Lecture 8: Probability theory

Probability theory started

Petersburg Paradox

Problem We throw dices until tail appears. If k times head came first, we get 2^k dollars. What is a fair fee to pay each time?

The paradox is that nobody would want to buy a 20 dollar entry fee. Mathematically, you win with probability 2^{-k} an amount of 2^k leading to an infinite expectation.
One solution is to look what happens if there is a limit K to the jackpot. The expected win is now about $\log_2(K)$.

Here is a related but more primitive problem. Why does this "Martingale strategy" not work?

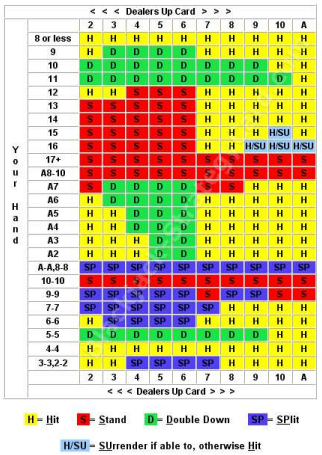
Problem We play roulette. Whenever we lose we double our input. We stop the first time we win.

Game 1: we enter a dollar and win. We stop playing.
Game 2: we enter a dollar and lose first. We bet 2 dollars. We lose again. We bet 4 dollars. Now we win. We have entered 7 dollars and won 8. We walk away with one dollar.

Playing Blackjack

Problem Is there an optimal strategy for blackjack?

There are many variations of the game. Depending on the game, there are tables telling in which situation to hit for a new card. Here is an example.



By looking at the history of the cards played (add the low and subtract the high cards), one can increase the odds.

Playing the Lottery

Problem What is the chance to hit 6 right in a lottery, where we chose 6 balls from 40?

$$\frac{6!34!}{40!} = 1/3838380$$

This is about 1 in 4 million.

There are variants of this. For Power Ball, we chose 5 from 49 and additionally one of 42 (thats the powerball). Are the odds better than for lotto?

$$\frac{5!44!}{49!42} = 1/80080128$$

The chance is one to 80 million, but there is also much more to win.