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Name:

LINEAR ALGEBRA AND VECTOR ANALYSIS

MATH 22A

Total:

Unit 13: Hourly 1 (III practice)

PROBLEMS

Problem 13R.1 (10 points):

Prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6$ for every positive integer n .

Problem 13R.2 (10 points):

a) (6 points) Row reduce $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

b) (4 points) Find A^2 .

Problem 13R.3 (10 points):

- a) (2 points) Parametrize the curve $y(x^2 + 1) = 1$.
- b) (2 points) Parametrize the spiral $r = \theta^2$.
- c) (2 points) Parametrize the graph $x = e^y = \exp(y)$
- d) (2 points) Parametrize the y -axis $x = 0$.
- e) (2 points) Parametrize the ellipse $(x - 1)^2/9 + (y - 4)^2/4 = 1$.



Problem 13R.4 (10 points):

Find the arc length of the curve

$$r(t) = [\sqrt{2} \log(t), \log(t)^2/2, \log(\log(t))]^T$$

for $e \leq t \leq e^2$.



Problem 13R.5 (10 points):

- a) (2 points) If v and w are two non-zero vectors, what is $|v \times w|/|v \cdot w|$?
b) (2 points) Finish this: if f is uniformly continuous on $[a, b]$, if

$$|x - y| \leq \dots, \text{ then } |f(x) - f(y)| \leq M_n.$$

- c) (2 points) If $x \cdot Bx + Ax - b = 0$ is a quadratic manifold. What object is B , What object is A ? Be specific. In case of vectors, distinguish between row and column vectors for example.
d) (2 points) Who found the formula $e^{i\theta} = \cos(\theta) + i \sin(\theta)$?
e) (2 points) Is it true that if f is a function on $[a, b]$ with $f(a) < 0$ and $f(b) > 0$, then $f'(x) = 0$ at some point?



Problem 13R.6 (10 points):

- a) (2 points) What is $(3 + i)^3$?
b) (2 points) What is $(-1)^i$?
c) (2 points) What is i^4 ?
d) (2 points) What surface is in cylindrical coordinates given as $z - r^2 = 1$?
e) (2 points) What set of points satisfies $\phi = \pi$?



Problem 13R.7 (10 points):

a) (5 points) You are given $r''(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \\ t \end{bmatrix}$ and $r(0) = (0, 0, 0)$ and $r'(0) = (0, 0, 0)$. Find $r(2\pi)$.

b) (5 points) What is the curvature of $r(t) = [2 \sin(t), 3 \cos(t), 0]$ at $t = 0$?



Problem 13R.8 (10 points):

a) (5 points) Find the parametrization of the surface $x^2 - y^2 + z^2 = 1$.

b) (5 points) Find the parametrization of the surface $x^2/4 + y^2/9 = 1$.



Problem 13R.9 (10 points):

a) (5 points) Find the dot product between the two matrices

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}.$$

b) (5 points) Find the angle between these two matrices.



Problem 13R.10 (10 points):

a) (5 points) What is the Jacobian matrix of the coordinate change

$$f(x, y, z) = \begin{bmatrix} x^2 \\ y^2x + yx^2 \\ z^3 \end{bmatrix}$$

b) (5 points) What is the distortion factor of f ?

