Name:

- Start by writing your name in the above box.

- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader can not be given credit.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

1) [T] [F] The vector $\vec{v} = \langle 1, 3, 5 \rangle$ is perpendicular to the plane $x + 3y + 5z = 1$.

Solution:
The normal vector to the plane can be read off from the coefficients.

2) [T] [F] The set of points which satisfy $x^2 - y^2 + z^2 - 2z + 1 = 0$ forms a double cone.

Solution:
Yes, it is a double cone, $z^2 - 2z + 1 = (z-1)^2$.

3) [T] [F] The set of points in $\mathbf{R}^3$ which have distance 1 from a point form a cylinder.

Solution:
The se is a sphere, not a cylinder.

4) [T] [F] The surface $-x^2 + y^2 + z^2 = 1$ is called a one-sheeted hyperboloid.

Solution:
Looking at the traces shows it.

5) [T] [F] The two vectors $\langle 2, 3, 0 \rangle$ and $\langle 6, -4, 5 \rangle$ are orthogonal to each other.

Solution:
Their dot product is zero.

6) [T] [F] Two nonzero vectors are parallel if and only if their dot product is 0.

Solution:
They are orthogonal if their dot product is 0.

7) $\boxed{\text{T}}$ $\boxed{\text{F}}$ The cross product is associative: $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \times \vec{w}$.

Solution:
Try the identity with $u = i, v = j, w = j$, where the right hand side is zero and the left hand side $-i$.

8) $\boxed{\text{T}}$ $\boxed{\text{F}}$ Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $(2, 3, 4)$.

Solution:
The vector $v = (2, 3, 4)$ is contained in the line and every vector in the line is parallel.

9) $\boxed{\text{T}}$ $\boxed{\text{F}}$ The line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{4}$ hits the plane $2x + 3y + 4z = 9$ at a right angle.

Solution:
The vector $(2, 3, 4)$ is in the line and perpendicular to the plane.

10) $\boxed{\text{T}}$ $\boxed{\text{F}}$ Two planes $ax + by + cz = d$ and $ux + vy + wz = e$ intersect in a line if $|\langle a, b, c \rangle \times \langle u, v, w \rangle| > 0$.

Solution:
The nonvanishing of the cross product assures that the two vectors are not parallel, which means that the planes are not parallel.

11) $\boxed{\text{T}}$ $\boxed{\text{F}}$ The equations $x - 2 = y - 3 = z - 4$ describe a line which contains the vector $\langle 1, 1, 1 \rangle$.

Solution:
This this is a special case of the symmetric equations.

12) $\boxed{\text{T}}$ $\boxed{\text{F}}$ In spherical coordinates, the equation $\cos(\phi) = \sin(\phi)$ defines a cone.

Solution:
It means that $tan(\phi)$ is constant which implies ϕ is constant equal to $\pi/4$.

13) **T** F — A point with spherical coordinates $(\rho, \theta, \phi) = (1, \pi/2, \pi/4)$ has cylinder coordinates $(r, \theta, z) = (1/\sqrt{2}, \pi/2, 1/\sqrt{2})$.

Solution:
It is the point $(0, 1/\sqrt{2}, 1/\sqrt{2})$.

14) T **F** — If in rectangular coordinates, a point is given by $(1, 0, -1)$, then its spherical coordinates are $(\rho, \theta, \phi) = (\sqrt{2}, \pi/2, 3\pi/4)$.

Solution:
The theta angle is wrong.

15) T **F** — The volume of a parallelepiped spanned by $(1,0,0), (0,1,0)$ and $(0,1,1)$ is equal to 2.

Solution:
It is 1.

16) T **F** — The vector projection of the vector $(2,4,5)$ onto the vector $(1,1,0)$ is $(3/2, 3/2, 0)$.

Solution:
Use the definition. The vector projection is $(3, 3, 0)$.

17) T **F** — If $g(x, y, z) = 0$ is a surface given implicitely, then $\vec{r}(u, v) = \langle u, v, g(u, v, g(u, v, 1)) \rangle$ is a parametrization of the surface.

Solution:
This was a practice exam problem.

18) T **F** — If $f = g(x, y)$ is a graph then $\vec{r}(u, v) = \langle u, v, g(u, v) \rangle$ is a parameterization of the surface.

Solution:
One of the 4 important cases.

19) [T] [F]   The distance from the point $P = (1,1,1)$ to the x axes is $\sqrt{2}$.

