

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work.
- Do not detach pages from this exam packet or unstaple the packet.
- Please try to write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- Problems 1-3 do not require any justifications. For the rest of the problems you have to show your work. Even correct answers without derivation can not be given credit.
- You have 180 minutes time to complete your work.

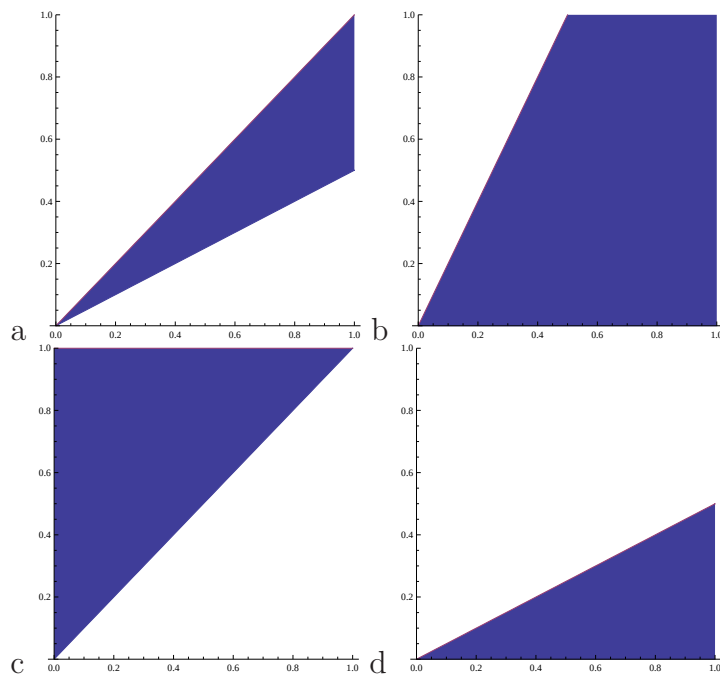
1		20
2		10
3		10
4		10
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11		10
12		10
13		10
14		10
Total:		150

Problem 1) (20 points)

- 1) ☐ T ☐ F The quadratic surface $x^2 + y - z^2 = -5$ is a hyperbolic paraboloid.
- 2) ☐ T ☐ F There are vectors $\vec{u}$ and $\vec{v}$ such that $|\vec{u} \times \vec{v}| > |\vec{u}||\vec{v}|$.
- 3) ☐ T ☐ F $\int_0^{2\pi} \int_0^5 r \, d\theta \, dr$ is the area of a disc of radius 5.
- 4) ☐ T ☐ F If a vector field $\vec{F}(x, y)$ satisfies $\text{curl}(\vec{F})(x, y) = Q_x - P_y = 0$ for all points (x, y) in the plane, then $\vec{F}$ is a gradient field.
- 5) ☐ T ☐ F The jerk of a parameterized curve $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ is parallel to the acceleration if the curve $\vec{r}(t)$ is a line.
- 6) ☐ T ☐ F The curvature of the curve $\vec{r}(t) = \langle 3 \sin(t), 0, 3 \cos(t) \rangle$ is twice the curvature of the curve $\vec{s}(t) = \langle 6 + 6 \sin(t), 6 \cos(t), 0 \rangle$.
- 7) ☐ T ☐ F The curve $\vec{r}(t) = \langle \sin(t), t^2, \cos(t) \rangle$ for $t \in [0, 10\pi]$ is located on a cylinder.
- 8) ☐ T ☐ F If a function $f(x, y)$ has the property that $f_x(x, y)$ is zero for all x, y , then f is the constant function.
- 9) ☐ T ☐ F If the unit tangent vector $\vec{T}(t)$ of a curve $\vec{r}(t)$ is always parallel to a plane Σ , then the curve is contained in a plane parallel to Σ .
- 10) ☐ T ☐ F If (x_0, y_0) is an extremum of $f(x, y)$ under the constraint $x^2 + y^2 = 1$, then the same point is an extremum of $10f(x, y)$ under the same constraint.
- 11) ☐ T ☐ F At a critical point (x_0, y_0) of a function $f(x, y)$ for which $f_{xx}(x_0, y_0) > 0$, the critical point is always a minimum.
- 12) ☐ T ☐ F If a vector field $\vec{F}(x, y)$ is a gradient field, and C is a closed curve which looks like a figure 8, then $\int_C \vec{F} \cdot d\vec{r}$ is zero.
- 13) ☐ T ☐ F If C is part of a level curve of a function $f(x, y)$ and $\vec{F} = \langle f_x, f_y \rangle$ is the gradient field of f , then $\int_C \vec{F} \cdot d\vec{r} = 0$.
- 14) ☐ T ☐ F The divergence of the gradient vector field $\vec{F}(x, y, z) = \nabla f(x, y, z)$ is always the zero function.
- 15) ☐ T ☐ F The line integral of the vector field $\vec{F}(x, y, z) = \langle x, y, z \rangle$ along a line segment from $(0, 0, 0)$ to $(1, 1, 1)$ is $3/2$.
- 16) ☐ T ☐ F The area of a region G can be expressed as a line integral along its boundary.
- 17) ☐ T ☐ F The flux of the vector field $\vec{F}(x, y, z) = \langle x, y, -z \rangle$ through the boundary S of a solid ellipsoid E is equal to the volume the ellipsoid.
- 18) ☐ T ☐ F If $\vec{F}$ is a vector field in space and S is a torus surface, then the flux of $\text{curl}(\vec{F})$ through S is 0.
- 19) ☐ T ☐ F If the divergence and the curl of a vector field $\vec{F}$ are both zero, then it is a constant field.
- 20) ☐ T ☐ F For any function f , the curl of $\vec{F} = \text{grad}(f)$ is the zero field $\langle 0, 0, 0 \rangle$.

