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- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work.
- Do not detach pages from this exam packet or unstaple the packet.
- Please try to write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- Problems 1-3 do not require any justifications. For the rest of the problems you have to show your work. Even correct answers without derivation can not be given credit.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
Total:		140

Problem 1) (20 points)
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- 1) 

T	F
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 The quadratic surface  $x^2 + y - z^2 = -5$  is a hyperbolic paraboloid.
- 2) 

T	F
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 There are vectors  $\vec{u}$  and  $\vec{v}$  such that  $|\vec{u} \times \vec{v}| > |\vec{u}||\vec{v}|$ .
- 3) 

T	F
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 $\int_0^{2\pi} \int_0^5 r \, d\theta \, dr$  is the area of a disc of radius 5.
- 4) 

T	F
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 If a vector field  $\vec{F}(x, y)$  satisfies  $\text{curl}(\vec{F})(x, y) = Q_x - P_y = 0$  for all points  $(x, y)$  in the plane, then  $\vec{F}$  is a gradient field.
- 5) 

T	F
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 The jerk of a parameterized curve  $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$  is parallel to the acceleration if the curve  $\vec{r}(t)$  is a line.
- 6) 

T	F
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 The curvature of the curve  $\vec{r}(t) = \langle 3 \sin(t), 0, 3 \cos(t) \rangle$  is twice the curvature of the curve  $\vec{s}(t) = \langle 6 + 6 \sin(t), 6 \cos(t), 0 \rangle$ .
- 7) 

T	F
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 The curve  $\vec{r}(t) = \langle \sin(t), t^2, \cos(t) \rangle$  for  $t \in [0, 10\pi]$  is located on a cylinder.
- 8) 

T	F
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 If a function  $f(x, y)$  has the property that  $f_x(x, y)$  is zero for all  $x, y$ , then  $f$  is the constant function.
- 9) 

T	F
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 If the unit tangent vector  $\vec{T}(t)$  of a curve  $\vec{r}(t)$  is always parallel to a plane  $\Sigma$ , then the curve is contained in a plane parallel to  $\Sigma$ .
- 10) 

T	F
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 If  $(x_0, y_0)$  is an extremum of  $f(x, y)$  under the constraint  $x^2 + y^2 = 1$ , then the same point is an extremum of  $10f(x, y)$  under the same constraint.
- 11) 

T	F
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 At a critical point  $(x_0, y_0)$  of a function  $f(x, y)$  for which  $f_{xx}(x_0, y_0) > 0$ , the critical point is always a minimum.
- 12) 

T	F
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 If a vector field  $\vec{F}(x, y)$  is a gradient field, and  $C$  is a closed curve which looks like a figure 8, then  $\int_C \vec{F} \cdot d\vec{r}$  is zero.
- 13) 

T	F
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 If  $C$  is part of a level curve of a function  $f(x, y)$  and  $\vec{F} = \langle f_x, f_y \rangle$  is the gradient field of  $f$ , then  $\int_C \vec{F} \cdot d\vec{r} = 0$ .
- 14) 

T	F
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 The divergence of the gradient vector field  $\vec{F}(x, y, z) = \nabla f(x, y, z)$  is always the zero function.
- 15) 

T	F
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 The line integral of the vector field  $\vec{F}(x, y, z) = \langle x, y, z \rangle$  along a line segment from  $(0, 0, 0)$  to  $(1, 1, 1)$  is  $3/2$ .
- 16) 

T	F
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 The area of a region  $G$  can be expressed as a line integral along its boundary.
- 17) 

T	F
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 The flux of the vector field  $\vec{F}(x, y, z) = \langle x, y, -z \rangle$  through the boundary  $S$  of a solid ellipsoid  $E$  is equal to the volume the ellipsoid.
- 18) 

T	F
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 If  $\vec{F}$  is a vector field in space and  $S$  is a torus surface, then the flux of  $\text{curl}(\vec{F})$  through  $S$  is 0.
- 19) 