Solution:
We do not need the distance formula, just look at the picture in the direction of the x-coordinate.

20) [T] [F]   The distance between the point $P = (1,1,1)$ and the xy plane is $\sqrt{2}$.
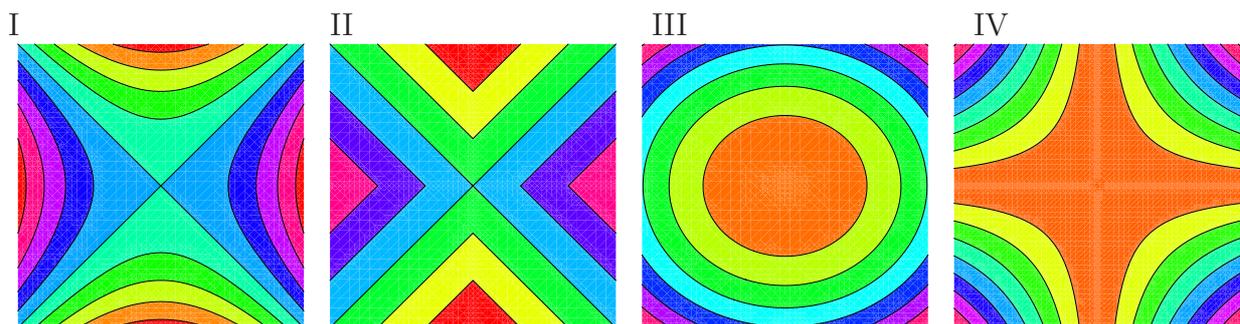
Solution:
It is 1.

	Total

Problem 2) (10 points)

a) (3 points) Match the contour maps with the corresponding functions $f(x,y)$ of two variables. Enter O if no figure matches. No justifications are needed.

I   II   III   IV

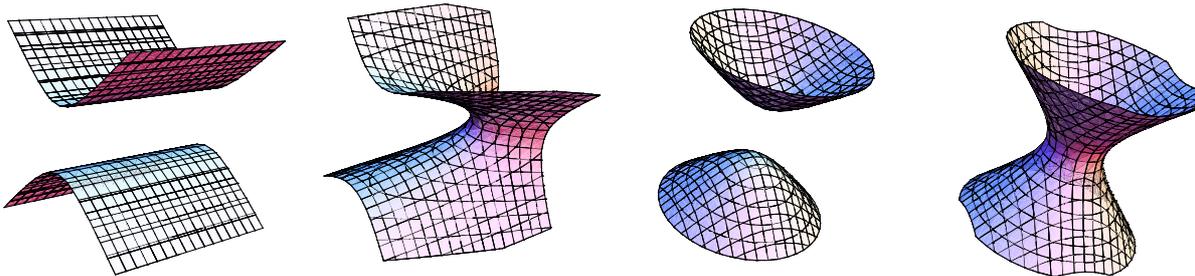| Enter I,II,III,IV or O	Function $f(x,y)$	Enter I,II,III,IV or O	Function $f(x,y)$
	$f(x,y) = x\cos(y)$		$f(x,y) = x^2 - y^2$
	$f(x,y) = 3x^2 + 4y^2$		$f(x,y) = |xy|$
	$f(x,y) = \cos(x)$		$f(x,y) = |x| - |y|$

b) (4 points) Match the quadrics with the functions. No justifications are necessary.
a   b   c   d

