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Name:

LINEAR ALGEBRA AND VECTOR ANALYSIS

MATH 22A

Total:

Unit 13: First Hourly Practice

PROBLEMS

Problem 13P.1 (10 points):

The **Fibonacci numbers** are defined recursively as follows: start with $F_0 = 0, F_1 = 1$ then define $F_{n+1} = F_n + F_{n-1}$, so that $F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5$ etc. Prove that

$$F_0 + F_1 + \cdots + F_n = F_{n+2} - 1$$

for every positive integer n .

Problem 13P.2 (10 points):

Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

- a) (4 points) Compute AB and $\text{rref}(AB)$.
- b) (4 points) Now row reduce both A and B and form $\text{rref}(A)\text{rref}(B)$.
- c) (2 points) Is the statement $\text{rref}(AB) = \text{rref}(A)\text{rref}(B)$ true for all A, B ?

Problem 13P.3 (10 points):

- a) (2 points) Parametrize the line through $(1, 1, 1)$ and $(4, 3, 1)$ in \mathbb{R}^3 .
- b) (2 points) Parametrize the ellipse $x^2/16 + y^2/25 = 1$ in \mathbb{R}^2 .
- c) (2 points) Parametrize the graph $y = x^5 + x$ in \mathbb{R}^2 .
- d) (2 points) Parametrize the circle $x^2 + (y - 2)^2 = 1, z = 4$ in \mathbb{R}^3 .
- e) (2 points) Parametrize the line $x = y = z$ in \mathbb{R}^3 .

Problem 13P.4 (10 points):

Find the arc length of the curve

$$r(t) = [t \cos(t^2), t \sin(t^2), t^2]$$

for $0 \leq t \leq 2$.

Problem 13P.5 (10 points):

- a) (2 points) What is the Heine-Cantor theorem?
- b) (2 points) Formulate the triangle inequality.
- c) (2 points) What is the Al Kashi identity?
- d) (2 points) Give the name of a nowhere differentiable function.
- e) (2 points) Is it true that a continuous curve $r(t)$ has a finite arc length?

Problem 13P.6 (10 points):

- a) (2 points) Find $(3 + i)(4 + 2i)$
- b) (2 points) What is $e^{i3\pi/4}$?
- c) (2 points) Convert from cylindrical $(r, \theta, z) = (2, \pi/2, 1)$ to Cartesian.
- d) (2 points) What are the spherical coordinates of $(1, \sqrt{3}, 2)$?
- e) (2 points) What surface is in spherical coordinates given as $\rho \sin(\phi) = 1$?

Problem 13P.7 (10 points):

- a) (5 points) You are given $r'''(t) = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ and $r(0) = (7, 8, 9)$ and $r'(0) = (1, 0, 0)$ and $r''(0) = (0, 1, 0)$. Find $r(1)$.
- b) (5 points) What is the curvature of $r(t) = [t, t + t^2, t + t^2 + t^3]$ at $t = 0$?

Problem 13P.8 (10 points):

- a) (5 points) Find a parametrization $r(u, v)$ of the cylinder $x^2 + z^2 = 9$.
- b) (5 points) Find $r(u, v)$ for the paraboloid $y^2 + 3z^2 = x$.

Problem 13P.9 (10 points):

- Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$.
- a) (2 points) The image of A is a plane. By using the cross product, write it as $ax + by + cz = d$.
 - b) (2 points) What is the first fundamental form $g = A^T A$?
 - c) (2 points) From a) you have $[a, b, c]^T = v \times w$. Find $\sqrt{a^2 + b^2 + c^2}$.
 - d) (2 points) Find the distortion factor $\|A\| = \sqrt{\det(A^T A)}$ of A .
 - e) (2 points) What theorem was involved to see $\|A\| = |v \times w|$?

Problem 13P.10 (10 points):

- a) (5 points) What is the Jacobian matrix df of the map

$$f(x, y, z) = [x^2 + y^2 + z^2, x + y, -x^2]^T ?$$
- b) (5 points) Find the distortion factor $\det(df)$.