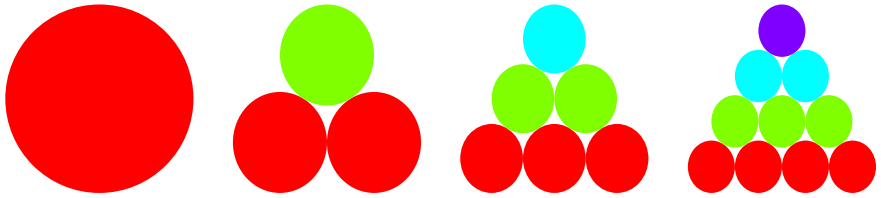
Lecture 6: Worksheets

We stack disks onto each other building n layers and count the number of discs. The number sequence we get are called **triangular numbers**.

1	3	6	10	15	21	36	45	...



n=1 n=2 n=3 n=4

This sequence defines a **function** on the natural numbers. For example, $f(4) = 10$.

1 Can you find $f(200)$? The task to find this number was given to Carl Friedrich Gauss in elementary school. The 7 year old came up quickly with an answer. How?

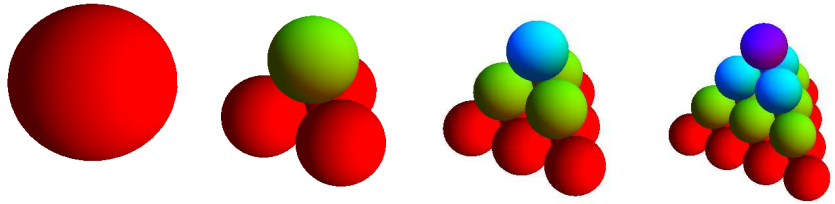
Carl-Friedrich Gauss, 1777-1855

Tetrahedral numbers

We stack spheres onto each other building n layers and count the number of spheres. The number sequence we get are called **tetrahedral numbers**.

1	4	10	20	35	56	84	120	...

Also this sequence defines a **function**. For example, $g(3) = 10$. But what is $g(100)$? Can we find a formula for $g(n)$?



n=1 n=2 n=3 n=4

2 Verify that $g(n) = n(n+1)(n+2)/6$, satisfies $Dg(n) = g(n) - g(n-1) = n(n+1)/2$.

3 **Problem:** Given the sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ which satisfies the rule $f(x) = f(x-1) + f(x-2)$. It defines a function on the positive integers. For example, $f(6) = 8$. What is the function $g = Df$, if we assume $f(0) = 0$?

4 **Problem:** Take the same function f given by the sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ but now compute the function $h(n) = Sf(n)$ obtained by summing the first n numbers up. It gives the sequence $1, 2, 4, 7, 12, 20, 33, \ldots$. What sequence is that?

5 **Problem:** Find the next term in the sequence

2 6 12 20 30 42 56 72 90 110 132 .

6 Find the next term in the sequence $3, 12, 33, 72, 135, 228, 357, 528, 747, 1$
To do so, compute successive derivatives $g = Df$ of f, then $h = Dg$ until you see a pattern.