This is part 1 (of 2) of the homework for the third week. It is due July 15 at the beginning of class.

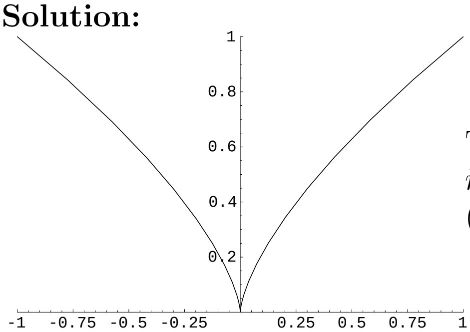
SUMMARY.

- $\vec{r}(t) = (x(t), y(t), z(t))$, $t \in [a, b]$ **curve** in space.
- $t \mapsto \vec{r}(t)$ is called **parametric representation** of the curve.
- $\vec{r}\,'(t) = (x'(t), y'(t), z'(t))$ **velocity**, $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$ **unit tangent vector**.
- $\vec{r}\,''(t) = (x''(t), y''(t), z''(t))$ **acceleration**.
- $\vec{R}(t) = \int_a^t \vec{r}(s)\ ds = (\int_a^t x(s)\ ds, \int_a^t y(s)\ ds, \int_a^t z(s)\ ds) + (C_1, C_2, C_3)$ **anti-derivative**, satisfies $\vec{R}\,'(t) = \vec{r}(t)$. (C_1, C_2, C_3) are arbitrary constants.

Homework Problems

1) (4 points)
Sketch the plane curve $\vec{r}(t) = (x(t), y(t)) = (t^3, t^2)$ for $t \in [-1, 1]$ by plotting the points for different values of t. Calculate its velocity $\vec{r}\,'(t)$ as well as its acceleration $\vec{r}\,''(t)$ at the point $t = 2$.

Solution:



The velocity is $\vec{r}\ '(t) = (3t^2, 2t)$. The acceleration is $\vec{r}\ ''(t) = (6t, 2)$. At the time $t = 2$ we have $\vec{r}\ '(2) = (12, 4)$ and $\vec{r}\,''(2) = (12, 2)$.

2) (4 points) A device in a car measures the acceleration $\vec{r}\ ''(t) = (\cos(t), -\cos(3t))$ at time t. Assume that the car is at the origin at time $t = 0$ and has zero speed at $t = 0$, what is its position $\vec{r}(t)$ at time t?

Solution:
$\vec{r}\,'(t) = (\sin(t), -\sin(3t)/3) + (C_1, C_2)$. Because the car has zero speed at time $t = 0$, we have $C_1 = C_2 = 0$.
From $\vec{r}\ '(t) = (\sin(t), -\sin(3t)/3)$, we obtain $\vec{r}(t) = (-\cos(t), +\cos(3t)/9) + (C_1, C_2)$. Because $\vec{r}(0) = (0, 0)$, we have $C_1 = 1, C_2 = -1/9$. $\vec{r}(t) = (-\cos(t) + 1, -\cos(3t)/9 - 1/9)$.

3) (4 points) Consider the curve $\vec{r}(t) = (x(t), y(t), z(t)) = (t^2, 1 + t, 1 + t^3)$.
a) (1) Verify that it passes through the point $(1, 0, 0)$.
b) (3) Find the velocity vector $\vec{r}\,'(t)$, the acceleration vector $\vec{r}\,''(t)$ as well as the jerk vector $\vec{r}\,'''(t)$ at the point $(1, 0, 0)$.

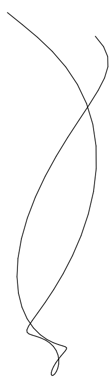
Solution:
a) Take $t = -1$.
b) $\vec{r}\,'(t) = (2t, 1, 3t^2)$,
$\vec{r}\,''(t) = (2, 0, 6t)$,
$\vec{r}\,'''(t) = (0, 0, 6)$
At time $t = -1$, we obtain $\vec{r}\,'(-1) = (-2, 1, 3)$,
$\vec{r}\,''(-1) = (2, 0, -6)$,
$\vec{r}\,'''(-1) = (0, 0, 6)$

4) (4 points) a) (2) Verify that the curve $\vec{r}(t) = (t\cos(t), 2t\sin(t), t^2)$ lies on the elliptic paraboloid $z = x^2 + y^2/4$.

b) (2) Use this fact to sketch the curve.

Solution:
a) Just plug in $x(t)^2 + y(t)^2 = z$.

b)



5) (4 points) Find the parameterization $\vec{r}(t) = (x(t), y(t), z(t))$ of the curve obtained by intersecting the cylinder $x^2 + y^2 = 9$ with the surface $z = xy$. Write down the formula for the velocity vector $\vec{r}\,'(t)$.

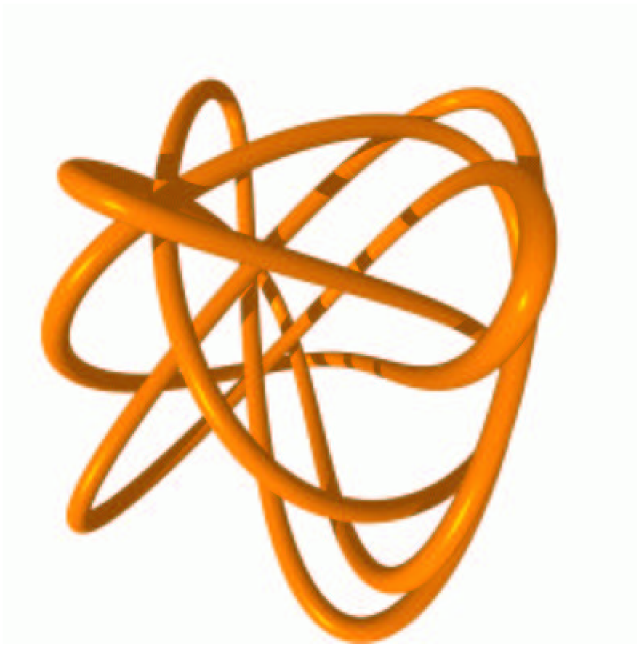
Solution:
We find first $x(t) = 3\cos(t), y(t) = 3\sin(t)$ using the first equation. Then get $z(t) = x(t)y(t) = 9\cos(t)\sin(t)$.
$\vec{r}(t) = (x(t), y(t), z(t)) = (3\cos(t), 3\sin(t), 9\cos(t)\sin(t)/2)$. The velocity vector is $\vec{r}'(t) = (x(t), y(t), z(t)) = (-3\sin(t), 3\cos(t), 9\cos^2(t) - 9\sin^2(t))$.

Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

1) A closed curve in space is called a **knot**. Consider the space curve $\vec{r}(t) = (\sin(3t), \cos(4t), \cos(5t))$. Find the smallest interval $[a, b]$ such that this curve is a knot. Sketch the curve.

Hint. Try first without technology. If needed, peek at the website http : //www.math.harvard.edu/jdg or type
ParametricPlot3D[{Sin[3t], Cos[4t], Cos[5t]}, {t, 0, 2Pi}]
in Mathematica.




2) How could one verify that it is not possible to deform the knot $\vec{r}(t) = (\sin(3t), \cos(4t), \cos(5t))$ into the trivial knot $\vec{r}(t) = (\cos(t), \sin(t), 0)$ in such a way that during the deformation, the curve can never selfintersect?

Hint. Look at the possible types of closed curves which don't intersect the knot. How many different types are there for the trivial knot or for the given knot?