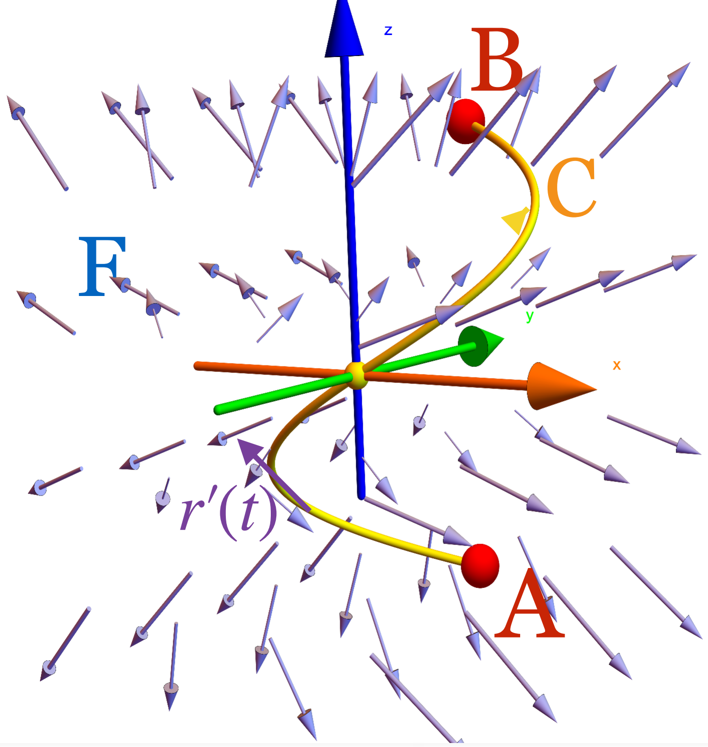
$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)dt$

