How to Solve It

a new aspect of mathematical method

With a new foreword by John H. Conway

G. POLYA

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third. *Carry out* your plan.

Carrying out your plan of the solution, *check each step.* Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth. *Examine* the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

How to Solve It

A New Aspect of Mathematical Method

G. Polya

With a new foreword by John H. Conway

A perennial bestseller by eminent mathematician G. Polya, *How to Solve It* will show anyone in any field how to think straight.

In lucid and appealing prose, Polya reveals how the mathematical method of demonstrating a proof or finding an unknown can be of help in attacking any problem that can be "reasoned" out—from building a bridge to winning a game of anagrams. Generations of readers have relished Polya's deft—indeed, brilliant—instructions on stripping away irrelevancies and going straight to the heart of the problem.

From reviews of the original edition:

"Every prospective teacher should read it. In particular, graduate students will find it invaluable. The traditional mathematics professor who reads a paper before one of the Mathematical Societies might also learn something from the book: 'He writes *a*, he says *b*, he means *c*; but it should be *d*.'"
—E. T. Bell, *Mathematical Monthly*, December 1945

"[This] elementary textbook on heuristic reasoning, shows anew how keen its author is on questions of method and the formulation of methodological principles. Exposition and illustrative material are of a disarmingly elementary character, but very carefully thought out and selected."
—Herman Weyl, *Mathematical Review*, October 1948

"Any young person seeking a career in the sciences would do well to ponder this important contribution to the teacher's art."
—A. C. Schaeffer, *American Journal of Psychology*, April 1946

GEORGE POLYA (1887–1985) was one of the most influential mathematicians of the twentieth century. His basic research contributions span complex analysis, mathematical physics, probability theory, geometry, and combinatorics. He was a teacher par excellence who maintained a strong interest in pedagogical matters throughout his long career. Even after his retirement from Stanford University in 1953, he continued to lead an active mathematical life. He taught his final course, on combinatorics, at the age of ninety. JOHN H. CONWAY is the John von Neumann Distinguished Professor of Mathematics at Princeton University. He was awarded the London Mathematical Society's Polya Prize in 1987. Like Polya, he is interested in many branches of mathematics, and in particular, has invented a successor to Polya's notation for crystallographic groups.

PRINCETON SCIENCE LIBRARY
psl.princeton.edu

From its publication of Albert Einstein's *The Meaning of Relativity* in 1922, Princeton University Press has been committed to bringing the writings of leading scientists to a broad professional and general audience. The Princeton Science Library now makes the best of these works available in inexpensive paperback editions for a new generation of scientists and teachers, students and nonspecialists.

Cover design by Carmina Alvarez-Gaffin

ISBN 0-691-11966-X

9 780691 119663 90000