Lecture 6: Worksheets

We stack disks onto each other building n layers and count the number of discs. The number sequence we get are called **triangular numbers**.

$$1 \quad 3 \quad 6 \quad 10 \quad 15 \quad 21 \quad 36 \quad 45 \quad ...$$

This sequence defines a **function** on the natural numbers. For example, $f(4) = 10$.

1   Can you find $f(200)$? The task to find this number was given to Carl Friedrich Gauss in elementary school. The 7 year old came up quickly with an answer. How?

Carl-Friedrich Gauss, 1777-1855

Tetrahedral numbers

We stack spheres onto each other building n layers and count the number of spheres. The number sequence we get are called **tetrahedral numbers**.

$$1 \quad 4 \quad 10 \quad 20 \quad 35 \quad 56 \quad 84 \quad 120 \quad ...$$

Also this sequence defines a **function**. For example, $g(3) = 10$. But what is $g(100)$? Can we find a formula for $g(n)$?

2   Verify that $g(n) = n(n+1)(n+2)/6$, satisfies $Dg(n) = g(n) - g(n-1) = n(n+1)/2$.

3   **Problem:** Given the sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ which satisfies the rule $f(x) = f(x-1) + f(x-2)$. It defines a function on the positive integers. For example, $f(6) = 8$. What is the function $g = Df$, if we assume $f(0) = 0$?
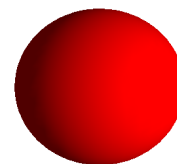
4   **Problem:** Take the same function f given by the sequence $1, 1, 2, 3, 5, 8, 13, 21, ...$ but now compute the function $h(n) = Sf(n)$ obtained by summing the first n numbers up. It gives the sequence $1, 2, 4, 7, 12, 20, 33, ...$ What sequence is that?

5   **Problem:** Find the next term in the sequence

$$2 \quad 6 \quad 12 \quad 20 \quad 30 \quad 42 \quad 56 \quad 72 \quad 90 \quad 110 \quad 132 \ .$$
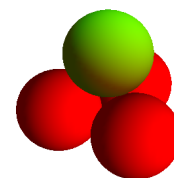
6   **Problem:** Find the next term in the sequence

$$3, 12, 33, 72, 135, 228, 357, 528, 747, 1020, 1353... \ .$$

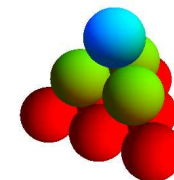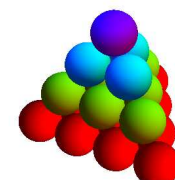To do so, compute successive derivatives $g = Df$ of f, then $h = Dg$ until you see a pattern.

n=1   n=2   n=3   n=4