A mathematical theorem

Objective

In this worksheet, we are exposed to a particular result in mathematics. Our task is to understand the theorem, place it into the landscape of mathematical fields and understand why all the conditions are necessary. If time permits, we learn why the theorem is true.

The theorem

We are given a quad-ruled paper, where one point is singled as the origin O. Assume we have a region G which has the property that the line segment between any two points in G is contained in G and for any point, also the point at O reflected point is in G. If the area of G is larger than 4, then there must exist a lattice point different from O inside G.

Questions

A) The theorem uses different concepts. There is a 1. A symmetry condition, 2. A convexity condition as well as 3. An area condition. In order to visualize them, we give for each condition an example of a region where the condition holds and for each condition an example where it does not hold.

Condition	Condition holds	Condition does not hold
Symmetry condition		
Convexity condition		
Area condition		

B) Draw some regions for which all conditions are true and which illustrate the theorem. Does the origin necessarily have to be in the region?
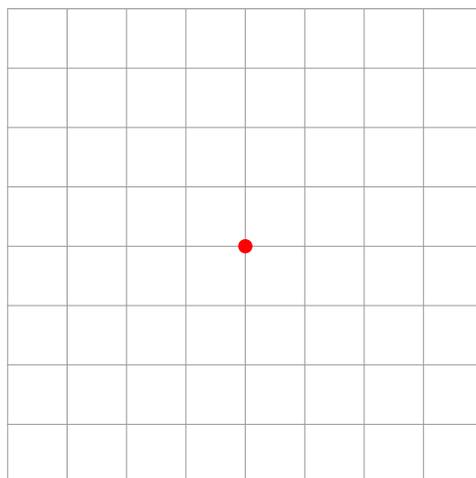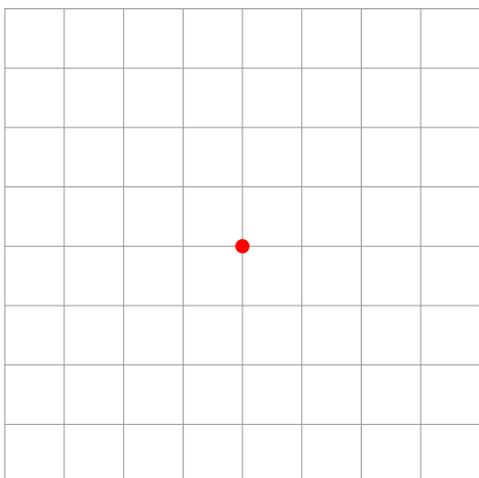
C) Which of the 12 mathematical topics are involved in this theorem?

Arithmetics	Geometry	Number theory
Algebra	Calculus	Logic
Probability	Topology	Analysis
Numerics	Dynamics	Algorithms

D) Lets find some examples which show that all three conditions are necessary. The area condition for example is necessary because we can draw a disc of radius 1 which is symmetric and convex but does not contain any other lattice point except O. Can you find a region of area larger than 4 which satisfies the convexity condition but not the symmetry condition and for which only the origin is a lattice point in the region? Can you find a region of area larger than 4 which satisfies the symmetry condition and not the convexity condition, for which only the origin is a lattice point in the region?