* wedge, whose volume is half the cube. Each corner of the object is made up of a *yáng mǎ* 陽馬, a pyramid on a square base constructed by joining one vertex of a cube to the four corners of a non-adjacent face. A little reflection will show that three identical *yangma* can be constructed simultaneously within a cube, so that each of then occupies one third of its volume: see the discussion of these two shapes in Shen (1999).

What is the minimum number of *chú tóng* that will enable us to form rectangular parallelepipeds of the resulting blocks? The answer is obvious, since the number must have a factor 3 to produce complete cubes from the *yangma*, and there are already an even number of both sorts of wedges. But the Nine Chapters evidently prefers to take 6 *chú tóng*, which produces a slightly neater procedure.

We begin then by reckoning up the elementary blocks in the *chú tóng*, introducing obvious abbreviations for convenience:

two cubes: 2c
four *yangma*: 4y
four sides *qiandu*: 4s
two end *qiandu*: 2e

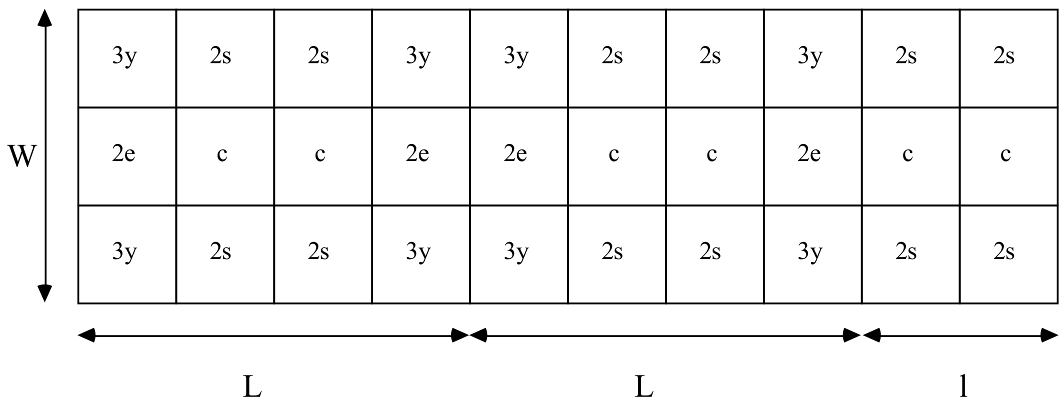
If we take six *chú tóng*, the totals of the shapes will thus be: 12c 24y 24s 12e

Recalling that the Nine Chapters expression for the volume is equivalent to:

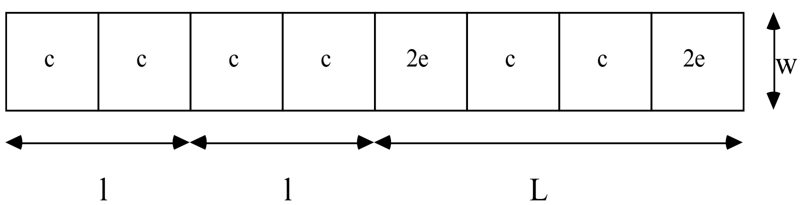
$$V = h/6 \{(2L + l) W + (2l + L)w\}$$

we may construct two rectangular blocks with the dimensions $h(2L + l) W$ and $h(2l + L)w$

The plan view of the first is:



and of the second:



In each case the number of each of the three elementary shapes used is marked inside each cube of which the two parallelepipeds are composed. Inspections will show that the total is indeed six times the number of each elementary shape in a *chú tóng*, and so the result has been established. The fact that we have treated each of the three elementary shapes independently means that the application of scaling factors to the length, breadth and height of the *chú tóng* will not damage the generality of the result.

In explaining this dissection, Liú Huī says (5: *Shāng gōng* 商功, Guō 1990, 289-290; compare Shen 1999, 289-290):

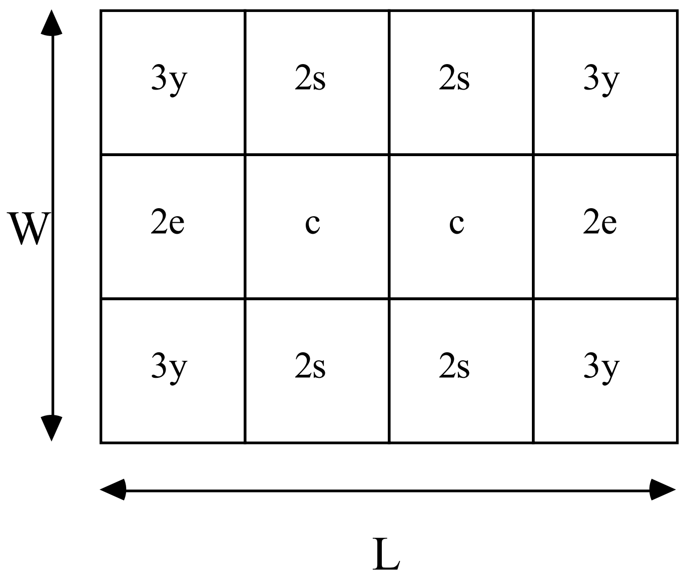
> **是為得中央立方各三。兩端塹堵各四。兩旁塹堵各六 [...] 得中央立方各三。兩端塹堵各二。并兩旁。三品棊皆一而為六。故六而一。即得。**
>
> This implies that [in the first figure] one gets three for each of the central cubes, four for each of the end *qiàn dǔ*, and six for each of the side *qiàn dǔ* [... in the second figure] one gets three for each of the central cubes, and two for each of the end *qiàn dǔ*. Combining the two, one gets six for each one of the three kinds of *qi*. So taking one for six, one obtains the result.

The procedure in the *Suàn shù shū* amounts to calculating the volume as:

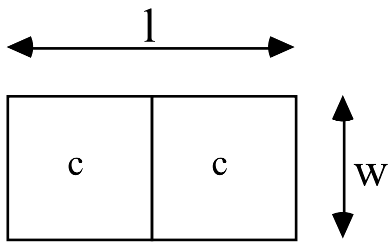
$$V = h/6 \{(WL + wl) + (l + L) W + (L + l) w\}$$

We therefore need to form four parallelepipeds as shown below:

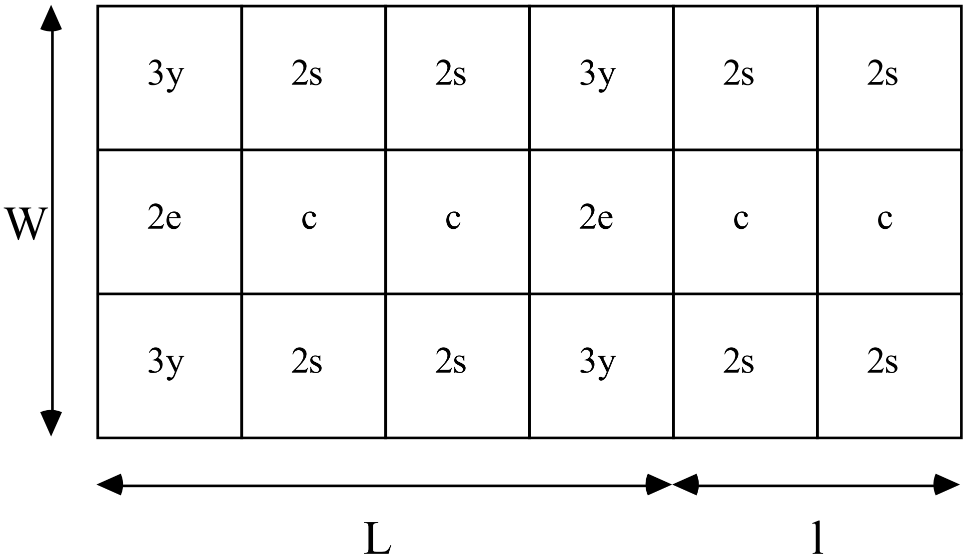
h(WL):



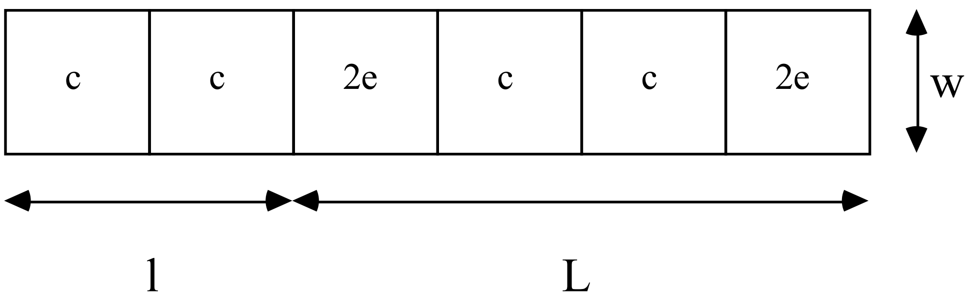
h(wl):



h(l + L) W:



h(L + l) w:



Once again, the totals are six times those in a single *chú tóng*, and the procedure is established.

Liú Huī himself gives two further equivalent algorithms, one explicitly justified by a different dissection - but neither resembles the *Suàn shù shū* procedure. Three points emerge from this discussion:

(1) It seems very likely that the basic dissection procedures known to Liú Huī were already in use at the time that this part of the *Suàn shù shū* material was written, since it is hard to see how else the result given could have been derived. Although the *yáng mǎ* is not named, the fact that the corner piece of the *chú tóng* is one third of the cube appears to have been known.

(2) The fact that Liú Huī mentions three other results, but not the one given in the *Suàn shù shū* suggests that he may not have met material (or at least not all of the material) from the *Suàn shù shū* tradition.

(3) While the Nine Chapters method can be seen as a more general version of that given for the *chú méng* described above, the *Suàn shù shū* method for the *chú tóng* seems formally unconnected to the procedure given here.

Shen (1999) 293 quotes a 12th C. AD Indian procedure that is close to that of the *Suàn shù shū* ; clearly this is most likely to be a case of independent discovery.

Checking the figures in the given example:

$w = 15$, $l = 30$

$W=20$, $L = 40$

$h = 15$

Now $V = h/6 \{(WL + wl) + (l + L) W + (L + l) w\}$

So in this case, $V = 15/6 \{(20 \times 40+15 \times 30) + 70 \times 20 + 70 \times 15)$

$= 15/6 \times (1250+1400+1050)$
$= 9250$

Péng Hào gets the answer 9800 (and seeks to emend the result because of this). But this appears to be because he makes W = 22, which is not in the text as he gives it.

S146,S147(long gap)

(58) A whirl of grain
A whirl of hulled grain has a height of 5 *chí*; its lower circumference is 3 *zhāng*; the volume is 125 *chí*. [There is] 1 *shí* for 2 *chí* 7 *cùn*. [So] it makes 46 *shí* 8/27 *shí* of hulled grain. The method for this: Let the lower circumference be multiplied by itself; multiply it by the height; Make 1 [from] 36. The greater volume is 4,500 *chí*.

