Lecture 1: Mathematical roots

Similarly, as one has distinguished the **canons of rhetorics**: memory, invention, delivery, style, and arrangement, or combined the **trivium**: grammar, logic and rhetorics, with the **quadrivium**: arithmetic, geometry, music, and astronomy, to obtain the seven **liberal arts and sciences**, one has tried to **organize all mathematical activities**.

Historically, one has distinguished **eight ancient roots of mathematics**. Each of these 8 activities in turn suggest a key area in mathematics:

counting and sorting	**arithmetic**
spacing and distancing	**geometry**
positioning and locating	**topology**
surveying and angulating	**trigonometry**
balancing and weighing	**statics**
moving and hitting	**dynamics**
guessing and judging	**probability**
collecting and ordering	**algorithms**

To morph these 8 roots to the 12 mathematical areas covered in this class, we complemented the ancient roots with calculus, numerics and computer science, merge trigonometry with geometry, separate arithmetic into number theory, algebra and arithmetic and turn statics into analysis.

Lets call this modern adaptation the

12 modern roots of Mathematics:

counting and sorting	**arithmetic**
spacing and distancing	**geometry**
positioning and locating	**topology**
dividing and comparing	**number theory**
balancing and weighing	**analysis**
moving and hitting	**dynamics**
guessing and judging	**probability**
collecting and ordering	**algorithms**
slicing and stacking	**calculus**
operating and memorizing	**computer science**
optimizing and planning	**numerics**
manipulating and solving	**algebra**

While relating **mathematical areas** with **human activities** is useful, it makes sense to select specific topics in each of this area. These 12 topics will be the 12 lectures of this course.

Arithmetic	numbers and number systems
Geometry	invariance, symmetries, measurement, maps
Number theory	Diophantine equations, factorizations
Algebra	algebraic and discrete structures
Calculus	limits, derivatives, integrals
Set Theory	set theory, foundations and formalisms
Probability	combinatorics, measure theory and statistics
Topology	polyhedra, topological spaces, manifolds
Analysis	extrema, estimates, variation, measure
Numerics	numerical schemes, codes, cryptology
Dynamics	differential equations, maps
Algorithms	computer science, artificial intelligence

Like any classification, this chosen division is rather arbitrary and a matter of personal preferences. The **2010 AMS classification** distinguishes 63 areas of mathematics. Many of the just defined main areas are broken off into even finer pieces. Additionally, there are fields which relate with other areas of science, like economics, biology or physics:

00 General	45 Integral equations
01 History and biography	46 Functional analysis
03 Mathematical logic and foundations	47 Operator theory
05 Combinatorics	49 Calculus of variations, optimization
06 Lattices, ordered algebraic structures	51 Geometry
08 General algebraic systems	52 Convex and discrete geometry
11 Number theory	53 Differential geometry
12 Field theory and polynomials	54 General topology
13 Commutative rings and algebras	55 Algebraic topology
14 Algebraic geometry	57 Manifolds and cell complexes
15 Linear/multi-linear algebra; matrix theory	58 Global analysis, analysis on manifolds
16 Associative rings and algebras	60 Probability theory and stochastic processes
17 Non-associative rings and algebras	62 Statistics
18 Category theory, homological algebra	65 Numerical analysis
19 K-theory	68 Computer science
20 Group theory and generalizations	70 Mechanics of particles and systems

22 Topological groups, Lie groups	74 Mechanics of deformable solids
26 Real functions	76 Fluid mechanics
28 Measure and integration	78 Optics, electromagnetic theory
30 Functions of a complex variable	80 Classical thermodynamics, heat transfer
31 Potential theory	81 Quantum theory
32 Several complex variables, analytic spaces	82 Statistical mechanics, structure of matter
33 Special functions	83 Relativity and gravitational theory
34 Ordinary differential equations	85 Astronomy and astrophysics
35 Partial differential equations	86 Geophysics
37 Dynamical systems and ergodic theory	90 Operations research, math. programming
39 Difference and functional equations	91 Game theory, Economics Social and Behavioral Sciences
40 Sequences, series, summability	92 Biology and other natural sciences
41 Approximations and expansions	93 Systems theory and control
42 Fourier analysis	94 Information and communication, circuits
43 Abstract harmonic analysis	97 Mathematics education
44 Integral transforms, operational calculus	

What are

fancy developments

in mathematics today? Michael Atiyah identified in the year 2000 the following **six hot spots**:

local	and	global
low	and	high dimension
commutative	and	non-commutative
linear	and	nonlinear
geometry	and	algebra
physics	and	mathematics

Also this choice is of course highly personal. One can easily add 12 other **polarizing** quantities which help to distinguish or parametrize different parts of mathematical areas, especially the ambivalent pairs which produce a captivating gradient:

regularity	and	randomness
integrable	and	non-integrable
invariants	and	perturbations
experimental	and	deductive
polynomial	and	exponential
applied	and	abstract

discrete	and	continuous
existence	and	construction
finite dim	and	infinite dimensional
topological	and	differential geometric
practical	and	theoretical
axiomatic	and	case based

An other possibility to refine the fields of mathematics is to **combine** different of the 12 areas. Examples are **probabilistic number theory**, **algebraic geometry**, **numerical analysis**, **geometric number theory**, **numerical algebra**, **algebraic topology**, **geometric probability**, **algebraic number theory**, **dynamical probability = stochastic processes**. Almost every pair is an actual field. Finally, lets give a short answer to the question: What is Mathematics?

Mathematics is the science of structure.

The goal is to illustrate some of these structures from a historical point of view.