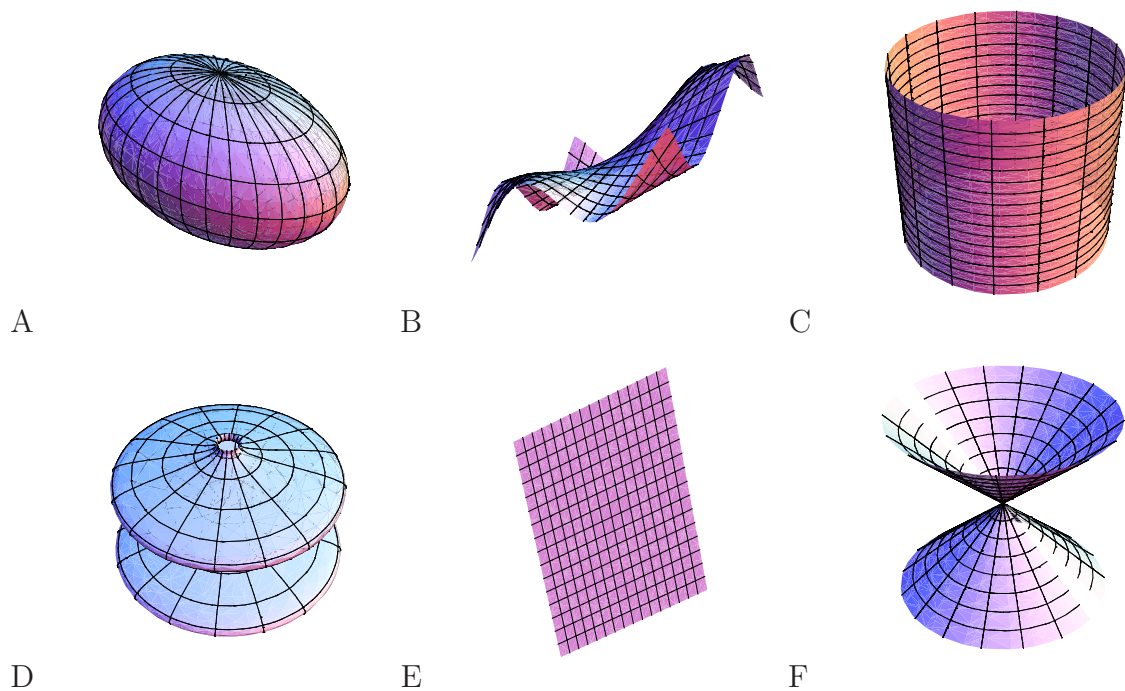
Problem 2) (10 points)

a) (4 points) Match the regions with the corresponding double integrals



Enter a,b,c,d	Function
	$\int_0^1 \int_{x/2}^x f(x, y) dy dx$
	$\int_0^1 \int_0^y f(x, y) dx dy$
	$\int_0^1 \int_0^{x/2} f(x, y) dy dx$
	$\int_0^1 \int_{y/2}^1 f(x, y) dx dy$

