Lecture 1: Mathematical roots

In the same way as one has distinguished the **canons of rhetorics**: memory, invention, delivery, style, and arrangement, or combined the **trivium**: grammar, logic and rhetorics, with the **quadrivium** arithmetic, geometry, music, and astronomy, to get the seven **liberal arts and sciences**, one has also tried to **organize all mathematical activities**.

Historically, one has distinguished **eight ancient roots of mathematics**. These are 8 activities, each of which suggest a key area in mathematics:

counting and sorting	**arithmetic**
spacing and distancing	**geometry**
positioning and locating	**topology**
surveying and angulating	**trigonometry**
balancing and weighing	**statics**
moving and hitting	**dynamics**
guessing and judging	**probability**
collecting and ordering	**algorithms**

To morph these 8 roots to the 12 mathematical areas we cover in this class, we complemented the ancient roots by calculus, numerics and computer science, merge trigonometry with geometry, separate arithmetic into number theory, algebra and arithmetic and change statics to analysis.

Lets call this more modern adaptation the

12 modern roots of Mathematics:

counting and sorting	**arithmetic**
spacing and distancing	**geometry**
positioning and locating	**topology**
dividing and comparing	**number theory**
balancing and weighing	**analysis**
moving and hitting	**dynamics**
guessing and judging	**probability**
collecting and ordering	**algorithms**
slicing and stacking	**calculus**
operating and memorizing	**computer science**
optimizing and planning	**numerics**
manipulating and solving	**algebra**

While relating **mathematical areas** with **human activities** is useful, it can make more sense to pair these 12 major areas with one or two examples of topics which appear in this area. These 12 topics will be the 12 lectures of this course.

Arithmetics	numbers and number systems
Geometry	invariance, symmetries, measurement, maps
Number theory	Diophantine equations, factorizations
Algebra	algebraic and discrete structures
Calculus	limits, derivatives, integrals
Set Theory	set theory, foundations and formalisms
Probability	combinatorics, measure theory and statistics
Topology	polyhedra, topological spaces, manifolds
Analysis	extrema, estimates, variation, measure
Numerics	numerical schemes, codes, cryptology
Dynamics	differential equations, maps
Algorithms	computer science, artificial intelligence

Of course, like any classification, this division is rather arbitrary and also a matter of personal preferences. The **2010 AMS classification** for example distinguishes 63 areas of mathematics. In MSC 2010, many of the main areas are broken off into even finer pieces. Additionally, there are fields which relate with other areas of science, like economics, biology or physics:

00 General
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear/multi-linear algebra; matrix theory
16 Associative rings and algebras
17 Non-associative rings and algebras
18 Category theory, homological algebra
19 K-theory
20 Group theory and generalizations

45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations, optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems

22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables, analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus

74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, math. programming
91 Game theory, Economics Social and Behavioral Sciences
92 Biology and other natural sciences
93 Systems theory and control
94 Information and communication, circuits
97 Mathematics education

What are hot spots in mathematics today? Michael Athiyah identified in the year 2000 the following **6 hot spots** in the development of mathematics:

local	and	global
low	and	high dimension
commutative	and	non-commutative
linear	and	nonlinear
geometry	and	algebra
physics	and	mathematics

Also this choice is of course highly personal. One can easily add an other 12 of such **polarizing** quantities which help to distinguish or parametrize different parts of mathematical areas, especially the ambivalent pairs which are "hot":

regularity	and	randomness
integrable	and	non-integrable
invariants	and	perturbations
experimental	and	deductive
polynomial	and	exponential
applied	and	abstract

discrete	and	continuous
existence	and	construction
finite dim	and	infinite dimensional
topological	and	differential geometric
practical	and	theoretical
axiomatic	and	case based

An other possibility to refine the fields of mathematics is to **combine** different of the 12 areas. Examples are **probabilistic number theory**, **algebraic geometry**, **numerical analysis**, **geometric number theory**, **numerical algebra**, **algebraic topology**, **geometric probability**, **algebraic number theory**, **dynamical probability = stochastic processes**. Almost every pair is an actual field. Finally, lets give a short answer to the question: What is Mathematics?

Mathematics is the science of structure.

The goal is to illustrate some of these structures from a historical point of view.