This is part 1 (of 3) of the homework which is due July 8 at the beginning of class.

SUMMARY.

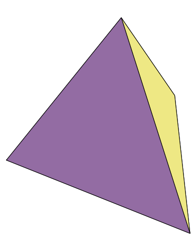
- $\vec{r}(t,s) = P + t\vec{v} + s\vec{w}$ **parametric equation** for a **plane**. $P = (x_0, y_0, z_0)$ a point $\vec{v}, \vec{w}$ are vectors.
- $ax + by + cz = d$, **implicit equation** for a **plane**.
- $\vec{r}(t) = P + t\vec{v}$ **parametric equation** for a line, P a point, $\vec{v}$ is a vector.
- $\frac{(x-x_0)}{a} = \frac{(y-y_0)}{b} = \frac{(z-z_0)}{c}$ **symmetric equation** for a line.

Homework Problems

1) (4 points)
a) (3) Find the distance between the point $(2,-1,2)$ and the plane $4x - 2y + z = 2$.
b) (1) If no absolute value is taken in the distance formula, what does the sign of the result say?

Solution:
a) The point $Q = (0,0,2)$ is on the plane. The scalar projection of $P - Q = (2,-1,0)$ onto the normal vector $(4,-2,1)$ of the plane is $10/\sqrt{21}$.
b) If $(P - Q) \cdot \vec{n}/|\vec{n}|$ is positive, then the point P is on side into which the normal vector points.

2) (4 points) A regular tetrahedron has vertices at the points $P_1 = (0,0,3)$,$P_2 = (0,\sqrt{8},-1)$, $P_3 = (-\sqrt{6},-\sqrt{2},-1)$ and $P_4 = (\sqrt{6},-\sqrt{2},-1)$. Find the distance between two edges which do not intersect.



Solution:
The vector $\vec{v} = (P_2 - P_1) = (0, 2\sqrt{2}, -4)$ is parallel to the first edge, the vector $\vec{w} = (P_4 - P_3) = (2\sqrt{6}, 0, 0)$ is parallel to the second edge. The cross product of $\vec{v}$ and $\vec{w}$ is $\vec{n} = (0, -8\sqrt{6}, -8\sqrt{3})$. The distance between the two edges is the scalar projection of $P_3 - P_1$ onto $\vec{n}$. It is $(P_3 - P_1).\vec{n}/|\vec{n}| = 2\sqrt{3}$.

3) (4 points) Find the equation for the plane which contains the three points $P = (1,2,3), Q = (3,4,4)$ and $R = (1,1,2)$.

Solution:
A normal vector $\vec{n} = (1, -2, 2) = (a, b, c)$ of the plane $ax + by + cz = d$ is obtained as the cross product of $P - Q$ and $R - Q$ With $d = \vec{n} \cdot P = 3$, we have the equation $x - 2y + 2z = 3$.

4) (4 points)
a) (2) Find the parametric equation for the line which passes through the points $P = (1, 2, 3)$ and $Q = (3, 4, 5)$.
b) (2) Find the symmetric equation for the same line.

Solution:
a) The vector $\vec{v} = (2, 2, 2)$ connects the two points. The parametric equation is $P + t\vec{v} = (1, 2, 3) + t(2, 2, 2) = (1 + 2t, 2 + 2t, 3 + 2t)$.
b) $(x - 1)/2 = (y - 2)/2 = (z - 3)/2$.

5) (4 points) Find a parametric equation for the line through the point $P = (3, 1, 2)$ that is perpendicular to the line $L : x = 1 + t, y = 1 - t, z = 2t$ and intersects this line.

Solution:
The point $Q = (1, 1, 0)$ is on the line. The vector $\vec{v} = (1, -1, 2)$ parallel to the line. We have $P - Q = (2, 0, 2)$. The vector $\vec{n} = \vec{v} \times (\vec{v} \times (P - Q)) = (-6, -6, 0)$ is the direction from P to the normal intersection with the line. The line can be given by $\vec{r}(t) = (3 - 6t, 1 - 6t, 2)$.

Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

1) How does one describe a three dimensional "hyperplane" in four dimensional space? Find a parametric description and an implicit description.

Solution:
The implicit description is $ax + by + zz + dw = e$. A three dimensional hyperplane is spanned by three vectors v_1, v_2, v_3. A general point is $P + t_1v_1 + t_2v_2 + t_3v_3$.

2) Can you find a line and a two dimensional plane in $\mathbf{R}^4$ which are not parallel and which do not intersect? How would you compute the distance between a line and a two dimensional plane in $\mathbf{R}^4$?

Solution:
Take the line $(1 + t, 0, 0, 0)$ and the plane $(0, 0, u, v)$. They are orthogonal but do not intersect. To compute the distance, we would have to find a vector $\vec{n}$ which is orthogonal to both the plane as well as the line, take a point P on the line and a point Q on the plane and take the scalar projection of $P - Q$ onto n.