Content:
This is the first of two seemingly independent versions of the procedure for calculating the volume of a cone; the other is 58. In addition this section gives a volume to mass conversion for grain, which enables the mass to be found from the

volume.

Parallels:
It is interesting that while the Nine Chapters gives a general statement of how the volume of a cone (literally 'round awl') is to be found, it also separately has a calculation dealing with the volume and mass of a conical pile of grain (5: *Shāng gōng* **商功**, Guō 1990, 285 and 293; Shen 1999, 267):

> **今有圓錐下周三丈五尺高五丈一尺問積幾何？**
>
> **荅曰**
> **一千七百三十五尺一十二分尺之五·**
>
> **術曰**
> **下周自乘以高乘之三十六而一**
>
> Now there is a cone; its lower circumference is 3 *zhāng* 5 *chí*; its height is 5 *zhāng* 1 *chí*. Question: how much is the volume?
>
> Answer:
> 1735 *chí* 5/12 *chí*.
>
> Method:
> the lower circumference is multiplied by itself; multiply it by the height; take one for 36.
>
> **今有委粟平地下周一十二丈高二丈問積及為粟幾何？**
>
> **荅曰**
> **積八千尺為粟二千九百六十二斛二十七分斛之二十六·**
>
> Now there is piled grain on level ground; its lower circumference is 12 *zhāng*; its height is 2 *zhāng*. Question: how much are the volume and the [amount of] grain that makes?
>
> Answer:
> the volume is 8000 *chí*; it make 2962 *hú* and 26/27 *hú* of grain

Mathematical note:
The thinking behind the algorithm used in the *Suàn shù shū* is revealed in the reference to the 'greater volume': This quantity is the volume of a cuboid of the same height as the cone, but with a side equal in length to the circumference. As Liú Huī points out, the factor of 1/36 follows from the fact that the volume of a pyramid on that square base would be ⅓ of the volume of the cuboid, and the area of the circular base of the cone is taken as 1/12 of the area of the base of the cuboid. The assumption here is that $\pi = 3$, as in all other problems in this text involving

circles.

S148

(59) A granary cover

A granary cover has a lower circumference of 6 *zhāng*; its height is 2 *zhāng*; this makes a volume in *chí* of 2000 *chí*. Method for multiplying: set out [a number] identical to the circumference; let them be multiplied together. Further multiply it by the height; let 36 form 1.

Content:
This is another procedure for a cone, though under a different name, but the language used here differs so greatly from that used in the previous section that the two procedures are clearly from different sources

S149.S150(long gap)

(60) A round pavilion

A round pavilion has an upper circumference of 3 *zhāng*; the larger circumference is 4 *zhāng*; the height is 2 *zhāng*. The volume is 2055 *chí* and 20/36 *chí*. Method: the upper circumference multiplies the lower circumference; the circumferences multiply themselves; let it all be combined; multiply it by the height; form 1 for 36. Now it is 2055 *chí* and 20 parts.

Content:
This is a frustum of a cone.

Parallel:
The Nine Chapters has a section close to this one (5: *Shāng gōng* **商功**, Guō 1990, 284; Shen 1999, 262-263):

> **今有圓亭**
> **下周三丈上周二丈高一丈問積幾何？**
>
> **荅曰**
> **五百二十七尺九分尺之七·**
>
> **術曰**
> **上下周相乘又各自乘并之以高乘之三十六而一**

Now there is a round pavilion; its lower circumference is 3 *zhāng*; its upper circumference is 2 *zhāng*; its height is 1 *zhāng*. Question: how much is the volume?

Answer:
527 *chí* 7/9 *chí*.

Method:
let the upper and lower circumferences be multiplied together; further let each be multiplied by itself; combine them; multiply it by the height; take 1 for 36.

S151,S152(end of strip obscure)

(60) A timber [shaped like] a well
A round timber, well-pit or other similar object: The circumference is 2 *zhāng* 4 *chí*; the depth is a *zhāng* and 5 *chí*; the volume is 720 *chí*. Method: register the circumference and multiply by itself; multiply it by the depth; form 1 for 12. One [method] says: multiply the circumference by the diameter; form 1 for 4. 100 and ½; Question (?) the diameter ......

Content:
This section describes the calculation of a cylindrical volume. It incorporates what are in effect two methods for finding the area of a circle, one based on the circumference alone and one based on circumference and diameter.

Parallel:
The Nine Chapters (5: *Shāng gōng* 商功, Guō 1990, 283; Shen 1999, 260-261) has:

今有圓堡壔周四丈八尺高一丈一尺問積幾何？

荅曰
二千一百一十二尺·

術曰
周自相乘以高乘之十二而一

Now there is a round piling; its circumference is 4 *zhāng* 8 *chí*; its height is 1 *zhāng* 1 *chí*. Question: how much is the volume?

Answer:
2112 *chí*.

Method:
The circumference is multiplied together with itself; multiply it by the height; take 1 for 12.

[Group 13: Circle and square]

S153

(62) A square from a round timber

To make a square timber from a round timber. The greater [is?] 4 *wéi* 2 *cùn* 14/25 *cùn*. How large is the square timber it makes? Reply: it is square 7 *cùn* ⅗ *cùn*. Method: take and 5-fold it to make the dividend; make it 1 for 7. Take 1 for 4

S154,S155(long gap)

(63) A round from a square timber

From square to make round. A timber is 7 *cùn* ⅗ *cùn* square How large is the round timber it makes? Reply: 4 *wéi* 2 *cùn* 14/25. Method: One face of the timber is the diameter of the round timber; take and 4-fold it to make the dividend. Let 7 be formed for 5.

Content:

Together with 62, this forms a pair of problems concerned in some way with passing from a square to a circle. The physical significance of the processes involved is however not obvious.

Mathematical note:

From the usage of the term in 36, we know that a *wéi* used as a measure of circumference is 10 *cùn*.

Thus the two measurements given here are:

round: 40 *cùn* + 2 *cùn* + 14/25 *cùn* = 1064/25 *cùn*.
square: 7 *cùn* ⅗ *cùn* = 38/5 *cùn*

So, noting that (1064/25) x (5/7) x (¼) = 38/5
and (38/5) x 4 x (7/5) = 1064/25

it does seem that the figures here are consistent, subject to minor amendments at the end of one of the procedures (see text). But what is the point of this calculation? The problem here is that the processes indicated by the figures do not match up with the section titles and the verbal descriptions they contain. Thus, taking the factor 7/5 as an approximation to $\sqrt{2}$, which seems highly probable, in the first case we divide some kind of circumference by 4 and also by $\sqrt{2}$. This would correspond to going from the perimeter of a square to its side, and then to half the length of its diagonal, with the second case reversing this. But somehow we are supposed to start from a round timber in the first case, and then get to a square, reversing this in the second case. For a review of attempts to deal with this confusion, none entirely convincing, see Duàn Yàoyǒng 段耀勇 and Zōu Dàhǎi 邹大海 (2003)

Parallel:
Problems of this kind appear to be the only cases in the *Suàn shù shū* in which there might have been some mention of the Pythagoras theorem: there are however no traces of it here. In the Nine Chapters, a problem about cutting a rectangular plank from a log is made to serve as an example of the application of Pythagoras (9: *Gōu gǔ* 句股, Guō 1990, 420; Shen 1999, 466). This example is not however cited here as a parallel, but as the reverse, showing the obvious absence from the *Suàn shù shū* repertoire of a method taken for granted in the Nine Chapters.

今有圓材徑二尺五寸欲為方版令厚七寸問廣幾何

荅曰
二尺四寸

術曰
令徑二尺五寸自乘以七寸自乘減之其餘開方除之即廣

Now there is a round timber of diameter 2 *chí* 5 *cùn*; it is desired to make a rectangular plank, making the thickness 7 *cùn*. Question: how much is the breadth?

Answer:
2 *chí* 4 *cùn*.

Method:
let the diameter 2 *chí* 5 *cùn* multiply itself; reduce it by the 7 *cùn* multiplied by itself; as for the remainder, reduce it by opening the square [i.e. find the square root], and that is the breadth.

S156(strip very obscure)

(64) A round timber

There is a round timber of 1 ... It is cut... market .... how big? Reply: 76 (?).......4 *cùn* and a half *cùn*. Method: ... multiplied by itself

S157(long gap)

1, then it is completed.

S158(long gap)

It enters two inches. Increase it and this is the larger number

Content:
Unfortunately all that can be said of this material is that it involves another round timber, which is cut in some way. The presence of the squaring procedure suggests that an area, or perhaps a volume is calculated. There seems to be no very obvious basis for the editor's decision to group these three strips together; S157 and S158 may simply be unrelated fragments.

[Group 14: Sides and areas with mixed numbers]

S159(3 character gap)

(65) Revealing the breadth

A field has a length of 30 *bù*; How much must the opened breadth be to make a 1 *mǔ* field? Say: it reveals 8 *bù* Method: take 30 *bù* as the divisor; take 240 *bù* as the dividend. For opening out the length [proceed] similarly to this.

Content:
This section is ostensibly concerned with the calculation of the missing side of a field of known area when the other side is known. We have already seen a similar problem in 47 - where however only the area and that fact that the field was square were known. In that case a square root was found by an approximate method, but here nothing is required but simple division. For some reason the calculation required in this case (240 /30) is not actually carried through. The final clause seems to suggest that a separate method need not be stated for the case when the length rather than the breadth is the unknown; however that is exactly what the next section does, which suggests that they come from different sources. Problems relating to areas and sides of fields continue in the sections with which the *Suàn shù shū* concludes.

S160.S161(long gap)

(66) Revealing the length

(a) The breadth is 23 *bù.* [What] is the revealed length to seek a field of 4 *mǔ?* Method: set out the number of *bù* in 4 *mǔ*; Make it that one gets 1 *bù* of length for every time it accommodates the *bù* of the breadth; for what does not fill a *bù*, designate the part by the breadth. Reversing it: let them be multiplied together; where there is a part of a *bù*; multiply the numerator of the part by the breadth; and where [the result] accommodates the number of the breadth, you get 1 *bù*.

S162,S163

(b) The breadth is 6/8 *bù*; one seeks a field of 4/7; its length is 16/21

(c) The breadth is 3/7 *bù*; one seeks a field of 2/4; its length is 1 *bù* and ⅙ *bù*.

(d) The method for seeking the length: the numerator of the breadth part multiplies the denominator of the area denominator to make the divisor; The area part numerator multiplies the breadth part denominator to make the dividend; [count] 1 *bù* for each time the dividend accommodates the divisor. Then one multiplies the breadth and length together. One always makes the denominators of the parts multiply together to make the divisor; the numerators of the parts multiply together to make the dividend [count] 1 for each time the dividend accommodates the divisor.

