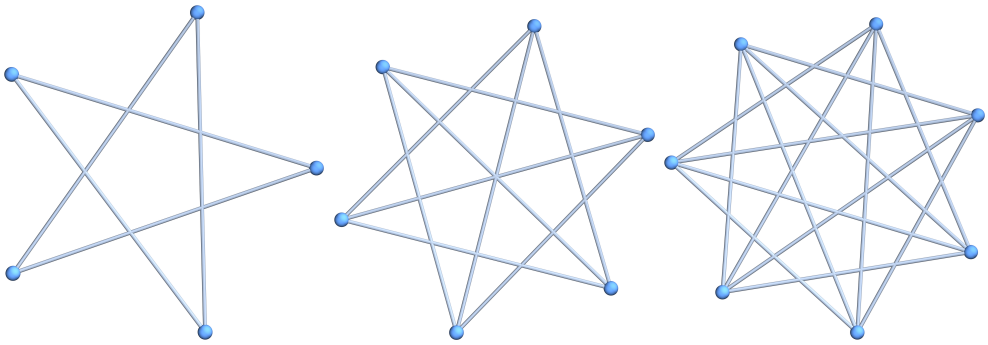
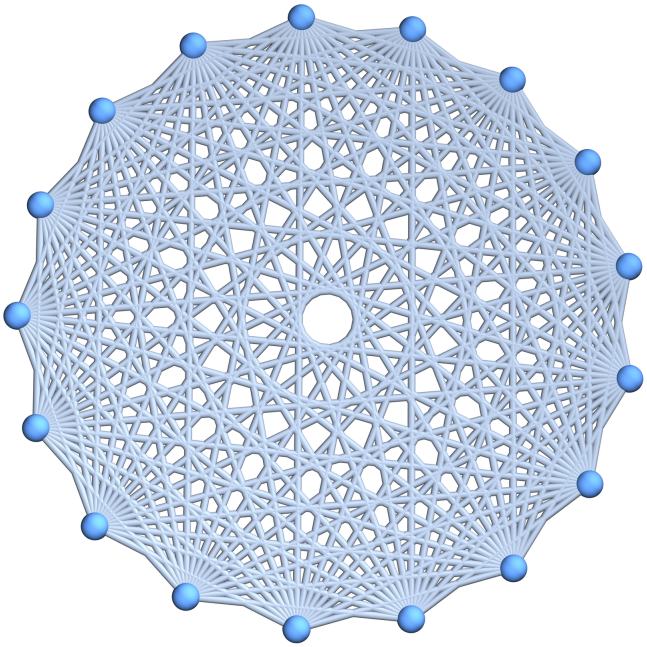
Lecture 1: Polygons and Polygrams

In the first week lecture we discuss a topic which has cultural historical and artistic connections. The topic illustrates an amazing **unity of mathematics**.

1 If you have n people sitting on a circular table who are already acquainted with their neighbors, how many additional connections need to be done? In other words, how many diagonals are there in a polygon with n sides? Lets look at small cases. How many diagonals are there in a pentagon, hexagon or heptagon?



2 Can you find a general formula? What would the answer be for a 17-gon?



3 We can also look at the number of triangles built by diagonals. There are 7 in the 7 gon. Counting is more difficult. For 17, there are already 442 triangles. Is there a way to get this without just counting? Here is a table. You filled out already the missing diagonal numbers in problem 1 and 2. Can you find a rule which allows to complete the table?

n-gon	vertices	diagonals	triangles	4-cliques	5-cliques	6-cliques
4	4	2				
5	5	...				
6	6	...	2			
7	7	...	7			
8	8	20	16	2		
9	9	27	30	9		
10	10	35	50	25	2	
11	11	44	77	55	11	
12	12	54	112	105	36	2
13	13					
14	14					
15	15					
16	16					
17	17					

5 The last questin is homework. Find out as much as you can about pentagons, hexagons, heptagons, octagons, nonagons. You will find relations with art, religion, mysticism, architecture, history or of course Mathematics. It does not have to be long.