Line Integral Theorem



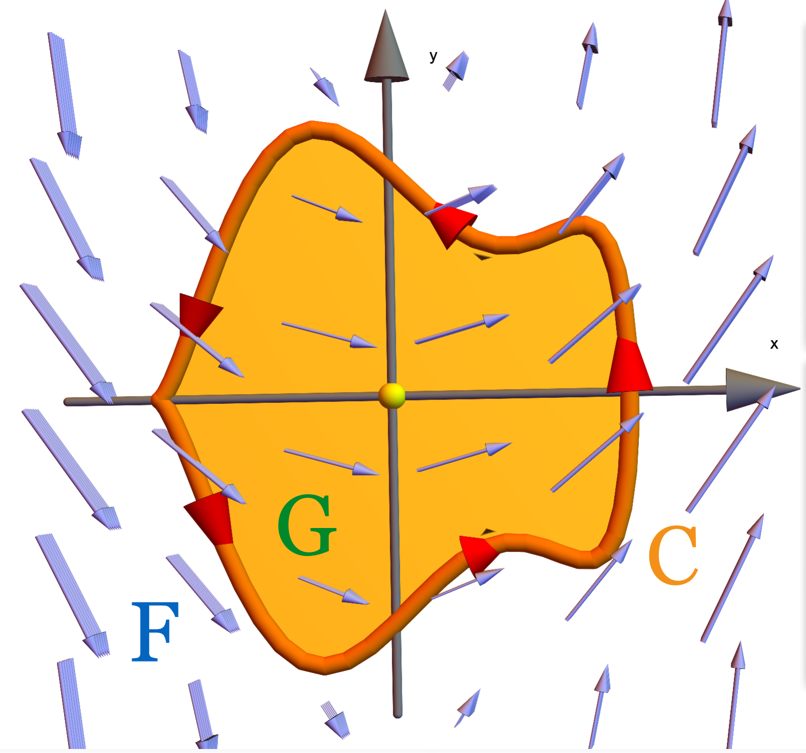
gradient fields are path independent

Line integral of gradient field over closed curve is zero

$$\int_C \nabla f \cdot dr = f(B) - f(A)$$

$\nabla f = [f_x, f_y, f_z]$, $\text{curl}[P,Q] = Q_x - P_y$

Green's Theorem



if curl(F)=0 then F is gradient field

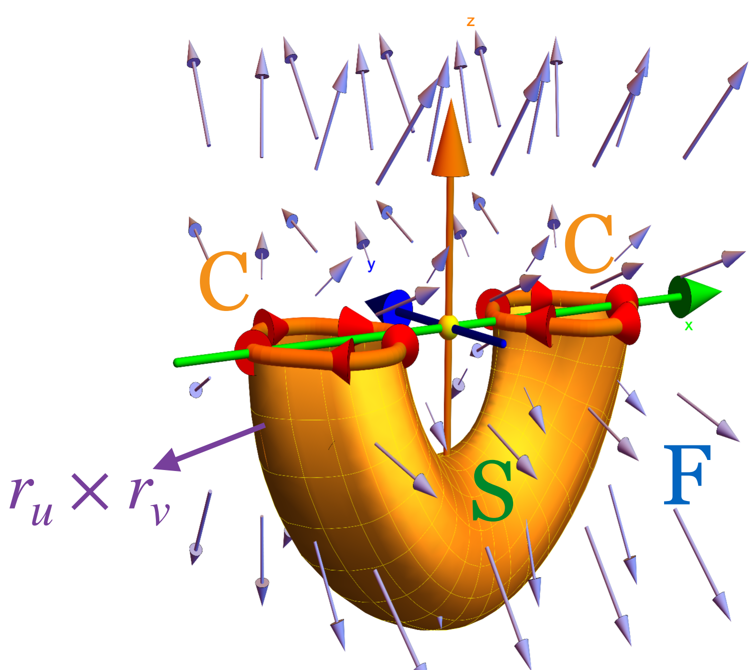
can be used to compute area

$$\iint_G \text{curl}(F)\ dA = \int_C F \cdot dr$$

Use to compute line integrals

$\iint_S F \cdot dS = \iint_G F(r(u,v)) \cdot r_u \times r_v dudv$

Stokes Theorem



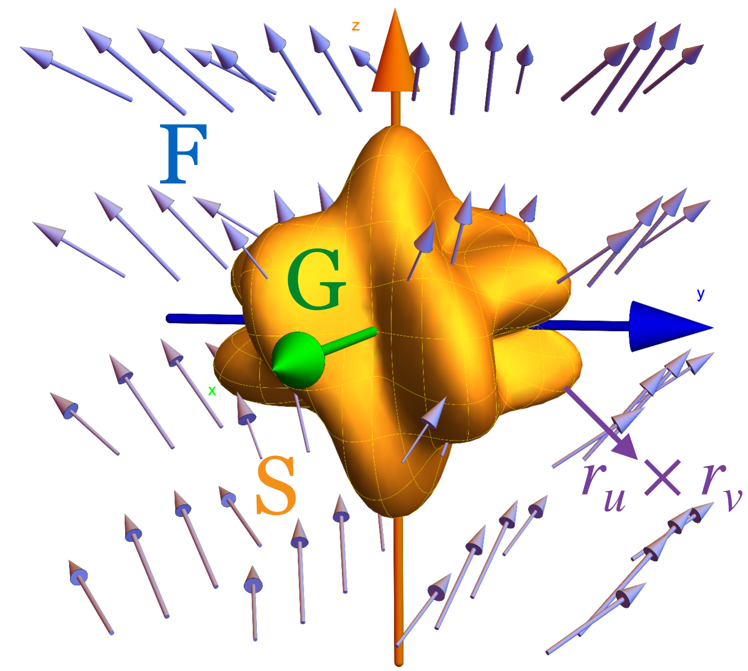
if curl(F)=0 then F is gradient field

flux integral of curl field over closed surface is zero

$$\iint_S \text{curl}(F) \cdot dS = \int_C F \cdot dr$$

$\text{curl}[P,Q,R] = [R_y - Q_z, P_z - R_x, Q_x - P_y]$, $\text{div}([P,Q,R]) = P_x + Q_y + R_z$

Divergence Theorem



can be used to compute volume

Flux integral of curl field over closed surface is zero

$$\iiint_G \text{div}(F)\ dV = \iint_S F \cdot dS$$

Use to compute flux integrals

$\text{curl}(\text{grad}(f)) = \text{div}(\text{curl}(F)) = 0$

Math 22a, Fall 2018, Oliver Knill, Harvard College