Lecture 2: Irrational numbers

Theorem: $\sqrt{3}$ is not rational.

Proof: $\sqrt{3} = p/q$ implies $3 = p^2/q^2$ or $3q^2 = p^2$. If we make a prime factorization, then on the left hand side contains an odd number of factors 3, while the right hand side contains an even number of factors 3. This is not possible.

Theorem: $\log_{10}(3)$ is irrational.

Proof. If $\log_{10}(3) = p/q$ then $3 = 10^{p/q}$ or $3^q = 10^p$. This is not possible because the right hand side is not divisible by 3, while the left hand side is.

Problem 1) Now its up to you:

a) Show that $\sqrt{17}$ is not rational.
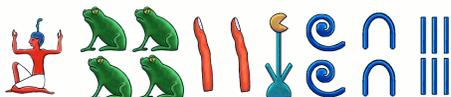b) Prove that $\log_{10}(3)$ is irrational.

Lecture 2: number systems

1. Egyptian system

Here are the symbols for 10, 100, 1000, 10000, 100000, 1000000:

Problem 2) Which integer does this **hieroglyph** represent? Remember: "The priest holds a frog in his finger. The flower fears a monkey bending a stick."

2. Babylonian cuneiforms

In 4000 BC in Mesopotamia region, cuneiform were imprinted on a wet clay tablets. An example is "Plimpton 322", a Clay tablet from 1800 BC. The Babylonians already contemplated the square root of 2. We have seen in the presentation the Clay tablet YBC 7289:

Problem 3. How would you write the number 1000 in the hexadecimal system? Take a chewing gum, and any hard object with a corner and use to write the number into the gum.
Problem 4. Clay tablet YBC 7289 is a especially interesting. We will look at the mystery in the lecture. Problem 4 is to figure out the secret of that tablet.

3. The Mayan system

The Mayan system is based on 20. Here are the first 20 numbers. Note that the Mayans used 0 too and had a place valued system.

Problem 5. How would you write the number 401 in the Mayan system?

4. The Roman Numerals

The letters I, V, X, L, C, D, M were of Etruscan origin and one speculates that the Etruscan numerals derive from tally marks or resemble hand signs.

The subtractive principle like $9 = IX, 90 = XC$ were hardly used by the Romans. They would write $VIIII, LIIII$ instead.

Problem 6. How would you write 129 using Roman system?