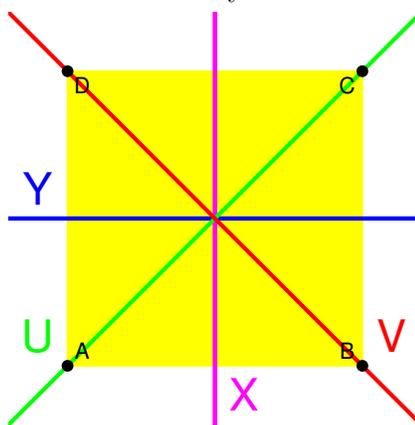


Lecture 5: Multiplication table and Sudoku

The symmetry group of a square

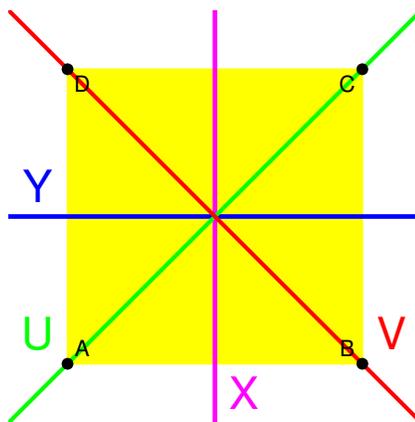
The symmetry group of the square has 8 elements. There are four rotations 0, 90, 180, 270, a reflection $X : (x, y) \rightarrow (-x, y)$ switching the sign of the x-coordinate, a reflection $Y : (x, y) \rightarrow (x, -y)$ switching the sign of the y-coordinate, a reflection $U : (x, y) \rightarrow (y, x)$ at the diagonal and a reflection $V : (x, y) \rightarrow (-y, -x)$ at the anti diagonal. In order to build the multiplication table, perform the operation on the left first then do the operation on the top. Make a picture or build a model so that you can do the operations. Here is the table without any entries:

*	0	90	180	270	X	Y	U	V
0								
90								
180								
270								
X								
Y								
U								
V								



1. Step. First fill in the rotation part. All rotations are counter clockwise. For example, rotating by 180 and then rotating by 270 gives you a rotation by $180 + 270 = 450 = 90$. You can use in this part that rotation in two dimensions is commutative and also use the **Sudoku rule** that all of the rotations have to appear in all of the rows and columns.

*	0	90	180	270	X	Y	U	V
0	0	90	180	270				
90	90	180	270	0				
180	180	270	0	90				
270	270	0	90	180				
X								
Y								
U								
V								

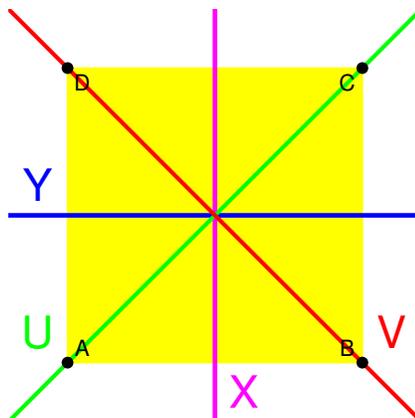


2. Step. Now fill in products of reflections. For example, $X * X = 0$, $X * Y = 180$, $X * V = 90$, $V * X = 270$. For example, in order to see $X * V = 90$, look what happens with the points A,B,C,D of the square. Do this for each multiplication. For example:

$$X * V : A \rightarrow B \rightarrow B, B \rightarrow A \rightarrow C, C \rightarrow D \rightarrow D, D \rightarrow C \rightarrow A .$$

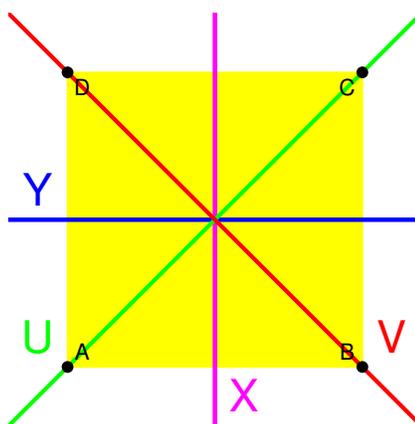
This means $ABCD$ goes to $BCDA$, which is a 90 degree rotation. You should reach something like

*	0	90	180	270	X	Y	U	V
0	0	90	180	270				
90	90	180	270	0				
180	180	270	0	90				
270	270	0	90	180				
X					0	180	270	
Y					180	0	90	
U						270	0	
V								0



2. Step continue. Sudoku! If you reach a stage like this, you can use Sudoku rules. Two reflections give a rotation and all rotations have to appear. You can now fill the rest without having to do the computations!

*	0	90	180	270	X	Y	U	V
0	0	90	180	270				
90	90	180	270	0				
180	180	270	0	90				
270	270	0	90	180				
X					0	180	270	90
Y					180	0	90	270
U					90	270	0	180
V					270	90	180	0

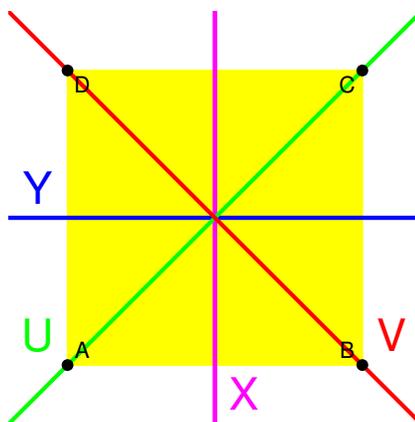


3. Step. Now combine rotations with reflections. That is the upper right part. For example $90 * X$. In order to see this look what happens with the square:

$$A \rightarrow B \rightarrow A, B \rightarrow C \rightarrow D, C \rightarrow D \rightarrow C, D \rightarrow A \rightarrow B$$

We see that $ABCD$ goes to $ADCB$ which means $90 * X = V$.

*	0	90	180	270	X	Y	U	V
0	0	90	180	270	X	Y	U	V
90	90	180	270	0	V			
180	180	270	0	90				
270	270	0	90	180				
X					0	180	270	90
Y					180	0	90	270
U					90	270	0	180
V					270	90	180	0



In a 4. Step, fill in the lower left cases like $X * 0 = X$. Continue filling and do not forget to use Sudoku rules when having enough info. In each row and each column, all the entries 0, 90, 180, 270, X, Y, U, V should appear. In the upper 4×4 block, only reflections should X, Y, U, V appear. These are mini Sudoku blocks. Reflect also a bit after having finished: You might notice for example for two reflections $X * Y = -Y * X$, where $-90 = 270$, $-180 = 180$.