Enter a,b,c,d here	Equation
	$x + y^2 - z^2 - 1 = 0$
	$y^2 - z^2 + 1 = 0$

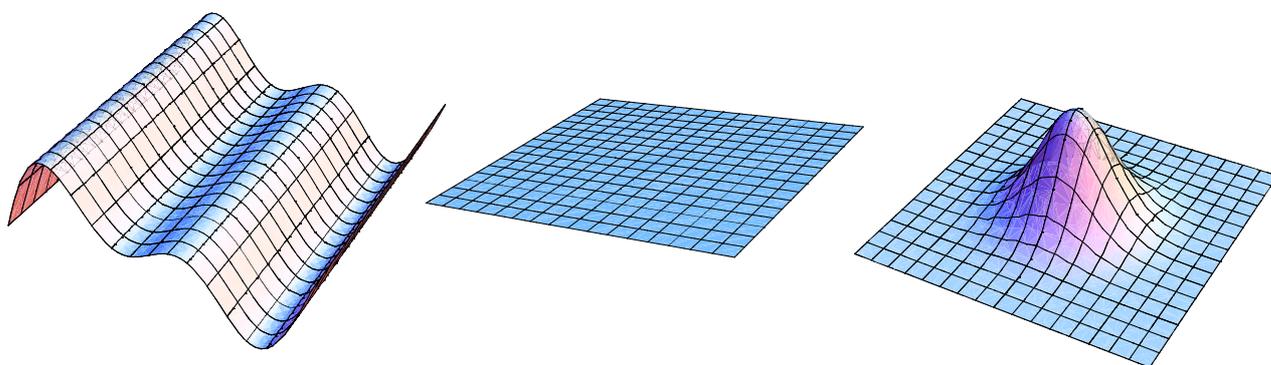| Enter a,b,c,d here	Equation
	$x^2 + y^2 - z^2 + 1 = 0$
	$x^2 + y^2 - z^2 - 1 = 0$

c) (3 points) Match the equation with their graphs. No justifications are necessary.

A   B   C

Enter A,B,C here	Equation	Enter A,B,C here	Equation	Enter A,B,C here	Equation
	$z = e^{-x^2-y^2}$		$z = x\cos(x)$		$z = x - y$

Solution:
$O, III, 0 \ I, IV, II$.

$b, a \ c, d$.

C, A, B.

Problem 3) (10 points)

Surfaces $z = f(x,y)$ which are graphs can be written implicitly as $g(x,y,z) = 0$, parametrized as $\vec{r}(u,v)$. For example, $z = \log(xy)$ is given by $g(x,y,z) = 0$ with $g(x,y,z) = z - \log(xy)$ or parametrized as $\vec{r}(u,v) = \langle u, v, \log(uv) \rangle$. Complete the following table by filling in the choices $A - J$ below. No justifications are needed in this problem.

$z = f(x,y)$ for	$g(x,y,z) = 0$	$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$
		$\langle v\cos(u), v\sin(u), v \rangle$
	$x + y - 2z = 0$	
$f(x,y) = x^2 - y^2$		
		$\langle \cos(u)\sin(v), \sin(u)\sin(v), \cos(v) \rangle, v < \pi/2$
	$z - \sin(xy) = 0$	
$f(x,y) = x$		

A) $f(x,y) = x - y$

B) $f(x,y) = x^2 + y^2$

C) $z - x^2 - y^2$

D) $\langle 1 + u + v, 1 + u - v, u \rangle$

E) $z - x^2 + y^2$.

F) $f(x,y) = \sqrt{1 - x^2 - y^2}$

G) $\langle u, v, u^2 - v^2 \rangle$

H) $x^2 + y^2 + z^2 - 1 = 0$

I) $\langle u, v, u \rangle$

J) $z - x = 0$

Solution:

0 0

0 0

E,G

F,H

0,0

J,I

Problem 4) (10 points)

A billard ball starts at $A = (1, 1, 0)$, travels along the vector $\vec{u} = \langle 2, -2, 0 \rangle$ to other point B where it bounces off an other ball. It travels from there along the vector $\vec{v} = \langle -3, 4, 0 \rangle$ to a third point C, where it bounces off a wall, rolling along the vector $\vec{w} = \langle 1, 1, 0 \rangle$ to its final destination D. In other words, you know A, $\vec{AB} = \vec{u}$, $\vec{BC} = \vec{v}$ and $\vec{CD} = \vec{w}$.

a) (5 points) What are the coordinates of the point D?

b) (5 points) Find the total distance traveled by the ball along the path A, B, C, D.

Solution:

a) $D = A + \vec{u} + \vec{v} + \vec{w} = (1, 4, 0)$.

b) $||u|| + ||v|| + ||w|| = \sqrt{8} + 5 + \sqrt{2} = 3\sqrt{2} + 5$.

Problem 5) (10 points)

a) (5 points) Find the symmetric equation of the line which contains the point $P = (3, 4, 1)$ and the point $Q = (5, 5, 5)$.

b) (5 points) What is the equation of the plane perpendicular to the line in a) which passes through the point $P = (3, 4, 1)$?