T	F
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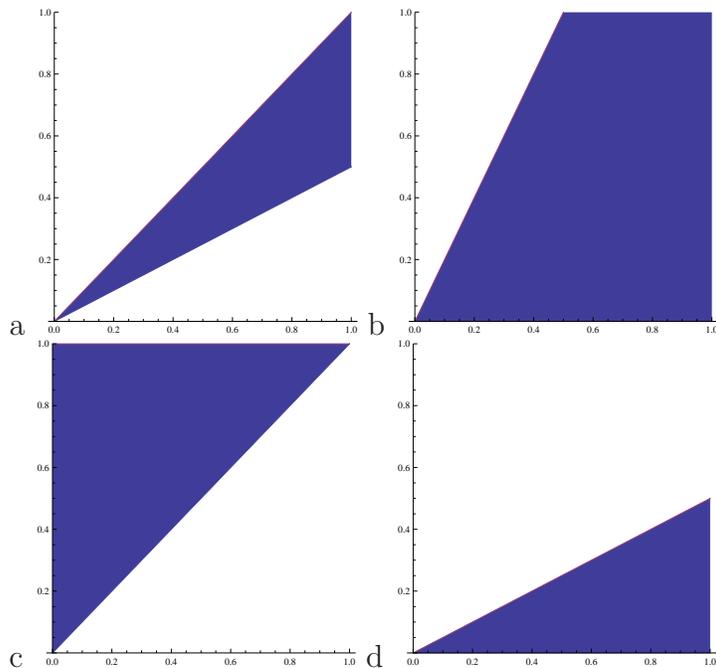
 If the divergence and the curl of a vector field  $\vec{F}$  are both zero, then it is a constant field.
- 20) 

T	F
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 For any function  $f$ , the curl of  $\vec{F} = \text{grad}(f)$  is the zero field  $\langle 0, 0, 0 \rangle$ .

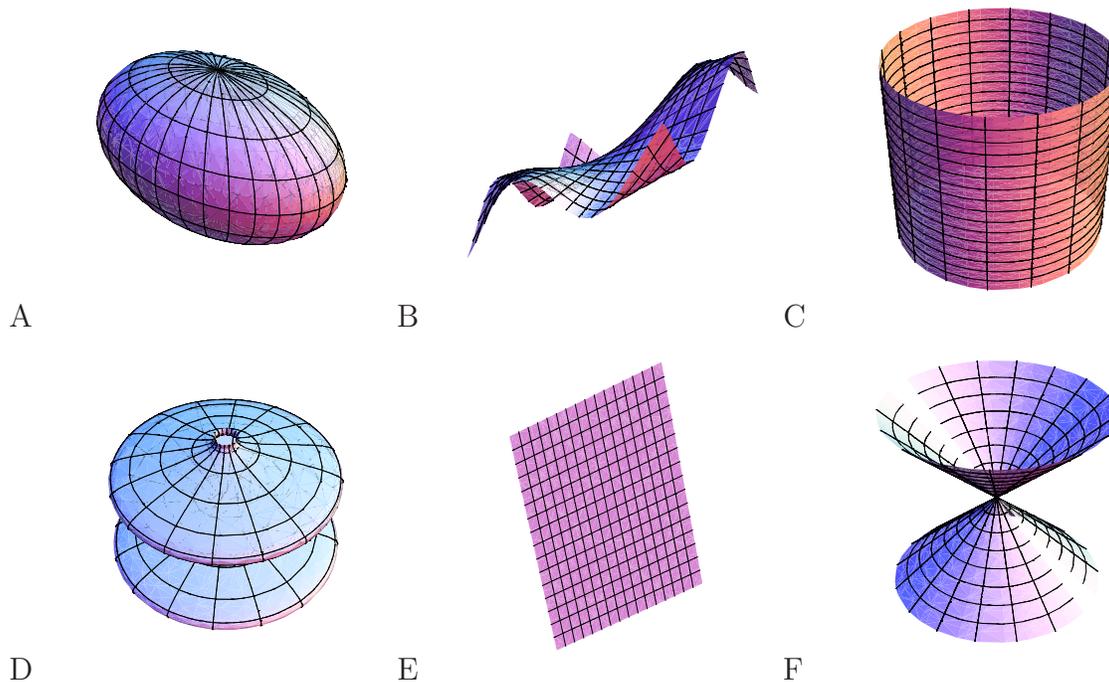
Problem 2) (10 points)

a) (4 points) Match the regions with the corresponding double integrals



Enter a,b,c,d	Function
	$\int_0^1 \int_{x/2}^x f(x, y) dy dx$
	$\int_0^1 \int_0^y f(x, y) dx dy$
	$\int_0^1 \int_0^{x/2} f(x, y) dy dx$
	$\int_0^1 \int_{y/2}^1 f(x, y) dx dy$

