The Mathematician Charles W. Cobb and the economist Paul H. Douglas (picture) found in 1928 empirically a formula $F(L,K) = bL^\alpha K^\beta$ giving the total production F of an economic system as a function of the amount of labor L and the capital investment K.

By fitting data, they got $b = 1.01, \alpha = 0.75, \beta = 0.25$. By rescaling the production unit we can get $b = 1$ and work with the formula:

$$F(L,K) = L^{3/4}K^{1/4}$$

Assume that the labor and capital investment are bound by the constraint $G(L,K) = L^{3/4} + K^{1/4} = 1$. Where is the production P maximal under this constraint?

$\nabla F(L,K) =$

$\nabla G(L,K) =$

Solve: $\nabla F(L,K) = \lambda \nabla G(L,K), G(L,K) = 1$: