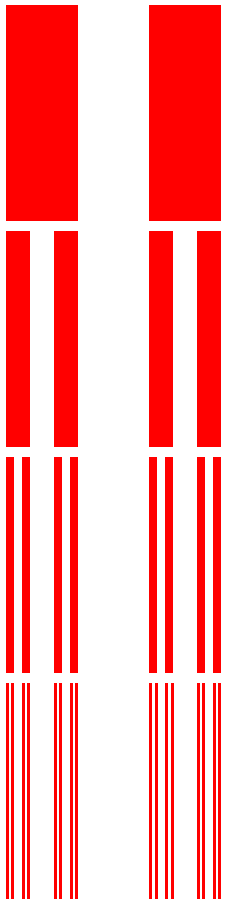
Lecture 10: Analysis

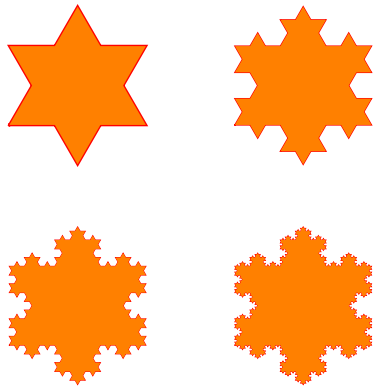
The Cantor Set



Analysis is a huge field. Our main theme for this lecture is the notion of a **fractal**. A fractal set is an object which has fractional dimension. It turns out that the subject of fractals enters many parts of analysis: spectral theory, harmonic analysis, calculus of variations, functional analysis/ But because these fields need some time to learn and explain, the subject of fractals looks like a nice entry point. The story becomes so very pictorial. While most presentations were more than 1 Gig, the presentation of today is a 4 Gigabyte file!

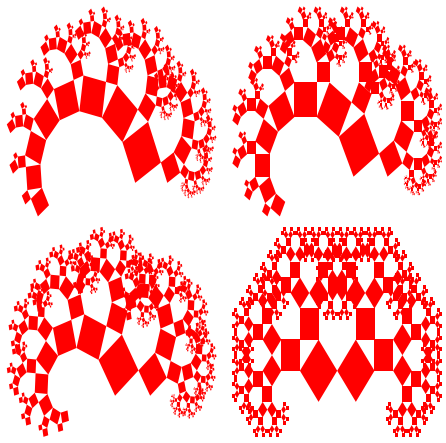
The prototype of a fractal is the **Cantor set** which was discovered in 1875. by **Henry Smith** Start with the unit interval. Cut the middle third, then cut the middle third from both parts then the middle parts of the four parts etc. What is left in the end is the Cantor set. We will see that its dimension is $\log(2)/\log(3)$.

The Koch Curve



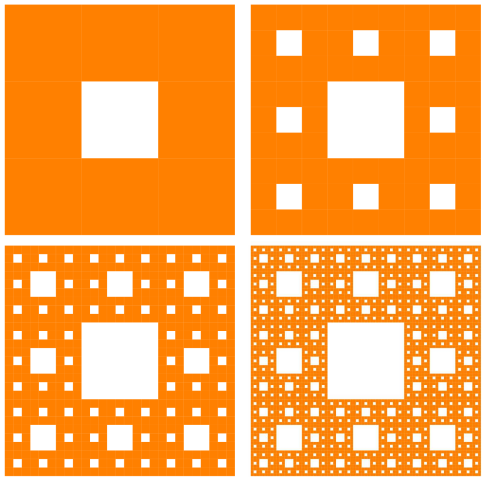
The **Koch snowflake** is an example of a fractal, where the dimension is between 1 and 2. It was first described by the Swedish mathematician Helge von Koch (1870-1924). The Koch curve was described by him in 1904. It is a simple model for a **snowflake**.

The Tree of Pythagoras



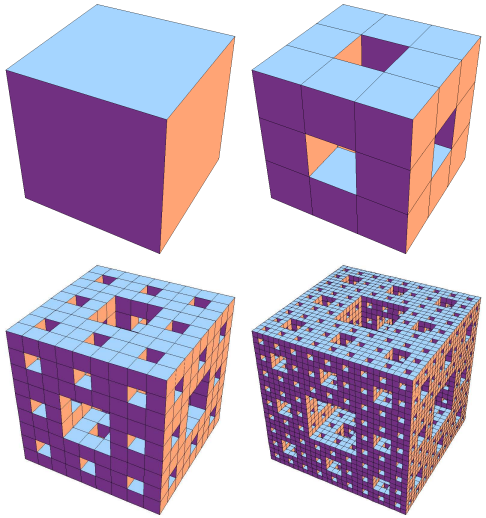
The **tree of Pythagoras** is an example of a fractal, where the dimension is between 1 and 2. It was first described by the Swedish mathematician **Helge von Koch** (1870-1924). The Koch curve was described by him in 1904. It comes close to actual **trees**. It inspired antenna designs.

The Sierpinski Carpet



The **Sierpinski carpet** is a fractal in the plane. Its dimension is $\log(8)/\log(2)$. It was described by **Waclav Sierpinski** in 1916.

The Menger Sponge



The **Menger sponge** is a fractal in space. Its dimension is between 2 and 3. It was first described by Karl Menger (1902-1985). Its dimension is $\log(20)/\log(3)$ which is about 2.7.

The Hofstadter Butterfly

The **Hofstadter butterfly** is an example of a fractal which appears in spectral theory. It was first described in 1976 and was popularized in Hofstadters book Goedel-Escher-Bach.