Content:
This section seems to be in an unsatisfactory state, and is certainly not a single integrated discourse. Subsection (a) fails to give the answer to the problem it poses, although it says clearly how the answer may be obtained. After 'reversing it' we are dealing with a simple check of the preceding calculation by multiplying together the given breadth and the derived length to get back to the original area with the proviso that fractions are involved in the derived length, though not in the breadth. Of course the denominator of any such fraction will be the number of *bù* in the breadth; it is unclear why we do not simply count the numerator as *bù* rather than multiplying and dividing by the breadth to reach that point. Subsections (b) and (c) appear to give further examples, with solutions. Section (d) is a clear and fairly complete statement of how the unknown side is to be calculated when fractions are involved, and how the result may be checked by multiplication.

S164,S165,S166(5 character gap)

(67) The lesser breadth

(a) The method for seeking the lesser breadth: first set out the breadth. Then say that in the lowest [place] there is a given part of a *bù.* Make one into that given amount; make a half into [its corresponding] amount, and make a third into [its corresponding] amount; accumulate the parts until you have exhausted the part sought; put them together to make the divisor; then register and set out a field of 240 *bù*; let that one for its part be made into [its corresponding] amount; take it as the 'accumulated *bù*'; reduce the accumulated *bù* by accommodating the divisor to obtain 1 *bù*; that which does not fill a *bù*, designate its part by the divisor. A further [method] says: reversing it, then multiply the length by the breadth; let it reverse to make a 240 *bù* field [which is] 1 *mǔ*. In the case where there is a length which is not divided into parts set it out and augment what is not divided into parts by the divisor; reversing, multiply it to make the smaller 10.

Content:
This subsection is clearly separate from what follows it, and deserves to be treated in its own right. The 'reversing it' section resembles what we saw previously, but the reference to 'a length which is not divided' is hard to interpret. The expression 'the smaller 10' seems to parallel a reference to a 'smaller 5' in section 10, where it referred to the denominator of the fraction 5/9: but it is not clear what fraction is referred to in this case.

S167

(b) The lesser breadth. The lesser breadth: if the width is 1 *bù* and half a *bù*; let 1 become 2 and let a half become 1; combine them as 3 to make the divisor; then set out 240 *bù*; for its part, take 1 as 2; reducing, obtain 1 *bù* of length [for each time it] accommodates the divisor; this makes a length of 160 *bù*; take and multiply [this] by 1 *bù* and a half *bù*.

S168(1 character gap)

If in the lowest [place] there is ⅓ take 1 as 6; a half as 3; ⅓ as 2; joining them: 11; one obtains a length of 130 *bù* and 10/11 *bù*; multiply it for a field of 1 *mǔ*.

S169(4 character gap)

If in the lowest [place] there is ¼; take 1 as 12; a half as 6; ⅓ as 4; ¼ as 3; joining them: 15; one obtains a length of 115 *bù* and 5/25 *bù*; multiply it for a field of 1 *mǔ*.

S170

If in the lowest [place] there is ⅕; take 1 as 60; a half as 30; ⅓ as 20; ¼ as 15; ⅕ as 12; joining them: 137; one obtains a length of 105 *bù* and 15/137 *bù*; multiply it for a field of 1 *mǔ*.

S171, S173(long gap)

If in the lowest [place] there is ⅙; take 1 as 60; a half as 30; ⅓ as 20; ¼ as 15; ⅕ as 12; ⅙ as 10; joining them: 147 one obtains a length of 97 *bù* and 141/147 *bù*; multiply it for a field of 1 *mǔ*.

S172,S182(long gap)

If in the lowest [place] there is ⅐; take 1 as 420; a half as 210; ⅓ as 140; ¼ as 105; ⅕ as 84; ⅙ as 70; ⅐ as 60; joining them: 1089; one obtains a length of 92 *bù* and 612/1089 *bù*; multiply it for a field of 1 *mǔ*.

S174,S175(10 character gap)

If in the lowest [place] there is ⅛; take 1 as 840; a half as 420; ⅓ as 280; ¼ as 210; ⅕ as 168; ⅙ as 140; ⅐ as 120; ⅛ as 105; joining them: 2283 to be the divisor; one obtains a length 88 *bù* and 696/2283 *bù*; multiply it for a field of 1 *mǔ*.

S176,S177(gap in middle of strip),S178(long gap to end of strip)

If in the lowest [place] there is $^{1}/_{9}$; take 1 as 2520; a half as 1260; ⅓ as 840; ¼ as 630; ⅕ as 504; ⅙ as 420; ⅐ as 360; ⅛ as 315; $^{1}/_{9}$ as 280; joining them: 7129 to be the divisor; one obtains a length of 84 *bù* and 5964/7129 *bù*; multiply it to form a field of 1 *mǔ*.

S179,S180,S181(fragmentary)

If in the lowest [place] there is $^{1}/_{10}$; take 1 as 2520; a half as 1260; ⅓ as 840; ¼ as 630; ⅕ as 504; ⅙ as 420; ⅐ as 360; ⅛ as 315; $^{1}/_{9}$ as 280; $^{1}/_{10}$ as 252; joining them: 7381 to be the divisor; one obtains a length of 81 *bù* and 6939/7381 *bù*; multiply it to form a field of 1 *mǔ*.

[S182 has been moved to follow S172]

Content:
This long section is entirely concerned with the problem of finding the side that will give a 1 *mǔ* field if the known side is 1 + ½ *bù*, 1 + ½ + ⅓ *bù*, 1 + ½ + ⅓ + ¼ *bù*, and so on up to a series ending in $^{1}/_{10}$.

Parallel:
The material here is closely paralleled by a series of procedures in the Nine Chapters (4: *Shǎo guǎng* **少廣**, Guō 1990, 251-257; Shen 1999, 199-202). Whereas here we stop at $^{1}/_{10}$, the Nine Chapters continue as far as $^{1}/_{12}$. For example, we have:

今有田廣一步半三分步之一四分步之一五分步之一六分步之一七分步之一八分步之一九分步之一十分步之一求田一畝問從幾何

荅曰
八十一步七千三百八十一分步之六千九百三十九

術曰
下有一十分以一為二千五百二十半為一千二百六十三分之一為八百四十四分之一為六百三十五分之一為五百四六分之一為四百二十七分之一為三百六十八分之一為三百一十五九分之一為二百八十十分之一為二百五十二并之得七千三百八十一以為法置田二百四十步亦以一為二千五百二十乘之為實實如法得從步

Now we have a field whose breadth is 1 *bù* and a half + ⅓ *bù* + ¼ *bù* + ⅕ *bù* + ⅙ *bù* + ⅐ *bù* + ⅛ *bù* + $^{1}/_{9}$ *bù* + $^{1}/_{10}$ *bù*. Question: how much is the length?

Answer:
81 *bù* and 6939/7381 *bù*. Method: In the lowest [place] there is $^{1}/_{10}$; take 1 as 2520; a half as 1260; ⅓ as 840; ¼ as 630; ⅕ as 504; ⅙ as 420; ⅐ as 360; ⅛ as 315; $^{1}/_{9}$ as 280; $^{1}/_{10}$ as 252; combine them to get 7381 to be the

> divisor; set out a field of 240 *bù*; take 1 in this case too as 2520, and multiply it to make the dividend; obtain a *bù* of length [each time] the dividend accommodates the divisor.

What, it may be asked, is the meaning of the term *Shǎo guǎng* **少廣** itself? Both the Tang commentator Lǐ Chúnfēng **李淳風** and his successor Lǐ Jí **李籍** offer what is basically the same explanation (4: *Shǎo guǎng* **少廣**, Guō 1990, 251 and 468; Shen 1999, 196). They point out that in all these problems the given breadth is less that the length which is the result of the calculation: in that sense, the breadth is 'lesser'. But see the discussion of the meaning of *Dà guǎng* below, section 61.

Mathematical note:
In three cases the fractions given here are not in their lowest terms, whereas the Nine Chapters reduces fractions systematically. In clause 101 the expression **卅十** for 40 is something of a curiosity.

S183(large portion illegible),S184 (2 character gap)

> (68) The greater breadth
> The breadth is 7 *bù* and ??/49 *bù* ....makes....64 *bù* and 273/343 *bù*. The method for increasing the breadth says: having set out the breadth and the length, in each case multiply the complete *bù* above by the denominator of the part; let the numerators of the parts follow them [in addition]. Let [the results for length and breadth] multiply one another to make the dividend; further in each case let the denominators of the parts multiply one another to make the divisor; obtain 1 *bù* [for each time] it accommodates the divisor; for that which does not fill the divisor, count it off by the divisor.

Content:
This is a straightforward account of how to multiply together two numbers, each with an integral and fractional part. The damage to the opening section is so extensive that there seems little point in trying to find a plausible restoration that will fit the remaining figures. The title of this section is clearly intended to be the inverse of the preceding section.

Parallel:
The Nine Chapters (1: *Fāng tián* **方田**, Guō 1990, 188-189; Shen 1999, 84) has a close parallel to the method given here:

> **大廣田術曰**
> **分母各乘其全分子從之相乘為實分母相乘為法實如法而一**
>
> The method for the field of greater breadth says:
> Each part's denominator multiplies its integral [part]; the numerators of the parts follow them [in addition]; multiply them together to make

> the dividend; the denominators of the parts multiply one another to make the divisor; [count] 1 for [each time] the dividend accommodates the divisor.

As for the significance of the term *Dà guǎng* **大廣**, one might have expected that it would be the inverse of the explanation of *Shǎo guǎng* **少廣** referred to in the annotation to 66. In that case, we would have to have been told that the breadth was greater than the width. However the actual explanation given by Lǐ Chúnfēng and Lǐ Jí (1: *Fāng tián* **方田**, Guō 1990, 188-189 and 461; Shen 1999, 84) is unrelated to what was said before. Lǐ Jí repeats Lǐ Chúnfēng's commentary, which points out that whereas earlier sections of the *Fāng tián* chapter had found areas from sides in whole numbers of *bù*, and then in fractions of *bù*, the discussion has now moved on to mixed numbers, that is with an integral and fractional part, so that the method given is in effect 'greater' since it subsumes the earlier two. This situation seems slightly unsatisfactory, but I cannot think of an obvious way of resolving it.

(S185, S186: see after S140)

S187,S188

(69) [Making] a field into *lǐ*

(a) The method for a field in *lǐ* says: if a *lǐ* multiplies a *lǐ* that is a *lǐ*; if the breadth and length are each 1 *lǐ*; set out 1, take and 3-fold it, then 3 [times] 5-fold it; so that makes a field of 3 *qǐng* and 75 *mǔ*. If the breadth and length are not equal, first let the *lǐ* multiply each other; when that is done, then take and 3-fold it, then 3 [times] 5-fold it, then it is complete. Now there is a breadth of 220 *lǐ* and a length 350 *lǐ*; it makes a field of 288,750 *qǐng*. When one lays out an allotted fief, you do it by this means.

