Lecture 2: irrational numbers

Objective

Theorem: $\sqrt{3}$ is not rational.
Proof: $\sqrt{3} = p/q$ implies $3 = p^2/q^2$ or $3q^2 = p^2$. If we make a prime factorization, then on the left hand side contains an odd number of factors 3, while the right hand side contains an even number of factors 3. This is not possible.

Theorem: $\log_{10}(3)$ is irrational.

Proof. $\log_{10})(3) = p/q$ implies $3 = 10^{p/q}$ or $3^q = 10^p$. Which is not possible. The right hand side is not divisible by 3 for example.

a) Show that $\sqrt{17}$ is not rational.
b) $\log_{10}(3)$ is irrational.

Lecture 2: number systems

1. Babylonian cuneiforms

The Babylonians already contemplated the square root of 2. We have seen in the presentation the Clay tablet YBC 7289. The most important document is "Plimpton 322", a Clay tablet from 1800 BC:

Look at the following example:

Cuneiform	Transliteration	Decimal value
⌁⌁	1, 15	75
⌁⌁	24, 13, 40	87,220

How would you write the number 1000 in this system? In 4000 BC in Mesopotamia region, cuneiform were imprinted on a wet clay tablets. I brought some clay. Lets write some Clay tablets (we can also use cream cheese).
Write 14'527 in the 60 system

2. The Mayan system

The Mayan system is based on 20. Here are the first 20 numbers. Note that the Mayans used 0

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

too and had a place valued system.   How would you write the number 401 in this system?

3. The Greek system

The Greeks used their alphabet with 24 letters with additional semitic letters (digamma, vau, qoph and koppa, sampi) to represent numbers. A comma in front of a letter would mean it would be multiplied by 1000. A dot would mean that the number in front would be multiplied by 10'000.

Cuneiform	Transliteration	Decimal value
⌁⌁	1, 15	75
⌁⌁	24, 13, 40	87,220

$$,\alpha\tau\lambda\alpha., \epsilon\sigma\iota\delta = 13315214$$

4. The Egyptian system
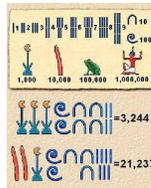
The Egyptians had a similar system as the romans but with fewer symbols.

5. The Roman system

The letters I, V, X, L, C, D, M were of Etruscic origin.

The subtractive principle like $9 = IX, 90 = XC$ were hardly used by the romans. They would write $VIIII, LIIII$ instead.

How would you write 129 in the roman system?

6. The Egyptian fractions

$$\overline{III} = \frac{1}{3}$$

$$\smile = \frac{1}{2} \quad \widehat{} = \frac{2}{3} \quad \widehat{II} = \frac{3}{4}$$

$$= \frac{1}{331}$$