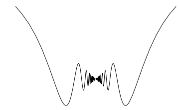
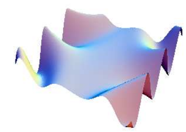
OBJECTS $f : \mathbf{R}^n \to \mathbf{R}^m$ IN MULTIVARIABLE CALCULUS — Maths21a, 2003

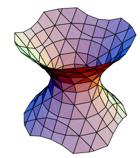
SCALAR FUNCTION (1D). ($n = 1, m = 1$). A function of one variable, where each $f(x)$ is a real number. The graph is a curve in the x, y plane. The derivative is $f'(x)$.



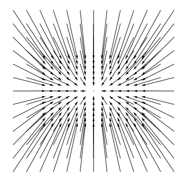
SCALAR FUNCTION (2D). ($n = 2, m = 1$). A function $f(x, y)$ defined in the plane is also called a scalar field. The graph of f is a curve in space (see figure). Level curves are curves in the plane. A derivative is $\nabla f(x, y)$, the gradient.



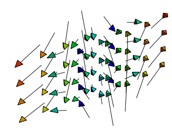
SCALAR FUNCTION (3D). ($n = 3, m = 1$). A function $f(x, y, z)$ defined in space is also called a scalar field. Its graph would be an object in 4D. Level surfaces are surfaces in space (see figure). A derivative is the gradient $\nabla f(x, y, z)$.



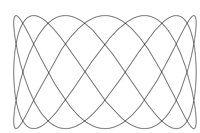
VECTOR FIELD (2D). ($n = 2, m = 2$) A function on the plane which attaches a vector $(P(x, y), Q(x, y))$ to each point (x, y). Derivatives are $\mathrm{curl}(F) = Q_x - P_y$ or $\mathrm{div}(F) = P_x + Q_y$ both scalar fields (real valued functions). Also **coordinate changes** $T(u, v) = (x(u, v), y(u, v))$ are the same mathematical object.



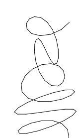
VECTOR FIELD (3D). ($n = 3, m = 3$) At each point in space, we attach a vector $F(x, y, z) = (P(x, y, z), Q(x, y, z), R(x, y, z))$ at each point (x, y, z). Derivatives are the curl $\mathrm{curl}(F)$, a vector field or the divergence $\mathrm{div}(F) = P_x + Q_y + R_z$, a scalar field (real valued functions). Also **coordinate changes** $T(u, v, w) = (x(u, v, w), y(u, v, w), z(u, v, w))$ are the same mathematical object.



CURVE (2D). ($n = 1, m = 2$) For each t is defined a point $r(t) = (x(t), y(t))$ in the plane. A derivative is $r'(t) = (x'(t), y'(t))$, the velocity.



CURVE (3D). ($n = 1, m = 3$) For each t, we have a point $r(t) = (x(t), y(t), z(t))$ in space. A derivative is $r'(t) = (x'(t), y'(t), z'(t))$, the velocity.



SURFACE (2D). ($n = 2, m = 3$) For each (u, v) there is a point $r(u, v) = (x(u, v), y(u, v), z(u, v))$ in space. The normal vector at a point $r(u, v)$ is $r_u(u, v) \times r_v(u, v)$.