S189,S190(long gap)

(b) One [method]: a *lǐ* if multiplied by a *lǐ* is a *lǐ*; 1 to 3 and then 3 [times] 5-fold it then that is the number of *qǐng* and *mǔ*.

(c) Further: a *lǐ* multiplied by a *lǐ* is a *lǐ*; below the *lǐ* then prepare 25, take and 3-fold them. and that is the number of *mǔ* for a *qǐng*.

(d) [Another method]: a breadth of 1 *lǐ* and a length of 1 *lǐ* makes a field of 3 *qǐng* 75 *mǔ*.

Content:
Subsection (a) describes, with an example, how a piece of land measured in *lǐ* can be reckoned as an area in *qǐng* and *mǔ*, where 1 *qǐng* = 100 *mǔ*; (b), (c) and (d) are passages repeating the same thing, presumably taken from different sources.

Mathematical note:
The slightly strange expression in clause 5 'take and 3-fold it, then 3 [times] 5-fold it' is in fact an instruction to multiply by $3 \times 5 \times 5 \times 5 = 375$, which is the factor required in the next clause, given that 3 *qǐng* and 75 *mǔ* = 375 *mǔ*. A similar

expression was seen in (16), where the expression **參倍** required a number to be 'thrice doubled', i.e. multiplied by 2 x 2 x 2.
In clause 18, we evidently produce 3 and 75 by starting with 1 and 25, and then tripling each number.

Parallel:
The Nine Chapters has a similar section to this in both title and content (1: *Fāng tián* **方田**, Guō 1990, 182; Shen 1999, 63). Since this is placed by the editors as the final section of the *Suàn shù shū* , it is a noteworthy contrast that the Nine Chapters material should be found in the first few lines of that book:

今有田廣一里從一里 問為田几何

荅曰
三頃七十五畝

今有田廣二里從三里 問為田几何

荅曰
二十二頃五十畝

里田術曰
廣從里數相乘得積里以三百七十五乘之即畝數

Now there is a field of breadth 1 *lǐ* and length 1 *lǐ*. Question: how much of a field does that make?

Answer:
3 *qǐng* 75 *mǔ*.

Now there is a field of breadth 2 *lǐ* and length 3 *lǐ*. Question: how much of a field does that make?

Answer:
22 *qǐng* 50 *mǔ*.

The method for a field in *lǐ* says:
the number of lǐ in the breadth and length multiply each other, and so one obtains the 'accumulated *lǐ*'. Multiply it by 375; then that is the number of *mǔ*.

Suàn shù shū 筭數書: text and editing notes

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Previous transcriptions and this version

The original text of the *Suàn shù shū* takes the form of columns of characters written from top to bottom, each one on a separate strip of bamboo. For the convenience of a modern reader, each column is represented here by an unbroken horizontal row of characters - hence the use of 'landscape' format in this part of my work. The characters were written on the strips in ink, using a version of the common scribal hand of the early imperial age. Although the style of the characters is different from modern Chinese handwriting it is not particularly difficult to read for anyone acquainted with classical Chinese. The text was published in three stages, through which the full contents of the text and its physical appearance were progressively made public. Since I had to work with what was available at each stage, the transcription given below was prepared in three corresponding steps:

(1) The simplified character version published in *Wénwù* 文物 in December 2000 was transcribed. This publication did not show where strips began or ended.
(2) The resulting electronic text was then transformed to full characters using a software utility. After correction of mechanical errors and omissions, this was compared with the full character transcription published by Péng Hào 彭浩 in 2001, which I obtained in the summer of that year, and revised accordingly. Péng Hào's book marks the start and end of strips, and gives very clear but reduced size photographs of some, but not all, of the strips. Some of these photographs are in colour.
(3) Finally in late 2001 I was able to compare the text with the full-size photographs of the strips published with the other material from Zhāngjiāshān 張家山 in Zhāngjiāshān (2001), together with the accompanying transcription and notes. My numbering of the strips follows the ordering in that publication, which differs slightly from the arrangement in (2001). After this stage, my electronic version of the text became in effect an independent edition based directly on the photographed strips, though made in the light of amendments suggested by

other editors.

Principles of this transcription

My aim has been to produce a transcription that will render the contents of the strips into modern orthography as fully and straightforwardly as possible, with the absolute minimum of apparatus needed to indicate obvious scribal errors or omissions in the original. Unlike the editors of Zhāngjiāshān (2001) I have not suggested insertions purely designed to regularise the diction of the text, for instance by inserting the name of the unit into a fraction when this is obvious from the context. Nor have I inserted passages which would be present if the text gave complete details of every procedure, but which are unnecessary if the reader is assumed to be familiar with mathematical practice. One of the main interests of the material before us is that it shows how informally structured early Han mathematical writing could be, compared (for instance) to the regular, fully explicit and internally consistent style of the text of the Nine Chapters as it has come down to the present day. Given the rare chance to read an ancient work like the *Suàn shù shū* without having to view it through the lens of generations of scribes and editors, it seems pointless to obscure its actual structure by attempting to reconstruct an idealised and regularised version of which the *Suàn shù shū* is assumed to be merely an imperfect copy. There is of course a difference between an informal writing style and inadvertent mistakes and omissions. Where it seems that these have occurred I have noted their presence and suggested emendations.

The usage of characters on the strips differs somewhat from the received literary tradition. This is not surprising, since this text dates from before the process of regularisation that was undertaken later in the Han dynasty. It is a commonplace that early Han readers were expected to use context to guide their reading in directions that modern orthography makes explicit; what is more, different parts of the text sometimes write what is clearly the same word in more than one way, e.g. 朮, 術 and 述. I have simply recorded the variant usage as it exists on the strips, and have commented only when there is a definite need to do so in order to justify my interpretation of the passage in question. Common examples of non-standard usages requiring interpretation are as follows:

有　read as 又 when used as a conjunction.
朱　read as 銖

朮 read as 術
直 read as 置
秏 read as 耗
賈 read as 價
毀 read as 穀

These readings are assumed without comment in my translations. In less common cases I have given the appropriate reading in a footnote when the character first appears. In another case I have decided that the function of a character in the original Chinese text is best served in English by not translating it at all, or at least not translating it as a word in its own right. This is *yūe* 曰, a character most familiar when it occurs before a passage of direct speech, as in the phrase in the Confucian Analects *zǐ yūe* 子曰 'The Master said ...' used to introduce the words of Confucius as recorded by his disciples. But in the task of translating the *Suàn shù shū* I have felt constrained to abandon this simple usage when faced with the frequent repetition of such introductory phrases as *shù yūe* 朮曰, *dà yūe* 荅曰, *dé yūe* 得曰, *yòu yūe* 有曰, or just plain *yūe* 曰. Here 曰 seems to have no sense of utterance, and to be little more than a punctuation mark equivalent to a colon in English, informing us that what comes next is the thing just referred to - 'the method', 'the answer' etc. As such I have therefore rendered it. However, when 曰 appears on its own (usually introducing the answer to a problem posed) I have avoided concealing its presence altogether and rendered it as 'Reply:'.

There are a number of characters on the strips that are not equivalent to any form found in modern dictionaries. I have made new characters to represent these in printed form, and have commented on their likely meaning in the context of my translation. In one such case, Zhāngjiāshān (2001) deals with the non-current form 𠦃 for 70 by translating it as 七十 throughout; the original character is retained here. In two other cases Zhāngjiāshān (2001) elides interesting features in the process of transcription. Firstly it omits altogether the small 'punctuation hooks' ˪ that appear in many sections of the original, while supplying modern punctuation, which I omit. Secondly, when the strips mark the repetition of a character by showing a 'ditto mark' ₌ , Zhāngjiāshān (2001) omits the mark and repeats the character (or group of characters) in question without comment. On the other hand Zhāngjiāshān (2001) does mark in the rarer round 'blob' • which occurs as a marker at some places in the text.

Where the strips are unreadable or broken, I follow Anon (2001) in using the usual conventional signs:

illegible character: □
broken strip: ⧅

To deal with omissions, emendations and corrections I use a somewhat simpler apparatus than Zhāngjiāshān (2001). Anything to be inserted is shown in square brackets thus [ ], while anything to be deleted or replaced by something else is shown in round brackets ( ). Thus a suggestion that A in the original text should be read as B would be shown as [B] (A), and a suggestion that an illegible character should be read as X is shown as [X] (□).

Since the text is written on bamboo strips, the physical form of the medium inevitably conditions the structure of the text. All whole strips are close to a standard 30 cm in length, and about 1.5 to 2.0 cm from the top and bottom of each strip, as well as about half-way down there appear marks representing the slight thickening where a node occurred in the length of bamboo from which the strip was cut. Several strips bear one to three characters above the upper node mark: these are clearly titles for what follows in the main body of the strip, and have been shown for clarity on a separate line from the main text on the rest of the strip. In a few cases there are also characters or blob marks below the lower node. Where this occurs, I draw attention to this feature by inserting the arbitrary mark ¶ to represent the position of the lower node. Finally, I have also noted the existence of blank space on a strip after the text ends, since this clearly suggests that there is a significant division in the text at that point. Where no such note is given, that indicates that the strip is filled with text down to the lower node.

■ 筭數書[1]

Strip 1
相乘
寸而乘寸=也ʟ乘尺十分尺一也ʟ乘十尺一尺也ʟ乘百尺十尺也乘千尺百尺也ʟ半 [分寸] (囗囗)[2] 乘尺廿分尺一也 ¶・楊

Strip 2
ʟ 三分寸乘尺卅分尺一也ʟ八分寸乘尺八十分尺一也 (gap to end of strip)

Strip 3
一半乘一半也ʟ乘半四分一也ʟ三分而乘一 ʟ 三分一也ʟ乘半六分一也ʟ乘三分九分一也ʟ四分而乘一也 ¶ 楊

Strip 4
四分一也乘半 [八] (卅)[3] 分尺一也ʟ四分寸乘尺四十分尺一也ʟ五分寸乘尺五十分尺一也六分寸乘尺六十分尺

Strip 5
一也ʟ七分乘尺丯 (八)[4] 分一也ʟ乘三分十二分一也ʟ乘四分十六分一也ʟ五分而乘一五分一也ʟ乘半十分一也

Strip 6
乘三分十五分一也ʟ乘四分廿分一也ʟ乘五分廿五分一也ʟ乘分之朮曰母乘母為法子相乘為實

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1. These characters, together with the preceding black mark, are found on the back of strip 6. They would have been visible from the outside when the strips were rolled up for storage, and apparently function as a title for the whole bundle.
2. The illegible characters are obvious from the sense and the pattern of the rest of the passage.
3. This graphically plausible emendation makes sense if we follow Zhāngjiāshān (2001) in assuming that this clause follows from what precedes.
4. This character is clearly inserted in error, since the text makes mathematical sense only if it is removed. Guō Shūchūn (2001) and his predecessors, working from the *Wénwù* transcription, seek to move all the characters from the preceding 四分寸 to 丯 to another position, following 卅分尺一也 in strip 2, for the sake of a more orderly text. But the flow of text on the strips makes it unlikely that a scribal error has occurred. In similar cases elsewhere I have not commented explicitly.

