Lecture 9: Topology of Alphabet

1. Objective

Topology identifies objects which can be morphed into each other. We want to explore here, how many topological types the letters in our alphabet have.

2. Warm-up with digits 0-9

1) The numbers $0, 4, 6, 9$ are topologically equivalent.

0 4 6 9

2) The numbers $1, 2, 3, 5, 7$ are topologically equivalent.

1 2 3 5 7

3) The number 8 is not topologically equivalent to any other digit.

8

3. Now the alphabet

Classify the alphabet according to topological equivalence.

ABCDE

FGHIJ

KLMNO

PQRST

UVW

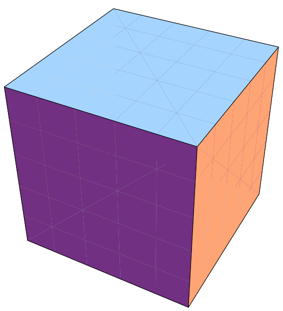
XYZ

Lecture 9: The Euler Characteristic

1. Objective

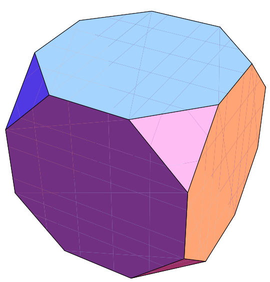
We meet here some regular and semi-regular polyhedra.

2. Euler Characteristic



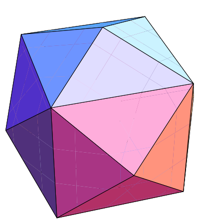
1) Compute the Euler Characteristic $V - E + F$ for the cube

Vertices V	Edges E	Faces F



2) Cut the corners of the cube. It will produce new faces and produce octagonal faces from the old faces. Count the Euler characteristic of the new object which is a semi-regular polyhedron.

Vertices V	Edges E	Faces F



3) Start again with the cube, but now cut each of the faces into 4 faces by drawing the diagonals in the squares. If the midpoints are lifted up a bit so that all triangles become equilateral, the new object is called a **stellation** of the cube. It is an other semi-regular polyhedron.

Vertices V	Edges E	Faces F

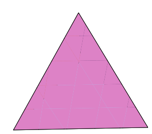
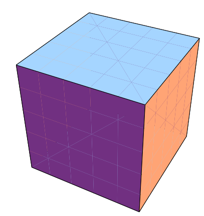
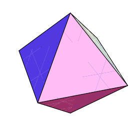
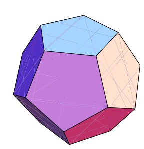
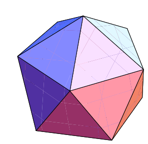
3. Duality

If we take a polyhedron and replace every face with a vertex in the middle of the face and every vertex with a face. We obtain an other polyhedron.

4) What is the dual of the cube?

5) What is the dual of the octahedron?

6) What is the dual of the tetrahedron?

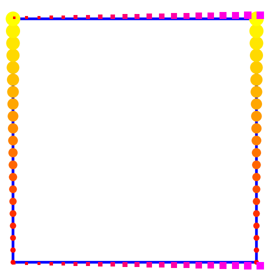
Lecture 9: Topology of Surfaces

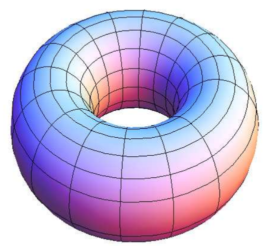
1. Objective

By identifying sides of a square we obtain models of compact surfaces: the **sphere**, the **torus**, the **projective plane** and the **Klein bottle**. We want to explore here the topology of these spaces, especially the simply conntectedness: can one pull any closed rope in this space to a point? Only the sphere is simply connected.

2. The torus

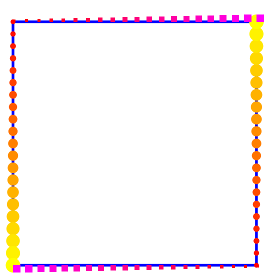
1) Draw some curves on the torus, which can not be pulled together to a point.

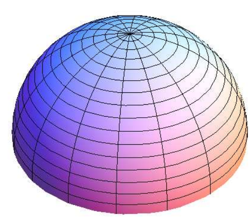




2. The projective plane

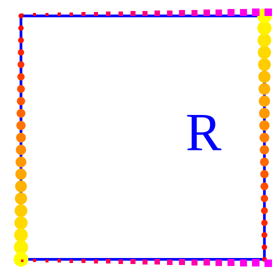
2) Draw a curve on the projective plane, which can not be pulled together to a point.

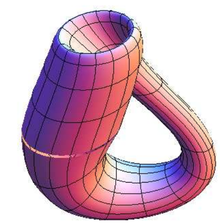




3. The Klein bottle

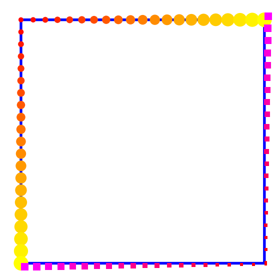
3) Move a letter R around on the Klein bottle, what happens with the letter as it moves over the boundary to the right and appears to the left?

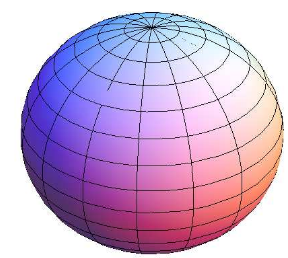




4. The sphere

4) Draw a curve on the sphere. Visualize that you can pull it together to a point.





5. Euler characteristic

If you have more time: By triangulating the space, we are also able to compute the Euler characteristic of these spaces. The Euler characteristic of the sphere is 2, the Euler characteristic of the torus is 0, the Euler characteristic of the projective plane is 1, the Euler characteristic of the Klein bottle is 0. Can you show this in the examples?