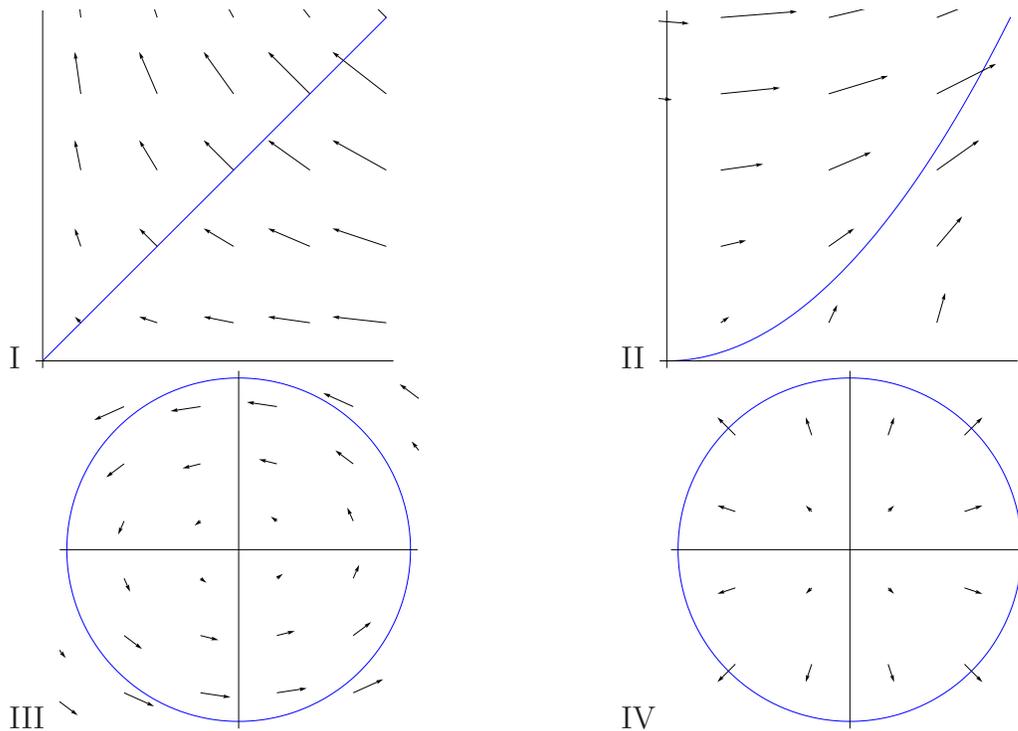
b) (6 points) Match the parametrized or implicit surfaces with their definitions



Enter A-F here	Function or parametrization
	$\vec{r}(u, v) = \langle \cos(u), \sin(u), v \rangle$
	$\vec{r}(u, v) = \langle u - v, u + 2v, 2u + 3v \rangle$
	$x^2 + y^2/3 + z^2/3 = 1$
	$\vec{r}(u, v) = \langle (\sin(v) + 1) \cos(u), (\sin(v) + 1) \sin(u), v \rangle$
	$z - x + \sin(xy) = 0$

Problem 3) (10 points)

a) (4 points) Match the vector fields and curves with the corresponding line integral



Enter I,II,III,IV	Line integral
	$\int_0^{2\pi} \langle \cos(t), \sin(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle dt$
	$\int_0^{2\pi} \langle -t, t^2 \rangle \cdot \langle 1, 1 \rangle dt$
	$\int_0^{2\pi} \langle t^2, t \rangle \cdot \langle 1, 2t \rangle dt$
	$\int_0^{2\pi} \langle -3 \sin(t), 3 \cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle dt$

b) (6 points) Fill in from following choice: "arc length", "surface area", "chain rule", "volume of parallelepiped", "area of parallelogram", "line integral", "flux integral", "curvature".

Formula	Name of formula or rule or theorem
$\int \int_R  \vec{r}_u \times \vec{r}_v  dudv$	
$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}'(t)$	
$\int_a^b  \vec{r}'(t)  dt$	
$\frac{ \vec{r}'(t) \times \vec{r}''(t) }{ \vec{r}'(t) ^3}$	
$ \vec{u} \cdot (\vec{v} \times \vec{w}) $	
$\int_0^1 \int_0^1 \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) dudv$	

Problem 4) (10 points)

Given the line  $x - 1 = y - 2 = z - 3$  and the point  $P = (8, 4, 5)$ . Find the equation

$$ax + by + cz = d$$

of the plane which contains the line and the point.