Strip 7
分乘
分乘分朮皆曰母相乘為法子相乘為實 (long gap to end of strip)

Strip 8
乘
少半乘少半九分一ʟ半步乘半步四分一也ʟ半步乘少半步六分一也ʟ少半乘大半九分二也ʟ五分乘五分廿

Strip 9
五分一ʟ四分乘四分十六分一ʟ四 [分][5] 乘五分廿分一ʟ五分乘六分卅分一也ʟ七分乘七分卌九分一也ʟ六分乘六分卅六分一也ʟ六

Strip 10
分乘七卌二分一ʟ七分乘八分五十六分一也 (long gap to end of strip)

Strip 11
一乘十=也ʟ十乘萬十萬也ʟ千乘萬千萬一乘十=萬=也ʟ十乘十萬百萬ʟ半乘千五百ʟ一乘百=

Strip 12
萬=ʟ十乘百萬千萬ʟ半乘萬五千ʟ十乘千萬也ʟ百乘萬百萬ʟ半乘百五十 (two character gap to end of strip)

Strip 13
矰[6]減分
增分者增其子ʟ減分者增其母 (long gap to end of strip)

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5. The insertion is needed for the sense, and also to follow the pattern of the rest of this section. The omission is evidently a scribal error.
6. This character seems to function identically to 增; it is not clear why it is used in this title, when the more common form is used in the body of the strip. In strip 166 it occurs in the body of the strip.

Strip 14
分當半者[7]
諸分之當半者倍其母ㄴ當少半者三其母ㄴ當四分者四其母ㄴ當五分者五其母當十百分者輒十

Strip 15
百其母如所欲分 (long gap to end of strip)

Strip 16
分半者
雖有百分以此進之 (long gap to end of strip)

Strip 17
約分
約分術曰以子除母₌亦除子₌母數交等者即約之矣ㄴ有曰約分朮曰可半₌之可令若₌干₌一₌・ 其一朮曰

Strip 18
以分子除母ㄴ 少以母除子₌母等以為法子母各如法而成一 (long gap to end of strip)

Strip 19
不足除者可半₌母亦半子 (long gap to end of strip)

Strip 20
・ 二千一十六分之百六十二約之百一十二分之九 (long gap to end of strip)

Strip 21
合分
合分術曰母相類子相從母不相類可倍₌可三₌可四₌可五₌可六₌ [子] (七)[8] 亦輒倍₌及三四五之如母₌相類

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7. To fit all four characters above the upper node, they appear in two columns.
8. The emendation is graphically plausible and replaces the pointless figure 7 with "numerator" as required by the sense.

Strip 22
者子相從其不相類者母相乘為法子互乘母并以為實ㄴ如法成一今有五分二ㄴ六分三

Strip 23
十 (一)[9] 分八ㄴ十二分七ㄴ三分二為幾何曰二錢六十分錢五十七ㄴ其術如右方五人分七錢少半=錢人得一錢卅

Strip 24
分錢十七術曰下三分以一為六即因而六人以為法亦六錢以為實ㄴ有曰母乘母為法子羨[10]乘母

Strip 25
為實=如法而一ㄴ其一曰可十=可九=可八=可七=可六=可五=可四=可三=可倍=母相類止ㄴ母相類子相從

Strip 26
徑分
徑分以一人命其實故曰五人分三有半少半各受卅分之廿三其朮曰下有少半以一為六以半為 [三] (一)[11] 以少半為二

Strip 27
并之為廿三即值[12] [人] (一)[13] 數因而六之以命其實ㄴ有曰朮曰下有半因而倍之下有三分因而三之下有四分因而四之

Strip 28
出金
有金三朱九分朱五今欲出其七分朱六問餘金幾何曰餘金二朱六十三分朱卌四其術曰母相乘

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9. This deletion makes the fraction 8/10 as required by the calculation.

10. 漢語大辭典 makes this the same as 羡, which has as one meaning 邪, here presumably 'diagonally'.

11. Although the strip is clear, this emendation is required to make sense of the figures.

12. The character is faint on the strip; if Zhāngjiāshān (2001) is correct in this reading, then 值 is evidently to be read as 置, like the more usual 直.

13. Zhāngjiāshān (2001) does not emend here, but the change is graphically plausible as well as yielding 'the number of men' as the sense requires.

Strip 29
也為法子互乘母各自為實以出除焉餘即餘也· 以九分朱乘三朱與小五相 [并] (除)[14] (7 character gap to end of strip)

Strip 30
今有金七分朱之三益之幾何而為九分七ㄴ曰益之六十三分朱廿二· 術曰母相乘為法子互乘母各自為

Strip 31
實以少除多餘即益也 (long gap to end of strip)

Strip 32
共買材
三人共材以賈[15]一人出五錢一人出三ㄴ一人出二錢· 今有嬴四錢欲以錢數衰分之出五者得二錢出三者

Strip 33
得一錢五分錢一出二者得五分錢四朮曰并三人出錢數以為法即以四錢各乘所出錢數如法得一錢

Strip 34
狐出關
狐貍犬出關租百一十一錢犬謂貍₌謂狐 [爾] (而) 皮倍我出租當倍弐[16]問出各幾何得曰犬出十五錢七分六

Strip 35
貍出卅一錢分五狐出六十三錢分三朮曰令各相倍也并之七為法以租各乘之為實₌如法得一 (6 character gap to end of strip)

Strip 36
狐皮
狐皮卅五弐[17]貍皮廿五弐犬皮十二弐偕出關₌并租廿五錢問各出幾何得曰狐出十二丰二分十一貍出八分卌

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14. The strip reads clearly, but the emendation is required, since addition rather than subtraction makes sense here.
15. This character is evidently to be read as 價; alternatively Guō Shūchūn (2001) reads 賈 as *gǔ*
16. This character is used here as the exclamatory 哉, rather than for 裁 as on strip 36.
17. Here this character is read as the measure word 裁.

Strip 37
九犬出四分十二朮 [曰][18] 并賈為法以租各乘賈為實 (long gap to end of strip)

Strip 38
負米
人負米不智[19]其數以出關₌三 [₌][20] 税之一已出餘米一斗問始行齎米幾何得曰齎米三斗三升四分三朮曰

Strip 39
直一（關）[21]而參倍為法有直米一斗而三之有三（倍）之而關數焉為實 (long gap to end of strip)

Strip 40
女織
鄰里有女〔善織日自倍〕（惡自喜）[22]也織曰自再五日織五尺問始織日及其次各幾何曰始織一寸六十二分寸卅八ㄴ次三寸六十

Strip 41
二分寸十四ㄴ次六寸六十二分寸廿八ㄴ次尺二寸六十二分寸五十六ㄴ次 [二] (一)[23] 尺五寸六十二分寸五十ㄴ朮曰直二ㄴ直四ㄴ直八ㄴ直十六ㄴ直

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18. Though the point is trivial, since 曰 is found in all other such cases after 朮, it does seem that the omission was a scribal slip.
19. Read here as 知.
20. This strip is clear at this point, and bears no ditto mark after 三. However this seems necessary for the sense, since we need 3 for the number of customs posts, as well as for the denominator in the fraction payable as duty.
21. This and the subsequent omission make somewhat better sense of this passage, which has to result in the factor (3*3*3)/(2*2*2).
22. As it stands these three characters make little sense in this context. The emendation is suggested partly on the basis of the wording in the related problem in the Nine Chapters, and partly since 善 and 倍 are graphically close to 惡 and 喜. This suggestion assumes that a scribe uncomprehending enough to make these substitutions (or reading a damaged text) also omitted 織日. Guō Shūchūn (2001) simply reads 惡 as equivalent to 甚, without further discussion.
23. This simple emendation is required by the calculation, in which each amount must be double the preceding.

Strip 42
卅二并以為法L以五尺偏[24]乘之各自為實=如法得尺不盈尺者十之如法一寸L不盈寸者以法命分 (5 character gap) ¶ 王已讎

Strip 43
并租
禾三步一斗麥四步一斗荅五步一斗今并之租一石L問租幾何得曰禾租四斗卌七分十二L麥租三斗分九

Strip 44
荅租二斗分廿六朮曰直禾三步〔麥〕（吏）[25]四步荅五步令禾乘麥為荅實荅乘禾為麥實L [麥乘荅為禾實L][26] 各 [異][27] (口)

Strip 45
直之以一石各乘之 [而十之][28] (口口口) 為實卌七為法一斗 (long gap to end of strip)

Strip 46
金賈
金賈兩三百一十五錢今有一朱問得錢幾何曰得十三錢八分一朮曰直一兩朱數以為法以錢數為實=如法得一錢

Strip 47
廿四朱一兩L三百八十四朱一斤L萬一千五百廿朱一鈞L四萬六千八十朱一石 (5 character gap to end of strip)

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24. Read as 徧, as in the Nine Chapters, 方程. Later usage in this context would have been 遍.

25. There is clearly a scribal error here, in a the form of a graphical confusion suggesting that the scribe was not paying attention to the meaning of the text, in which each type of grain is named in turn.

26. This seems an essential addition to make the procedure work, since each dividend needs to be specified: it may be that the scribe copied only two of the three parallel phrases required before going on to the rest of the text. On the other hand, once the first two amounts are known, the third could have been found by subtraction from the total.

27. The restoration of 異 for an illegible character is based on the parallel in strip 134. Alternatively, Guō Shūchūn (2001) suggests 副, which has the same effect mathematically.

28. Zhāngjiāshān (2001) prefers to supply 禾麥荅 here. Multiplication by 10 is required to obtain *dǒu* from *shí*.

Strip 48
舂粟
稾粟一石舂之為八斗八升當益秏粟幾何曰 [一] (二) 斗三升十一分升 [七] (八)[29] 朮曰直所得米升數以為法有直一石

Strip 49
米粟升數而以秏米升數乘之如法得一升 (long gap to end of strip)

Strip 50
銅秏
鑄銅一石秏七斤八兩今有銅一斤八兩八朱問秏幾何得曰一兩十 [二] (一) 朱百卌四分朱 [七十二] (九十一)[30] 朮曰直一石朱數為法亦直七斤八兩者

Strip 51
朱數以一斤八兩八朱者朱數乘之如法一朱 (long gap to end of strip)

Strip 52
傳馬
傳馬日二[31]匹共芻稾二石令芻三而稾二今馬一匹前到問予芻稾各幾何曰予芻四斗ʟ稾二斗泰半斗朮曰直芻三稾

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29. Following the text, we are to perform the calculation 100*12/88 to find the number of *shēng* of unprocessed grain which will compensate for a deficit of 12 *sheng* of processed grain, given that 100 *sheng* unprocessed yields 88 *shēng* processed. The result is 13 and 56/88 *shēng* = 13 and 7/11 *shēng*, hence the two graphically plausible emendations suggested here. For reasons that are not clear, Anon (2001) wishes to insert a new clause that would make the calculation relate to the processing of 188 *shēng*, with a different emendation of the result in consequence. Guō Shūchūn (2001) also has a significant insertion, and makes it plain that this follows from his reading of **秏粟** as the name of a type of grain, rather than the simpler 'wasted grain'.

