This is part 1 (of 3) of the weekly homework. It is due Tuesday, July 1 at the beginning of class. The website will contain information on how to make use of the challenge problem. Don't turn them in together with the regular homework.

SUMMARY.

- $d((x,y,z),(u,v,w)) = \sqrt{(x-u)^2+(y-v)^2+(z-w)^2}$ **distance**.
- $(x-a)^2+(y-b)^2+(z-c)^2 = r^2$ equation of a **sphere** with **center** (a,b,c) and **radius** r.
- **completion of the square**:
  $x^2+ax = b \Leftrightarrow x^2+ax+a^2/4 = b+a^2/4 \Leftrightarrow (x+a/2)^2 = b+a^2/4$.

Homework Problems

1) (4 points) Describe and sketch the set of points (x,y,z) in $\mathbf{R}^3$ represented by

a) $x+y+z=1$
b) $x^2+z^2=4$
c) $y^2=1$
d) The distances of (x,y,z) to $(0,0,0)$ and $(0,0,6)$ is both equal to 5.

Solution:
a) A plane through the points $(1,0,0),(0,1,0),(0,0,1)$.
b) A cylinder with the y axes as a center and radius 2.
c) The union of two planes $y=1$ and $y=-1$.
d) A circle of radius 4 in the plane $z=3$.

2) (4 points) Find the distance from the point $P=(2,1,5)$

a) to the z axes.
b) to the xz-coordinate plane.
c) to the origin.
d) to the point $Q=(2,1,1)$.

Solution:
a) A general point (x,y,z) has distance $\sqrt{x^2+y^2}$ from the z-axes. The point P has distance $\sqrt{5}$ from the z-axis.
b) A general point (x,y,z) has distance y from the xz-plane. In this case, 1.
c) The distance to the origin is $\sqrt{x^2+y^2+z^2} = \sqrt{30}$.
d) $d(P,Q) = \sqrt{(2-2)^2+(1-1)^2+(5-1)^2} = 4$.

3) (4 points)

a) Find the equation of the sphere with center $(6,5,-2)$ and radius 5.
b) Describe the traces of this surface, its intersection with each of the coordinate planes.

Solution:
a) $(x-6)^2+(y-5)^2+(z+2)^2=25$.
b) $z=0$: intersection with xy-plane: $(x-6)^2+(y-5)^2=25-4=21$ is a circle.
$y=0$: intersection with xz-plane: $(x-6)^2+(z+2)^2=0$ is a point $(6,0,-2)$.
$x=0$: intersection with yz-plane: $(y-5)^2+(z+2)^2=25-36=-11$ is empty.

4) (4 points) Find the center and radius of the sphere $x^2+y^2+z^2-2x+4y=1$.

Solution:
a) Completion of the square gives $(x^2-2x+1)+(y^2+4y+4)+z^2=(x+1)^2+(y+2)^2+(2z)^2=1+1+4=6$, so that the sphere is centered at the point $(-1,-2,0)$ and has radius $r=\sqrt{6}$.

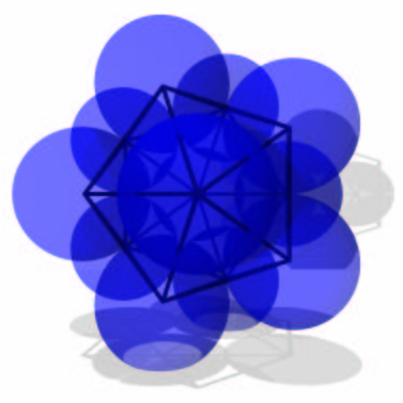
5) (4 points) Four spheres of equal radius are located in space so that any pair of two spheres touch. The centers of three spheres are known to be $(\sqrt{2},0,0),(0,\sqrt{2},0),(0,0,\sqrt{2})$ and the forth sphere with coordinates $(-a,-a,-a)$ is located in the octant $\{x<0,y<0,z<0\}$. Find the radius of the spheres as well as the center of the fourth sphere.

Solution:
The distance between two of the three spheres is 2. The fourth sphere has to have the coordinates $P=(-a,-a,-a)$. In order that this sphere touches the first spheres, we must have $(a+\sqrt{2})^2+a^2+a^2=4$. Solving the quadratic equation gives $a=\sqrt{2}/3$.

Remarks

(You don't need to read these remarks to do the problems.)

Remark to problem 5) Mathematicians call the touching of sphere also "kissing". An old mathematical problem is to determine the **"kissing number"** of a sphere. How many spheres of radius 1 can you arrange around a given sphere S of radius 1 such that any of them touches the sphere S. Newton correctly believed that the kissing number was 12. Proofs that a 13'th sphere can not be squeezed in were only given in the 19'th and 20'th century. Packing 12 spheres around a central one can be realized if they kiss the central sphere S at the vertices of a **icosahedron**.



An other famous problem is the **Kepler problem** which asks about the densest sphere packing in space. A solution of this problem has only been announced a few years ago. The packing of 4 spheres you have looked at in the homework is a first small part of the densest packing. Can you guess, how it should continue?
The challenge problems are optional: at the end of this course you can either turn in a few solutions to challenge problems instead of doing the computer laboratory. Note that some of the challenge problems are hard or even very hard.

Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

1) Show that the set of points in the plane for which the sum of the distances from two points $(-1,0)$ and $(1,0)$ is constant=3 forms an **ellipse**: $x^2/a^2 + y^2/b^2 = 1$.

   Hint: Start with $\sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + y^2} = 3$ and bring it into the form of an ellipse.

2) An other distance in the plane is defined by $d((x,y),(u,v)) = |x-u| + |y-v|$. It is called the **taxi metric** or **Manhattan metric** because a taxi driver in a town like Manhattan , where all streets are parallel either to the x or y axes experiences this distance between two points. How does an ellipse look like in this metric? You can assume that the ellipse is defined as the set of points (x,y) which have the property that the sum of the distances to $(-1,0)$ and $(0,1)$ is 4.

   Hint: You can explore this problem by taking a paper with a grid. Choose two points on the x axes a few grid-points apart and look at all grid-points which satisfy the requirement.