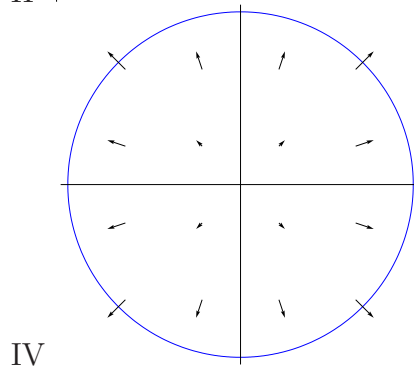
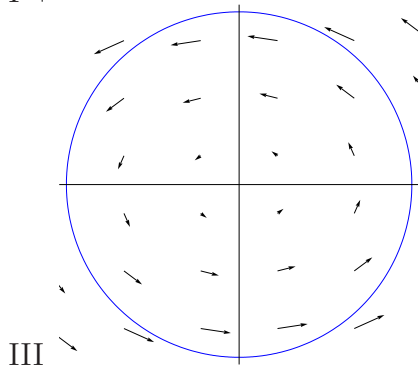
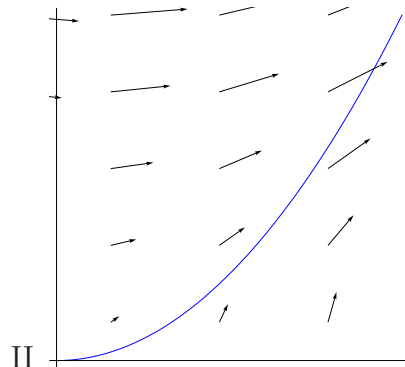
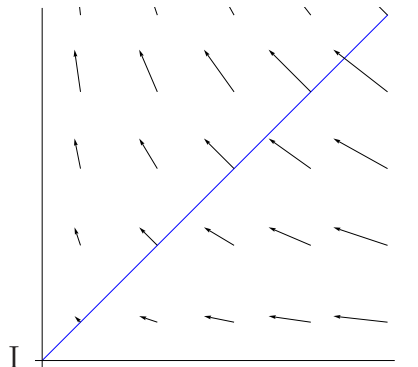
b) (6 points) Match the parametrized or implicit surfaces with their definitions



Enter A-F here	Function or parametrization
	$\vec{r}(u, v) = \langle \cos(u), \sin(u), v \rangle$
	$\vec{r}(u, v) = \langle u - v, u + 2v, 2u + 3v \rangle$
	$x^2 + y^2/3 + z^2/3 = 1$
	$\vec{r}(u, v) = \langle (\sin(v) + 1) \cos(u), (\sin(v) + 1) \sin(u), v \rangle$
	$z - x + \sin(xy) = 0$
	$x^2 + y^2 - z^2 = 0$

Problem 3) (10 points)

a) (4 points) Match the vector fields and curves with the corresponding line integral



Enter I,II,III,IV	Line integral
	$\int_0^{2\pi} \langle \cos(t), \sin(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle dt$
	$\int_0^{2\pi} \langle -t, t^2 \rangle \cdot \langle 1, 1 \rangle dt$
	$\int_0^{2\pi} \langle t^2, t \rangle \cdot \langle 1, 2t \rangle dt$
	$\int_0^{2\pi} \langle -3 \sin(t), 3 \cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle dt$

b) (6 points) Fill in from following choice: "arc length", "surface area", "chain rule", "volume of parallelepiped", "area of parallelogram", "line integral", "flux integral", "curvature".

Formula	Name of formula or rule or theorem
$\int \int_R \vec{r}_u \times \vec{r}_v du dv$	
$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}'(t)$	
$\int_a^b \vec{r}'(t) dt$	
$\frac{ \vec{r}'(t) \times \vec{r}''(t) }{ \vec{r}'(t) ^3}$	
$ \vec{u} \cdot (\vec{v} \times \vec{w}) $	
$\int_0^1 \int_0^1 \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) du dv$	

Problem 4) (10 points)

Given the line $x - 1 = y - 2 = z - 3$ and the point $P = (8, 4, 5)$. Find the equation

$$ax + by + cz = d$$

of the plane which contains the line and the point.

Problem 5) (10 points)