30. Following the text, the calculation should give a result of 5256/144 *zhū* = 1 *liǎng* 12 *zhū* 72/144 *zhū*. Péng Hào notes that the result in the text is incorrect, and seeks to emend it to **一兩十二朱二分朱一**, thus excising **百卌四分朱九十一**. However, it seems simpler to emend to **一兩十二朱百卌四分朱七十二**, which is graphically plausible, and simply assumes that the fraction is not in its lowest terms.

31. Anon (2001) emends **二** to **三**, which seems to miss the point that an extra horse has to be fed from the rations of two *shí* originally intended for only two horses. But the calculation is not affected.

Strip 53
二并之以三馬乘之為法以二石乘所直各自為實 (long gap to end of strip)

Strip 54
婦織
有婦三人長者一日織五十尺中者二日織五十尺少者三日織五十尺今織有攻[32]五十尺問各受幾何尺其得

Strip 55
曰長者受廿五尺中者受十六尺有十八分尺之十二ι少者受八尺有十八分尺之六其朮曰直一直二直三而各幾[33]

Strip 56
以為法有十而五之以為實如法而一尺不盈尺者以法命分・三為長者實二為中者一為少者 (5 character gap) ¶ 楊已讎

Strip 57
羽矢
羽二喉[34]五錢今有五十七分侯 [卅] (卌)[35] 七問得幾何曰得一錢百一十四分錢丰一朮曰二乘五十七為法以五乘卅七為實

Strip 58
如法一錢不盈以法命分 (long gap to end of strip)

Strip 59
桼[36] 錢
桼斗卅五錢今有四十分斗五問得幾何錢曰得四錢八分錢三朮曰以卌為法以五乘卅五為實＝如法

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32. Read as 功.
33. Zhāngjiāshān (2001) suggests that 各幾 be emended to并, which does not seem graphically plausible, even if the text is a little cryptic as it stands.
34. Read this and the subsequent 侯 as 猴.
35. The strip does appear to read 卅, but this emendation is necessary to produce the correct result: (37/57)*5/2 = 185/114 = 1 and 71/114, as stated.
36. Read as *qī* 漆 on this strip and below on strip 66.

Strip 60
得一錢 (damaged; three visible characters at the top, then seems to have a long gap before the break) ⧅

Strip 61
繒幅
繒幅廣廿二寸袤十寸賈廿三錢今欲買從利廣三寸袤六十寸問積寸及賈錢各幾何曰八寸十一

Strip 62
分寸二賈十八錢十一分錢九朮曰以廿二寸為法以廣從相乘為實₌如法得一寸亦以一尺寸數為法以

Strip 63
所得寸數乘一尺賈錢數為實₌如法得一錢 (long gap to end of strip)

Strip 64
息錢
貸錢百息月三今貸六十錢月未盈十六日歸計息幾何得曰廿五分錢廿四朮曰計百錢一月積

Strip 65
錢數以為法直貸錢以一月百錢息乘之有以日數乘之為實如 [法][37] 得息一錢 (long gap to end of strip)

Strip 66
㱃[38]桼[39]
⿰金桼一斗㱃水三斗而槃[40]㱃水二斗七升即槃問餘⿰金桼水各幾何曰餘⿰金桼卅七分升卅餘水二升卅七分

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37. The insertion of this character is required unless the expression is to be defective; presumably there has been a scribal slip, possibly also involving a dropped ditto mark after the preceding 實. The strip reads very distinctly at this point.
38. Read this non-standard character as *yìn* 飲 "to give drink to someone", here referring to pouring water into the lacquer.
39. Read both this and the later non-standard ⿰金桼 as *qī* 漆.
40. Read this non-standard character as *pán* 盤.

Strip 67
升七・ 朮曰以二斗七升者同一斗卅七也為法有直廿七ㄴ十升者各三之為實₌如法而一 (long gap to end of strip)

Strip 68
稅田
稅田廿四步ㄴ八步一斗租三斗今誤券三斗一升問幾何步一斗得曰七步卅 [一] (七)[41] 分步廿三而一斗ㄴ朮曰三斗一升者為法

Strip 69
十稅田令如法一步 (long gap to end of strip)

Strip 70
程竹
程曰竹大八寸者為三尺簡百八十三今以九寸竹為簡₌當幾何曰為二百五簡八分簡七朮曰以八寸為法 (4 character gap to end of strip)

Strip 71
程曰八寸竹一箇為尺五寸簡三百六十六今欲以此竹為尺六寸簡₌當幾何曰為三百 [卌] (廿)[42] 三ㄴ八分簡一朮曰以十六寸為法

Strip 72
醫
程曰醫治病者得六十筭囗囗廿筭囗囗程囗弗 (damaged and only partly decipherable)[43]

Strip 73
得六十而負幾何曰負十七筭二百六十九分筭十一其朮曰以今得筭為法令六十乘負筭為實 (space for five characters, possibly obliterated))

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41. Although the strip clearly reads 七, emend 七 to 一 since the divisor is clearly 31, and this is graphically plausible as a scribal error, given the old form of 七 on the strip is a horizontal line with a short central vertical stroke.
42. The strip reads 廿, but this graphically plausible emendation is required, since the calculation is clearly 366 x 15/16 = 5490/16 = 343 1/8
43. There is no room in the gaps here for the major insertions proposed by Guō Shūchūn (2001).

Strip 74
石衛[44]
石衛之朮曰以所賣₌[45]為法以得錢乘一石數以為實其下有半者倍之少半者三之有斗升斤兩朱者亦皆

Strip 75
破其上令下從之以為法錢所乘亦破如此 (long gap to end of strip)

Strip 76
賈鹽
今有鹽一石四斗五升少半升賈取錢百五十欲石衛之為錢幾何ㄴ曰百三錢四百卅 [六] 分錢九十 [二] (五)[46] 朮

Strip 77
曰三鹽之數以為法亦三一石之升數以錢乘之為實 (long gap to end of strip)

Strip 78
絲練
以級[47]絲求練因而十二之除十六而得一 (long gap to end of strip)

Strip 79[48]
挐脂
有米三斗問用脂 [水] (米)[49] 各幾何為挐幾何曰用脂六斤水四升半升為挐脂十斤十二兩十九朱五分朱一為挐米

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44. Read this non-standard character as *lǜ* 率.
45. Zhāngjiāshān (2001) suggests that the repeat of this character should be read as *mǎi* 買. It would also be possible to read as *jià* 價.
46. This emendation and the preceding insertion of 六 are needed to produce the correct result to the calculation specified: 150*100/(436/3) = 45000/436 = 103 92/436. Scribal error seems likely here.
47. The emendation of 級 to 絡 is graphically plausible, and also attractive in view of the parallel from the Nine Chapters.
48. The transpositions proposed for the material in this section by Guō Shūchūn (2001) are incompatible with the flow of text on the strips.
49. Emend thus since it is the amounts of fat and water that are calculated in the following. The change is graphically plausible.

Strip 80
一斗水一斗半 [斗] (升)[50] 崖脂廿斤為挈脂卅六斤今有峭脂五斤問用米水為挈各幾何得曰用米二 [升] (斗)半升水

Strip 81
三 [升] (斗) 四分升三為挈九斤术曰以廿為法直水十五米十挈卅六以五乘之為實₌如法得水米各一升挈一斤

Strip 82
不盈以法命分其以挈米崖亦一兩得峭九分之五也 (long gap to end of strip)

Strip 83
取程
取程十步一斗今乾之八升問幾何步一斗 (問)[51] 得 [曰] (田)[52] 十二步半一斗术曰八升者為法直一 [斗] (升)[53] 步數而十之如法一步 (競)[54]

Strip 84
程卅七步得禾十九斗七升問幾何步一斗得曰減田 (十) 一步有 [百] 九十七分步 [百] 丰 [三] (九) (步) 而一斗[55]

Strip 85
取程五步一斗今乾之一斗一升欲減田令一斗得曰減田十一分步五术曰以一斗（一）[56]升數乘五步令十一而一

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50. This emendation and the others on this and the following strip are needed to produce the correct results - which are simply a quarter of the amounts of rice and water previously given as needed to process 20 *jīn* of fat. The numbers are graphically plausible, and the ancient forms of **斗** and **升** differ by only one small stroke. Zhāngjiāshān (2001) emends differently, for reasons that are not immediately clear.
51. The deleted character makes no sense here.
52. This graphically plausible emendation yields a typical result statement.
53. Emend thus, since the calculation clearly needs the number of *bù* yielding one *dǒu*, and the forms of the characters on the strips are easily confused.
54. Though clear on the strip, this character seems to make little sense here. It is possible that it is used in the attested sense of *qū* **趨** 'seek after, pursue' (**漢語大辭典**), in which case it may be equivalent to the *qǔ* **取** 'collect' we would have expected before the following *chéng* **程**. But this is highly conjectural.
55. The strip reads clearly, but the emendations given here are needed if the calculation is to be $37*10/197 = 370/197 = 1$ and 173/197 as the problem requires; the final **步** is out of place. An arithmetical rather than a scribal error seems more plausible as an explanation of the present state of the text. Zhāngjiāshān (2001) wants to emend so as to yield the result 35 and 24/197, the amount when 1 and 173/197 is subtracted from 37, which seems unnecessarily complicated.
56. Although this character is clear on the strip it needs to be deleted if the text is to state the calculation correctly as $10*5/11$. This yields 4 and 6/11, which when

Strip 86
耗租
耗租產多乾少曰取程七步四分步一斗今乾之七升少半升欲取一斗步數朮曰直十升以乘七〔步〕（斗）[57]四分步 ¶ ·

Strip 87
一如乾成一數也曰九步卌四分步卅九而一斗程它物如此 (long gap to end of strip)

Strip 88
程禾
程曰禾黍一石為粟十六斗泰半斗舂之為糲₌米₌一石₌[58]為糳[59]₌米₌九斗₌為毀米八斗 (2 character gap) ¶ 王

Strip 89
程曰稻禾一石為粟廿斗舂之為米十斗為毀粲米六斗泰半斗麥十斗麴三斗(3 character gap)

Strip 90
程曰麥菽荅麻十五斗一石稟毀糳者以十斗為一石 (long gap to end)

Strip 91
取枲程
取枲程十步三韋束一今乾之廿八寸問幾何步一束朮曰乾自乘為法∟生自乘有以生一束步數乘之為

Strip 92
實₌如法得十一步有九十八分步卌七而一束 (long gap to end)

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subtracted from 5 gives 5/11 as stated.

