

## Methodology and Pedagogy

### History and Math

Both for teaching and learning, the difficulty of a mathematical subject is often reflected in its history. The struggles of research mathematicians who developed and formalized a topic have parallels with the struggle of students and teachers in the modern classroom. For example, when students are exposed to the concept of limits and series at first, they go through a similar process as the pioneers of calculus did when they developed the subject. It is a part of the mathematicians activity to find or invent new structure. This struggle goes on even today, where new flavors of calculus are developed and studied. In this course, we consider each week at a different subject and pinpoint in each topic a place, where something interesting happened.

Why look at the history at all? Wouldn't it be better to just teach the latest, most refined method? There is a well known pedagogical principle described in Eves book which helps to guide though some material: **"A student should be taught a subject pretty much in the order in which the subject developed over the ages."** Like any pedagogical rule, it should not be taken literally. But there are often parallels between learning obstacles today and research obstacles in the past, when a particular subject had been developed.

### The case method

We use the "case method" methodology and cover in every lecture a different part of mathematics. While the range of mathematical topics is broad, we work each week with a specific topic and particular example. The case method is somehow diametrically opposed to an encyclopedic approach. Whenever I have been taught with this method, from high school to graduate courses, I found the method effective, especially in the long term. As a student, I had been more involved and I can still remember the specific story and could use these "cases" as crystallization points for the entire subject. I have used the method already in a dynamical systems course. In a time when knowledge explodes fast and a plethora of possibilities are offered online, teaching requires both to be broad as well as care for details. The dilemma of combining these two can be achieved with a "short story approach". It is well established and was used especially at the Harvard business school, where "discussions focused on real-world situations" was considered a good way to prepare students. In our case, the "real word situations" are "historical highlights".

### Pedagogy

Besides the material, the pedagogy will play an important role.

While all of the lectures are suited for a high school level, we will discuss also pedagogy and think about it. Here are examples of points to discuss:

1. Topic choice: Are the illustrations helpful?
2. Lecture goals: How would this be implemented in class?
3. Worksheet clarity: Does it lead to a stimulating work?
4. Quiz difficulty: is the structure of the quiz adequate?
5. Alternatives: Are there similar topics in the area?
6. Sources: what books and articles are available.
7. Technology: are demonstrations or experiments possible?

Teaching mathematics using historical pointers has tradition. The process of introducing history in the teaching of mathematics is described by **Fulvia Furinghetti** as follows:

1. Know the sources.
2. Single out a topic suitable for the class.
3. Assess the needs of the class.
4. Plan the lecture: aims, context and resources
5. Realize the project
6. Evaluate the activity.

### The material

We try to condensed a specific moment in a specific and concrete lecture. Since we have 12 lectures besides the overview lecture, we pick 12 mathematical topics.

Topic	Transition
Arithmetics	From integers to complex numbers
Geometry	From planimetry to differential geometry
Number theory	From perfect numbers to Diophantine equations
Algebra	From symmetries to group theory
Calculus	From differences to differentials
Set theory	From paradoxes to undecidability
Probability	From combinatorics to stochastic processes
Topology	From curves and surfaces to invariants
Analysis	From calculus of variations to fractals
Cryptology	From codes to cryptography
Dynamics	From astronomy to differential equations
Computer science	From algorithms to artificial intelligence

### Structure of each lecture

A **presentation part** tell some stories and introduce the material. In a discussion, will talk about the material as well as about how the presentation was done and how to improve it. Participants of the course will be able to take home a revised version of the presentation.

We work in groups on a concrete **worksheet** with a problem material. After a presentation of the results by one of the groups, we discuss the worksheet and how to improve it. Participants will be able to take home a source of the worksheet for their own use.

In a final **quiz part**, we work in groups on a short quiz testing the material seen in the presentation. We will grade the results discuss the effectiveness of the test and how to improve it. Again, participants will also get a revised source of the quiz.