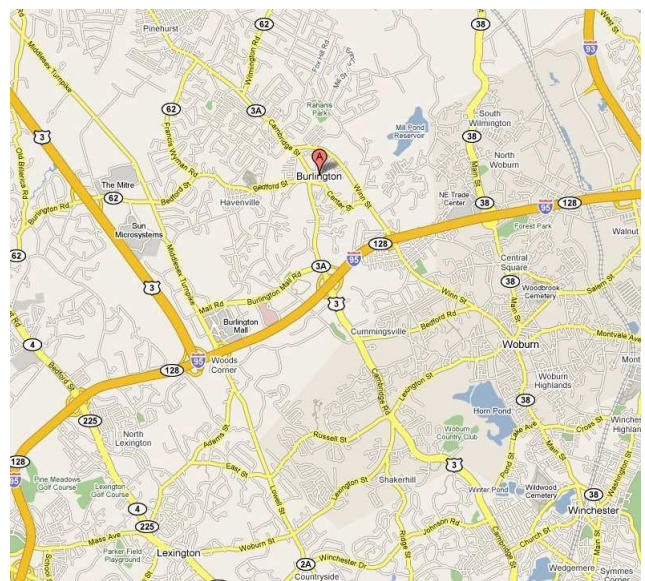
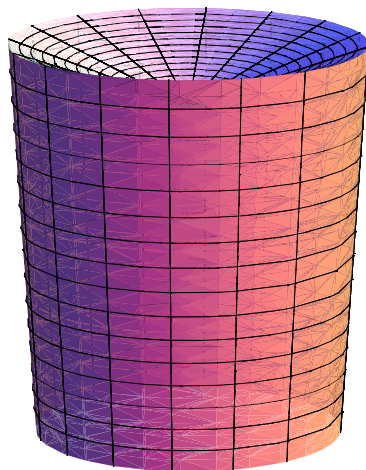
Find all the critical points of the function $f(x, y) = y^3 - 3y^2 + 4x + x^2 - 3$ and classify them by telling whether they are local maxima, local minima or saddle points.

Problem 6) (10 points)

The hyperbolic paraboloid $x^2 - y^2 - 3z = 0$ contains the point $P = (1, 1, 0)$ and the point $Q = (3, 0, 3)$. Find the tangent planes to the surface at P and Q and find a parametrization $\vec{r}(t)$ of the line of intersection of these two planes.

Problem 7) (10 points)

A water reservoir in Burlington, MA (the map to the right is centered there) is bounded by a solid cylinder $x^2 + y^2 \leq 1$. It has as the roof the cone $x^2 + y^2 = (z - 6)^2$ and is bounded from below by the xy -plane $z = 0$. What is the volume of the reservoir?



Problem 8) (10 points)

Find the maxima and minima of the function $f(x, y) = x^2 - y^2$ on the parabola $x + y^2 = 1$ using the Lagrange multiplier method.

Problem 9) (10 points)

Compute the surface area of the surface $\vec{r}(u, v) = \langle u^3, v^3, u^3 - v^3 \rangle$ parametrized so that (u, v) is in the unit disc.

Problem 10) (10 points)

Evaluate the following double integral

$$\int_0^2 \int_{x/2}^1 \cos(y^2) \, dy \, dx .$$

Problem 11) (10 points)

Find the value of the line integral

$$\int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) \, dt ,$$

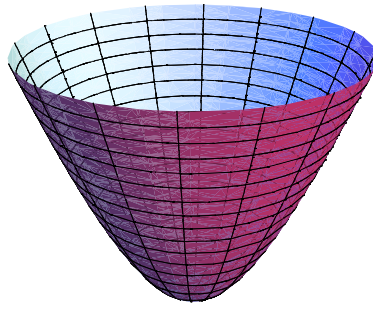
where $\vec{F}(x, y) = \langle y + \sin(\cos(x)), -2x \rangle$ and C is the boundary of the unit circle traversed in the counter clockwise direction.

Problem 12) (10 points)

Find the value of the flux integral

$$\int \int_S \text{curl}(\vec{F})(\vec{r}(u, v)) \cdot \vec{r}_u \times \vec{r}_v \, du \, dv$$

where $\vec{F}(x, y, z) = \langle -y, x, z \rangle$ and S is the part of the two-sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ which satisfies $1 < z < 2$ and which is oriented so that the normal vector points downwards on S .

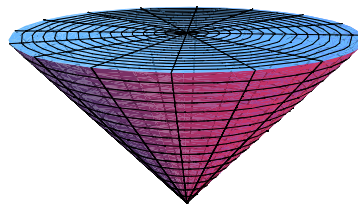


Problem 13) (10 points)

Let E be the solid which is bounded on the side by the cone $S_1 : x^2 + y^2 = z^2, 0 < z < 1$ and on top by the disc $S_2 = x^2 + y^2 \leq 1, z = 1$. Let $\vec{F}(x, y, z) = \langle 1 + 4x, 2 - 5y, 3 + 2z \rangle$. Find the value of the flux integral

$$\int_S \vec{F}(\vec{r}(u, v)) \cdot \vec{r}_u \times \vec{r}_v \, du dv ,$$

where S is the union of the two surfaces S_1 and S_2 . The normal vector of S is oriented outwards on $S_1 \cup S_2$.



Problem 14) (10 points)

Find the volume of the solid piece of **cheese** bound by the cylinder $x^2 + y^2 = 1$, the planes $y - z = 0$ (bottom boundary) and $y + z = 0$ (top boundary) which is on the quadrant $x \geq 0$ and $y \leq 0$.