57. The correction of the unit is obvious, given that the quantity was correctly stated earlier on.

58. As elsewhere, it is the number-unit pairing 一石 that is to be repeated.

59. Since this has the same upper part as 鑿 *zuò* but with the grain radical below, I therefore guess the reading *zuò.* It seems to be identical in usage to the later 糳.

Strip 93
誤券
租禾誤券者朮曰毋升者直稅田數以為實而以券斗為一以石為十并以為法如法得一步其券有 [斗][60] 者直與 (two character gap) ¶ ·

Strip 94
田步數以為實而以券斗為一以石為十并以為法如法得一步其券有升者直與田步數以為實而以

Strip 95
券之升為一以斗為十并為法如·[法][61] 得一步 (long gap to end)

Strip 96
租吳[62]券
田一畝租之十步一斗凡租二石四斗今誤券二石五斗欲益耎其步數問益耎幾何ㄴ曰九步五分步三而一斗ㄴ朮

Strip 97
曰以誤券為法以與田為實 (long gap to end)

Strip 98
粺毀
米少半升為粺十分升之三九之十而一米少半升為毀米十五分升之四八之十而一米少半升為麥半升· 三之二而一麥少 ¶ 楊

Strip 99
半升為粟廿七分升之十九母 [十子十之九而一麥少半] (口口口口口口口口口口)[63] 升為米九分升之二ㄴ參母再子二之三而一麥少半升為

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60. This insertion is seems essential if the phrase is not to be grammatically defective, and to ensure that it follows the later parallel dealing with the case when the smallest unit is a *shēng.*
61. This character is needed for the sense; possibly the blob was intended as a contraction for it?
62. Read as 誤.
63. The restoration of 十子十之九而一麥少半 for the lacuna caused by damage is clearly justified by the consistent mathematical pattern of this section.

Strip 100
粺五分升之一∟十五母九子∟九之十五而一∟麥少半升為毀卌五分升之八∟十五母八子 (long gap to end)

Strip 101
粺米四分升之一為粟五十四分升之廿五∟廿七母∟五十子∟ (one character gap) 粺米四分升之一為米十八分升之五∟九母∟十子∟粺米 ¶ 楊

Strip 102
四分升之一為毀米九分升之二∟九母∟八子∟粺四分升之一為麥十二分升之五∟九母∟十五子∟毀米四分升之一為米

Strip 103
十六分升之五∟八母∟十子∟毀四分升之一為粺卅二分升九∟八母∟九子∟毀米四分升之一為麥卅二分升之十五∟八母∟

Strip 104
五十子∟毀米四分升之一為粟卌八分升之廿五∟廿 [四] (五)[64] 母∟五十子 (long gap to end)

Strip 105
秏
粟一石秏一斗二升少半升稟米少半升者得粟七百八十九分升之五百∟稟一升者得粟一升二百六十三分升 ¶ 楊

Strip 106
之二百卅七∟稟一斗者得粟一斗九升有二百六十三分升之三∟稟一石者得粟十九斗有二百六十三分升之卅 (2 character gap)

Strip 107
粟石秏五升∟稟米少半升者得粟百卄一分升之百∟稟一升者得粟一升有二百八十五分升之二百十[65]五∟稟一斗者 ¶ 楊

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64. The character 五 is quite clear on the strip, but the emendation is essential for the conversion to work.
65. Zhāngjiāshān (2001) reads this as 卄, but it must be 十 for the calculation to work; the state of the strip at this point would allow either reading.

Strip 108
得粟[66]十七升有二百八十五分升之百五十 [五][67] ʟ稟一石者得粟十七斗五升有二百八十五分升之百廿五 (5 character gap)

Strip 109
粟為米
麻麥菽荅三而當米二ʟ九而當粟十ʟ粟五為米三ʟ米十為粺九ʟ為毀八ʟ麥三而當稻粟四ʟ禾粟 ¶ 楊

Strip 110
五為稻粟四 (long gap to end of strip)

Strip 111
粟求米
粟求米三之五而一ʟ粟求麥九之十而一ʟ粟求粺廿七之五十而一ʟ粟求糳[68]廿四之五十而一ʟ米求 ¶ 楊

Strip 112
粟五之三而一 (long gap to end of strip)

Strip 113
粟求米
粟求米因而三之五而成一· 今有粟一升七分三當為米幾何ʟ曰為米七分升六術曰母相乘為法以三

Strip 114
乘十為實 (long gap to end of strip)

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66. In Zhāngjiāshān (2001) 粟 is followed by 稟, but this character is not on the strip.
67. This insertion is required for the calculation to work.
68. Read as 毇.

Strip 115
米求粟
以米求粟因而五之三成一今有粟七 ⿴囗分[69] 升六當為粟幾何曰為米一升七分升三術曰母相

Strip 116
乘為法以五乘 [六為實] (口口口)[70] (long gap to end of strip)

Strip 117
米粟并
有米一石粟一石并提之問米粟當各取幾何曰米主取一石二斗十六分 [斗] (升)[71] 八粟主取七斗十六分 [斗] (升) 八朮

Strip 118
曰直米十斗六斗并以為法以二石扁[72]乘所直各自為實六斗者粟之米數也 (9 character gap)

Strip 119
粟米并
米一粟二凡十斗精之為七斗三分升一朮曰皆五米粟并為法五米三粟以十斗乘之為實 ¶ 王

Strip 120
⧅口口口得幾何曰粟口口口口卅口口口米⧅

Strip 121
⧅ [并粟米并粟米各五升問米主當] (口口口口口口口口口口口口口)[73] 得幾何得曰米六升四分升之一朮曰直米五升 ¶ 楊

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69. Read as 分.
70. The restoration of the illegible characters is required by the calculation.
71. In both instances in this strip, this graphically plausible emendation is required to make sense of the figures.
72. Read as 徧.
73. The restoration is justified by the calculation which follows as well as the parallels from previous strips.

Strip 122
粟=五=升=為米三升并米五升者八以為法乃更直五升而十之令如法粟米各一升 (6 character gap)

Strip 123
⧅囗囗二斗五升其朮曰直米粟五米三粟 ¶ 楊

Strip 124
⧅并以為法⧅ 并米粟各乘之自為實=如法而成一(long gap to end)

Strip 125
...... 石五十有......

Strip 126
負炭
負炭山中日為成炭七斗到車次一日而負炭道車到官一石今欲道官往之負炭中負炭遠到官

Strip 127
問日到炭幾何曰日得炭四斗十 [七] (一) 分 [斗] (升)[74] 二朮曰取七斗者十之得七石七日亦負到官即取十日與七日并

Strip 128
為法如法得一斗 (long gap to end of strip)

Strip 129
盧唐
程曰一日伐竹六十個一日為盧唐十五ㄴ一竹為三盧唐欲令一人自伐竹因為盧唐一日為幾何曰為十三

Strip 130
盧唐四分之三朮曰以六十為法以五十五乘十五為實 (long gap to end)[75]

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74. The emendations are required to give the denominator of 17 needed for the calculation, and to give the correct unit. The original characters are graphically similar.
75. Although the figures in this section do not work out properly, this is in my view more likely to be due to mistakes in the original calculation rather than to a

Strip 131
羽矢
程一人一日為矢卅羽矢廿ㄴ今欲令一人為矢且羽之一日為幾何日為十二朮曰并矢羽以為法以矢羽相乘為實 (10 character gap to end)

Strip 132
行
甲行五十日今今日壬申問何日初行朮曰問壬申何旬也曰甲子之旬也既道甲數到壬九日直九有增

Strip 133
分錢
分錢人二而多三人三而少二問幾何人錢幾何得曰五人錢十三贏不足互乘 [子] (母)[76] 為實 [母] (子) 相從為法皆贏若

Strip 134
不足 [子] (母) 互乘母而各異直之以子少者除子多者餘為法以不足為實[77] (long gap to end)

Strip 135
米出錢
糲米二斗三錢 [粺] (糲)[78] 三斗二錢今有糲粺十斗賣得十三錢問糲粺各幾何曰粺七斗五分三

Strip 136
糲二斗五分二朮曰令偕糲也錢贏二令偕粺也錢不足六少半同贏不足以為法以贏乘十斗為粺以不

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copying error: see translation.

76. This and similar revisions in strips 133 and 134 (correcting a scribal error?) seem necessary to deal with the fact that it should be the actual amounts of excess or deficit that are treated as the 母 'denominators' which should be added to make a divisor, and the rates of spending that are the 子 'numerators' cross-multeipled by them. In the rest of strip 134 the fact that it is the difference of the 子 that is formed strengthens this impression.

77. These last five characters do not make good sense here. Zhāngjiāshān (2001) agrees, but suggests moving them elsewhere after deletions and insertions which seem excessively confident.

78. This is the simplest emendation that makes the calculation work. However, as Zhāngjiāshān (2001) points out, the result makes *lì* grain more expensive than *bài* grain, which is not usual. It is equally possible to exchange the names of the two types of grain in the emended text, as Zhāngjiāshān (2001) suggests.

Strip 137
足乘十斗為糲皆如法一斗 (long gap to end of strip)

Strip 138
米斗一錢三分錢二黍斗一錢半錢令以十六錢買米黍凡十斗問各幾何用錢亦各幾何

Strip 139
得曰米六斗黍四斗米錢十黍六朮曰以贏不足令皆為米多三分錢二皆為黍少錢下有三分

Strip 140
以一為三命曰 [多二] (各而)[79] 少三并多而少為法更異直二ㄴ三以十斗各乘之即貿其得如法一斗 (7 character gap)

Strip 185[80]
方田
田一畝方幾何步ㄴ曰方十五步卅一分步十五朮曰方十五步不足十五步方六十步有徐[81]十六步曰并贏不足以為法ㄴ不足

Strip 186
子乘贏母贏子乘不足母并以為實复之如啟廣之朮 (long gap to end of strip)

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79. These two emendations make sense out of an otherwise meaningless phrase. The first is perhaps the result of a graphic confusion, while the second (suggested by Péng Hào but not adopted in Zhāngjiāshān (2001)) may be a phonetic slip.

80. The numbering of this and the following strip reflects the fact that the Zhangjiashan (2001) editors place them later; I prefer to take them with other strips bearing material relating to the Rule of False Position.