Solution:
a) $(x - 3)/2 = (y - 4)/1 = (z - 1)/4$.
b) We know the direction of the normal vector $(2, 1, 4)$. Therefore, the plane has the equation $2x + y + 4z = d$. Plugging in the point P gives $d = 14$. The solution is
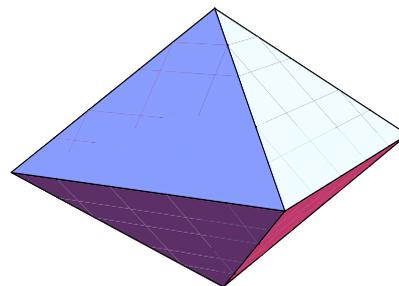
$$2x + y + 4z = 14 .$$

Problem 6) (10 points)

We look at a polyhedron which has the shape of a scaled octahedron. Its vertices are $A = (1, 1, 0), B = (-1, 1, 0), C = (-1, -1, 0), D = (1, -1, 0), E = (0, 0, 1), F = (0, 0, -1)$.

a) (5 points) Parametrize the line L passing through A,E and the line K passing through B,F.

b) (5 points) Find the distance between these two lines L and K.
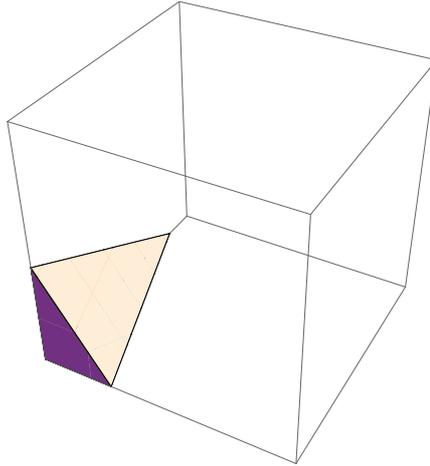

Solution:
a) $L : r(t) = (1, 1, 0) + t(-1, -1, 1)$ and $K : r(t) = (-1, 1, 0) + t(1, -1, -1)$
b) Use the distance formula $AB \cdot n/|n|$, where $n = (-1, -1, 1) x (-1, -1, 1) = (2, 0, 2)$. The answer is $4/\sqrt{8} = \sqrt{2}$.

The plane $3x + y + 2z = 6$ cuts out a triangle T from the octant $x > 0, y > 0, z > 0$. This triangle as well as the coordinate planes bound a solid E in space.

a) (5 points) Find the area of this triangle $T = ABC$.

b) (5 points) The volume of the solid E is known to be 1/6'th of the volume of the parallelepiped spanned by $\vec{OA}, \vec{OB}, \vec{OC}$. Find the volume of E.

Solution:
a) The area is $|\vec{AB} \times \vec{AC}|/2 = ||\langle 9, 3, 6 \rangle||/2 = \sqrt{126}$.
b) The volume is $\vec{OA} \cdot (\vec{OB} \times \vec{OC})/6 = 6$.

a) (2 points) Find the dot product of $\langle 3, 4, 5 \rangle$ and $\langle 3, 2, 1 \rangle$.

b) (2 points) Find the vector projection of the vector $\langle 4, 5, 6 \rangle$ onto the vector $\langle 3, 3, 3 \rangle$.

c) (2 points) Do the vectors $\langle 1, -1, -1 \rangle$ and $\langle 4, -5, 6 \rangle$ form an acute or obtuse angle?

d) (2 points) What is the triple scalar product of $\vec{i} + \vec{j}, \vec{j}$ and $\vec{i} - \vec{j}$?

e) (2 points) Find the cross product of $\langle 1, 1, 2 \rangle$ and $\langle 3, 4, 5 \rangle$.

Solution:
a) 22
b) $\langle 5, 5, 5 \rangle$
c) $\vec{v} \cdot \vec{w} > 0$, the angle is acute.
d) $(1, 1, 0) \cdot (0, 1, 0) \times (1, -1, 0) = 0$.
e) $\langle -3, 1, 1 \rangle$

Problem 9) (10 points)

---

a) (5 points) Parametrize the plane containing the three points $A = (1, 1, 1)$, $B = (1, 3, 2)$ and $C = (3, 4, 5)$.

b) (5 points) Parametrize the sphere which is centered at $(1, 1, 1)$ and has radius 3.

Solution:
a) $\vec{r}(s, t) = \langle 1, 1, 1 \rangle + t \langle 0, 2, 1 \rangle + s \langle 2, 3, 4 \rangle$.
b) $\vec{r}(\theta, \phi) = \langle 1 + 3 \cos(\theta) \sin(\phi), 1 + 3 \sin(\theta) \sin(\phi), 1 + 3 \cos(\phi) \rangle$.