Problem 5) (10 points)

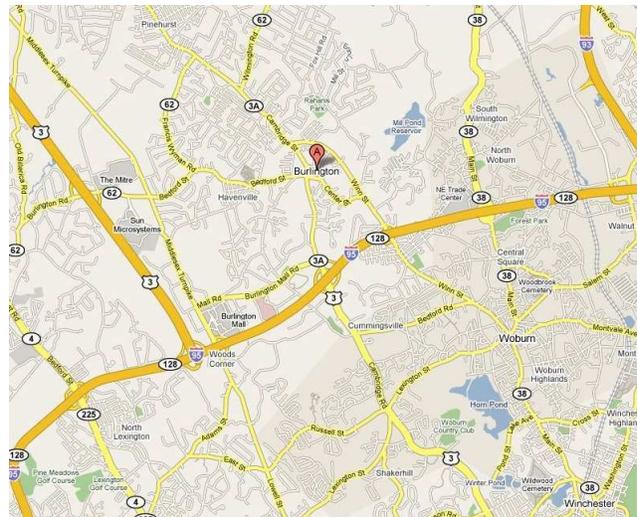
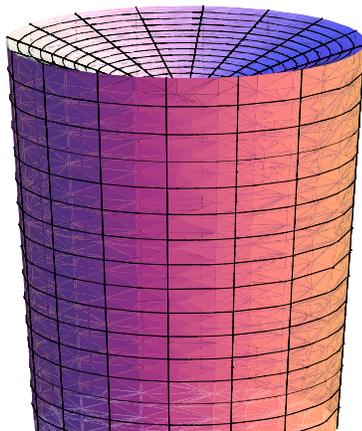
Find all the critical points of the function  $f(x, y) = y^3 - 3y^2 + 4x + x^2 - 3$  and classify them by telling whether they are local maxima, local minima or saddle points.

Problem 6) (10 points)

The hyperbolic paraboloid  $x^2 - y^2 - 3z = 0$  contains the point  $P = (1, 1, 0)$  and the point  $Q = (3, 0, 3)$ . Find the tangent planes to the surface at  $P$  and  $Q$  and find a parametrization  $\vec{r}(t)$  of the line of intersection of these two planes.

Problem 7) (10 points)

A water reservoir in Burlington, MA (the map to the right is centered there) is bounded by a solid cylinder  $x^2 + y^2 \leq 1$ . It has as the roof the cone  $x^2 + y^2 = (z - 6)^2$  and is bounded from below by the  $xy$ -plane  $z = 0$ . What is the volume of the reservoir?



Problem 8) (10 points)

Find the maxima and minima of the function  $f(x, y) = x^2 - y^2$  on the parabola  $x + y^2 = 1$  using the Lagrange multiplier method.

Problem 9) (10 points)

Compute the surface area of the surface  $\vec{r}(u, v) = \langle u^3, v^3, u^3 - v^3 \rangle$  parametrized so that  $(u, v)$  is in the unit disc.

Problem 10) (10 points)

Evaluate the following double integral

$$\int_0^2 \int_{x/2}^1 \cos(y^2) dy dx .$$

Problem 11) (10 points)

Find the value of the line integral

$$\int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt ,$$

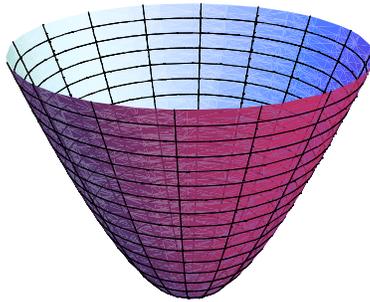
where  $\vec{F}(x, y) = \langle y + \sin(\cos(x)), -2x \rangle$  and  $C$  is the boundary of the unit circle traversed in the counter clockwise direction.

Problem 12) (10 points)

Find the value of the flux integral

$$\iint_S \text{curl}(\vec{F})(\vec{r}(u, v)) \cdot \vec{r}_u \times \vec{r}_v du dv$$

where  $\vec{F}(x, y, z) = \langle -y, x, z \rangle$  and  $S$  is the part of the two-sheeted hyperboloid  $x^2 + y^2 - z^2 = -1$  which satisfies  $1 < z < 2$  and which is oriented so that the normal vector points downwards on  $S$ .



Problem 13) (10 points)

Let  $E$  be the solid which is bounded on the side by the cone  $S_1 : x^2 + y^2 = z^2, 0 < z < 1$  and on top by the disc  $S_2 = x^2 + y^2 \leq 1, z = 1$ . Let  $\vec{F}(x, y, z) = \langle 1 + 4x, 2 - 5y, 3 + 2z \rangle$ . Find the value of the flux integral

$$\int_S \vec{F}(\vec{r}(u, v)) \cdot \vec{r}_u \times \vec{r}_v \, du dv ,$$

where  $S$  is the union of the two surfaces  $S_1$  and  $S_2$ . The normal vector of  $S$  is oriented outwards on  $S_1 \cup S_2$ .