81. Read as 餘.

Strip 141
除
美[82]除其定[83]方丈高丈二尺其除廣丈袤 [五] (三) 丈 [六] (九)[84] 尺其一旁毋高積三千三百六十尺术曰廣積卅尺 [乘] (除)[85] 高以其

Strip 142
(廣)[86] 袤乘之即定 [六成一][87] (long gap to end of strip)

Strip 143
鄆都
鄆都下厚四尺上厚二尺高五尺袤二丈責[88]百卅三尺少半尺术曰倍上厚以下厚增之以高及袤乘之六成一

Strip 144
芻
芻童及方闕下廣丈五尺袤三丈上廣二丈袤四丈高丈五尺積九千二百五十尺术曰上廣袤下廣袤各自乘又上

Strip 145
袤從下袤以乘上廣下袤從上袤以乘下廣皆并 [以高][89] 乘之六成一 (long gap to end)

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82. Read as yán 羨,since we are clearly dealing with the ramped tomb-entrance excavation called by that name in the Nine Chapters (5: Shang gong 商功, Guō 1990, 288), for which the graphically similar 美 appears to have been substituted by a scribe unfamiliar with this relatively rare character..
83. Zhāngjiāshān (2001) suggests that this *dìng* 定 might be read as *dǐng* 頂 , here referring to the vertical face of the end of the cutting. This makes more sense than the original, and is phonetically plausible.
84. According to the procedure, we are told that the accumulated widths are 30 *chí*, and that this is multiplied by the height, and then by the width, all divided by 6: 30*12*39/6 = 2340. However the result given in the text is 3360. We may either emend this, or change a figure in the data. The second course is adopted here, since the changes needed seem graphically more plausible, so that we have 30*12*56/6 = 3360.
85. Zhāngjiāshān (2001) suggests this emendation, which makes the link required in the calculation.
86. This character is clearly superfluous for the purposes of the calculation.
87. This or an equivalent insertion is essential to complete the algorithm, although the strip clearly lacks these characters. I follow the model of later sections.
88. Read as 積.
89. This or an equivalent insertion is required in order to make the calculation work, although the strip reads clearly at this point.

Strip 146
旋粟
旋米高五尺下周三丈積百廿五尺・二尺七寸而一石為米卌六石廿七分石之八其述曰下周自乘以高

Strip 147
乘之卅六成一・大積四千五百尺 (long gap to end)

Strip 148
囷蓋
囷蓋下周六丈高二丈為積尺二千尺乘之=述曰直如其周令相乘也ㄴ有以高乘之卅六成一

Strip 149
睘亭
圜亭上周三丈大周四丈高二丈積二千五十五尺卅六分尺廿朮曰下周乘上周=自乘皆并以高

Strip 150
乘之卅六成〔一〕[90]今二千五十五尺分廿 (long gap to end)

Strip 151
井材
圜材穽若它物周二丈四尺深丈五尺積七百廿尺朮曰耤[91]周自乘以深乘之十二成一一曰以

Strip 152
[周] (口)[92] 乘徑四成一・ 一百半問徑口口 (long gap, strip very obscure and might have borne further text)

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90. The insertion is needed to complete the algorithm, following the model repeated elsewhere.
91. Read as 藉.
92. The strip is obscure, but the emendation is plausible.

Strip 153
以睘材方
以圜材為方材曰大四韋二寸廿五分寸十四為方材幾何曰方七寸五分寸三術曰因而五之為實令七而一四 [而一][93]

Strip 154
以方材睘
以方為圜曰材方七寸五分寸三為圜材幾何曰四韋二寸廿五分十四· 術曰方材之一面即

Strip 155
圜材之徑也因而四之以為實令五而成 [七] (一)[94] (long gap to end of strip)

Strip 156
睘材
有圜材一 (?) 斷之口口市口口口口口口口大幾何曰 (?) 十 (?) 六 (?) 口口四寸半寸述曰口自乘以[95]

Strip 157
一即成 (long gap to end of strip)

Strip 158
入二寸益之即大數已 (long gap to end of strip)

Strip 159
啟廣
田從[96]卅步為啟廣幾何而為田一畝曰啟八步朮曰以卅步為法以二百卌步為實啟從亦如此 (3 character gap to end of strip)

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93. The missing characters appear to have been omitted because the scribe ran out of space at the end of the strip.

94. This graphically plausible emendation is the only one required to sustain my view of the way the calculations on strips 153-155 work. [$\sqrt{2}$ = 7/5, escribed and inscribed square etc.]

95. This strip is almost indecipherable in parts; readings followed by (?) are conjectural.

96. In such contexts read as *zōng*, equivalent to 縱.

Strip 160
啟從
廣廿三步為啟從求田四畝ᴸ朮曰直四畝步數令如廣步數而得從一步不盈步者以廣命分ᴸ复之令相乘也

Strip 161
ᴸ有分步者以廣乘分子如廣步數得一步 (long gap to end)

Strip 162
廣八分步之六求田 [七] (一)[97] 分之四其從廿一分之十六廣七分步之三求田四分步之二其從一步六分步之

Strip 163
一求從朮廣分子乘積分母為法積分子乘廣分母為實=如法一步ᴸ節[98]以廣從相乘凡 (凡)[99] 令分母相乘為法分子相乘為實¶=如法一[100]

Strip 164
少廣
救[101]少廣之術曰先直廣即曰下有若干 [分] (步)[102] 以一為若干以半為若干ᴸ以三分為若干積分以盡所救分同

Strip 165
之以為法即耤[103]直田二百卌步亦以一為若干以為積步除積步如法得從一步不盈步者以法命其分ᴸ有曰复

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97. This graphically plausible emendation is required for the calculation to work.
98. Read as 即.
99. The repetition of this character makes no sense and is presumably a scribal error.
100. The scribe was evidently determined to finish the text on this strip, since he took the exceptional step of cramming the ditto mark and last three characters into the space below the node. Even then he omitted the unit 步 which would normally have followed 如法一: see the parallels in strips 42, 51,58, 69, 83, 137, 140 and earlier in 163 itself.
101. Read here and later in this strip as 求.
102. Bearing in mind the detailed applications of this general statement below (e.g. 下有三分 in strip 168) it seems evident that this graphically plausible emendation is appropriate, even though 步 is quite clear on the strip.
103. Read as 藉.

Strip 166
之即以廣乘從令复為二百卌步田一畝其從有不分者直如法繒不分复乘之以為小十 (5 character gap to end of strip)

Strip 167
少廣[104]=一步半步以一為二∟半為一同之三以為法即值二百卌步∟亦以一為二除如法得從一步∟為從百六十步因以一步半步¶乘[105]

Strip 168
下有三分以一為六半為三=分為二同之十一得從百卅步有十一分步之十乘之田一畝 (one character gap)

Strip 169
下有四分以一為十二半為六三分為四=分為三同之廿五得從百一十五步有廿五分步之五乘之田一畝 (4 character gap)

Strip 170
下有五分以一為六十∟半為卅∟三分為廿∟四分為十五=分為十二同之百卅七得從百五步有百卅七分步之十五乘之田一畝

Strip 171
下有六分以一為六十∟半為卅∟三分為廿∟四分為十五=分為十二∟六分為十同之百卌七得從九十七步有百卌七分

Strip 173
步 [之] (五)[106] 百卌一乘之田一畝[107] (long gap to end of strip)

Strip 172
下有七分以一為四百廿半為二百一十∟三分為百卌∟四分為百五=分為八十四∟六分為丰七分為六十同之千八十九得從九十二

(Strip 173: moved to after Strip 171)

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104. These two characters are written below the upper node, but appear to be a title.
105. This last character is written below the lower node, which enables the scribe to get the whole text into the strip.
106. This graphical plausible emendation makes this strip into the missing conclusion of strip 171.
107. Zhāngjiāshān (2001) suggests a long insertion into this strip to make it follow on from strip 172. But it seems easier to read the second character (which is damaged) as 之 rather than 五, which would make this the missing ending of strip 171. This would of course leave us with a missing ending for the text on strip 172.

Strip 182
步有千八十九分步之六百一十二乘之田一畝 (long gap to end of strip)[108]

Strip 174
下有八分以一為八百卌半為四百廿三分為二百八十∟四分為二百一十∟五分為百六十八∟六分為百卌∟七分為百廿∟八分為百五同之 [二千] (于)[109]

Strip 175
二百 [八] (口)[110] 十三以為法得從八十八步有二千二百八十三分步之六百九十六乘之田一畝 (10 character gap to end of strip)

Strip 176
下有九分以一為二千五百廿∟半為千二百六十∟三分為八百 [卌] (卅)[111] ∟四分為六百卅∟五分為五百四∟六分為四百廿∟七分為三百六十∟八分為三百一十五

Strip 177
∟九分為二百八十同之七千一百廿九以為法 (long clean empty section in middle of strip here) 得從八十四步有七千一百廿九分步之

Strip 178
五千 [九] (七)[112] 百六十四乘之成田一畝 (long gap to end of strip)

Strip 179
下有十分以一為二千五百廿∟半為千二百六十∟三分為八百卌∟四分為六百卅∟五分為五百四∟六分為四百廿∟七分為三百六十∟八分

108. This strip is clearly the missing end of the previous strip, and should be moved to this position.
109. The strip is not easily legible here; the emendation is necessary for the calculation to work.
110. The character is illegible, but 八 is required by the calculation.
111. The strip is hard to read at this point; this emendation is both graphically plausible and required to make the calculation work.
112. The strip is hard to read at this point; this emendation is both graphically plausible and required to make the calculation work.

Strip 180
為三百一十五∟九分為二百八十∟十分為二百五十二同之七千三百八十一以為法得從八十一步有七千三百八十一分步

Strip 181[113]
之六千 [九] (八) 百 [卅九] (□□□) 乘之成田一畝

(Strip 182: moved to after Strip 172)

Strip 183
大廣
廣七步卌九分步之□□□□□□□□□□□□□□□□□為□六十四步有三百卌三分步之二百卄三大廣朮曰直廣從而各以其分母

Strip 184
乘其上全步令分子從之令相乘也為實∟有各令分母相乘為法∟如法得一步不盈步以法命之 (2 character gap to end of strip)

(In Zhangjiashan (2001) strips 185 and 186 are placed here. I have however preferred to take them as following strip 140, on the grounds that they belong best in the group of strips which, like these, deal with applications of the Rule of False Position)

Strip 187
里田
里田朮曰里乘里=也廣從各一里即直一因而三之有三五之即為田三頃卄五畝∟其廣從不等者先以里相乘已

Strip 188
乃因而三之有三五之乃成今有廣二百廿里從三百五十里為田廿八萬八千七百五十頃直提封以此為之

Strip 189
一曰里而乘里=也壹三而三五之即頃畝數也∟有曰里乘里=也以里之下即予廿五因而三之亦其頃

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113. This strip is a mere whisp, on which little can be made out in the photograph. I adopt the emendations made in Zhāngjiāshān (2001), which are required to make the calculation work.

Strip 190
畝數也曰廣一里從一里為田三頃卄五畝 (long gap to end of strip)

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