

## Lecture 2: irrational numbers

### Objective

**Theorem:**  $\sqrt{2}$  is not rational.  
 Proof:  $\sqrt{2} = p/q$  implies  $2 = p^2/q^2$  or  $2q^2 = p^2$ . If we make a prime factorization, then on the left hand side contains an odd number of factors 2, while the right hand side contains an even number of factors 2. This is not possible.

- a) Show that  $\sqrt{17}$  is not rational.
- b)  $\log_{10}(3)$  is irrational.

**Theorem:**  $\log_{10}(3)$  is irrational.

Proof.  $\log_{10}(3) = p/q$  implies  $3 = 10^{p/q}$  or  $3^q = 10^p$ . Which is not possible. The right hand side is not divisible by 3 for example.

## Lecture 2: number systems

### 1. Babylonian cuneiforms

The Babylonians already contemplated the square root of 2. We have seen in the presentation the Clay tablet YBC 7289. The most important document is "Plimpton 322", a Clay tablet from 1800 BC:

Cuneiform	Transliteration	Decimal value
𒌦𒍪	1, 15	75
𒌦𒍪𒌦𒍪	24, 13, 40	87,220

Look at the following example:

How would you write the number 1000 in this system? In 4000 BC in Mesopotamia region, cuneiform were imprinted on a wet clay tablets. I brought some clay. Lets write some Clay tablets.

### 2. The Greek system

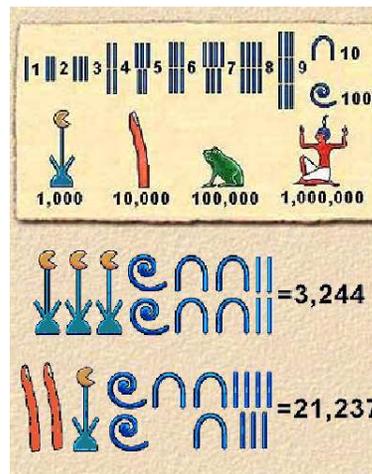
The Greeks used their alphabet with 24 letters with additional semitic letters (digamma, vau, goph and koppa, sampi) to represent numbers. A comma in front of a letter would mean it would be multiplied by 1000. A dot would mean that the number in front would be multiplied by 10'000.

Cuneiform	Transliteration	Decimal value
𒌦𒍪	1, 15	75
𒌦𒍪𒌦𒍪	24, 13, 40	87,220

, ατλα., εσιδ = 13315214

How would you write the number 40432 in this system?

### 3. The Egyptian system



The Egyptians had a similar system as the romans but with fewer symbols.

Problem: The following stone carving was found at Karnak. What number is it?

ϣ

### 4. The Roman system

The letters *I, V, X, L, C, D, M* were of Etruscic origin.

The subtractive principle like  $9 = IX, 90 = XC$  were hardly used by the romans. They would write *VIIII, LIIII* instead.

How would you write 129 in the roman system?

### 5. The Egyptian fractions

$$\overline{\text{III}} = \frac{1}{3}$$

$$\overline{\text{II}} = \frac{1}{2} \quad \overline{\text{I}} = \frac{2}{3} \quad \overline{\text{II}} = \frac{3}{4}$$

$$\overline{\text{IIII}} = \frac{1}{331}$$

1) Write 14'527 in the 60 system