7/28/2011 SECOND HOURLY **Maths 21a, O.Knill, Summer 2011**

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Provide details to all computations except for problems 1-3.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) True/False questions (20 points), no justifications needed

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F The partial differential equation $u_t = 5u_x$ is called a transport equation.

2) T F If $f(x,y)$ has a saddle point at $(0,0)$ and $g(x,y)$ has a saddle point at $(0,0)$ then the function $f(x,y) + g(x,y)$ has a saddle point at $(0,0)$.

3) T F A function $f(x,y)$ with a two local maxima also also a local minimum.

4) T F If $f_{xyy} = f_{yxx}$ holds everywhere in the plane then f is constant.

5) T F The tangent line to the function $x^2 + y^2 = 5$ at the point $(2,1)$ is $4x + 2y = 10$.

6) T F Fubini's theorem implies that for any function $f(x,y)$ of two variables, we have $\int_0^1 \int_3^4 f(x,y)\ dxdy = \int_0^1 \int_3^4 f(y,x)\ dxdy$.

7) T F If a function $f(x,y)$ has a local maximum at $(0,0)$ then $D_{\vec{v}} f(0,0) = 0$ for every unit vector v.

8) T F If $f(x,y) = \sqrt{x^2+y^2}$ then $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\ dxdy$ is the area of the unit disc.

9) T F If $f(x,y,z) = c$ contains all the coordinate planes, then $(0,0,0)$ is a critical point of $f(x,y,z)$.

10) T F If $f_{xx}(x_0,y_0)$ is positive at a critical point (x_0,y_0) then f has a local minimum at (x_0,y_0).

11) T F If $\nabla g = 0$, then f has is a local maximum or minimum of f under the constraint $g = 0$.

12) T F By writing $\nabla f = \langle a, b \rangle$, we see the inequality $-\sqrt{a^2+b^2} \leq a\cos(\theta) + b\sin(\theta) \leq \sqrt{a^2+b^2}$.

13) T F If $f(x,y,z)$ has a critical point under the constraint $g(x,y,z) = 0$, then we have infinitely many solutions. Only if we add two constraints like $g(x,y,z) = 0, h(x,y,z) = 0$ we get finitely many solutions.

14) T F If the entropy $-x\log(x) - y\log(y) - z\log(z) = S(x,y,z)$ is maximal under the constraint $x + y + z = 1$, then $x = y = z$.

15) T F The equation $u_{tt} = u^2 u_t$ is a partial differential equation, not an ordinary differential equation.

16) T F The integral $\int\int_R |\nabla f(x,y)|\ dxdy$ is equal to the surface area of the graph of f above the region R.

17) T F Fubini's theorem assures that $\int_0^1 \int_0^\theta r dr d\theta = \int_0^{\pi/2} \int_0^r r\ d\theta dr$.

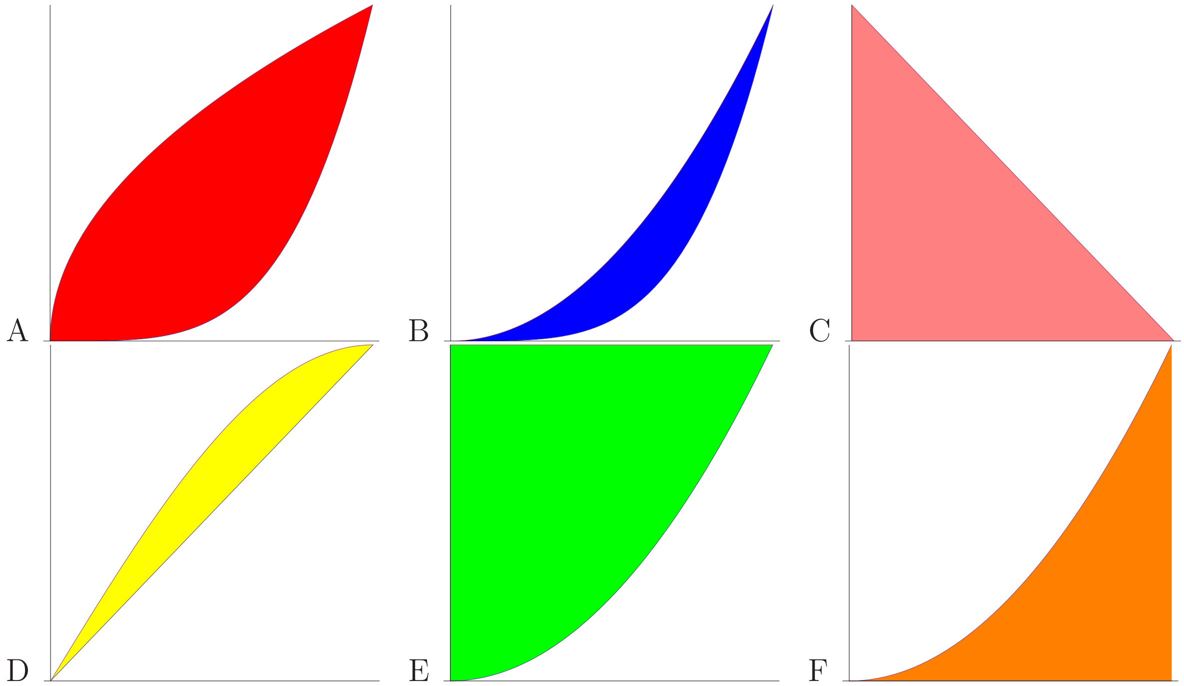
18) T F The region $x^2 + y^2 \leq 1, x > 0$ is both a type I and type II region.

19) T F The partial differential equation $u_t u_x + u_{xx} = 0$ is called the Burgers equation.

20) T F Let (x_0,y_0) be a maximum of $f(x,y)$ under the constraint $g(x,y) = 0$. Then$|\nabla f(x_0,y_0) \cdot \nabla g(x_0,y_0)| = |\nabla f(x_0,y_0)| \cdot |\nabla g(x_0,y_0)|$.

Problem 2) (10 points) No justifications are needed

a) (6 points) Match the regions with the double integrals. If none applies, put O.



Enter A-F	Integral of Function $f(x,y)$
	$\int_0^1 \int_x^{\sin(\pi x/2)} f(x,y)\ dydx$
	$\int_0^1 \int_0^{x^2} f(x,y)\ dxdy$
	$\int_0^1 \int_{x^4}^{\sqrt{x}} f(x,y)\ dydx$

Enter A-F	Integral of Function $f(x,y)$
	$\int_0^1 \int_{x^2}^1 f(r,\theta)r\ drd\theta$
	$\int_0^1 \int_{x^4}^{x^2} f(x,y)\ dxdy$
	$\int_0^1 \int_0^{1-x} f(x,y)\ dydx$

b) (4 points) Various concepts are related to the gradient. Match them. Each of the formulas $A - F$ match exactly one spot:

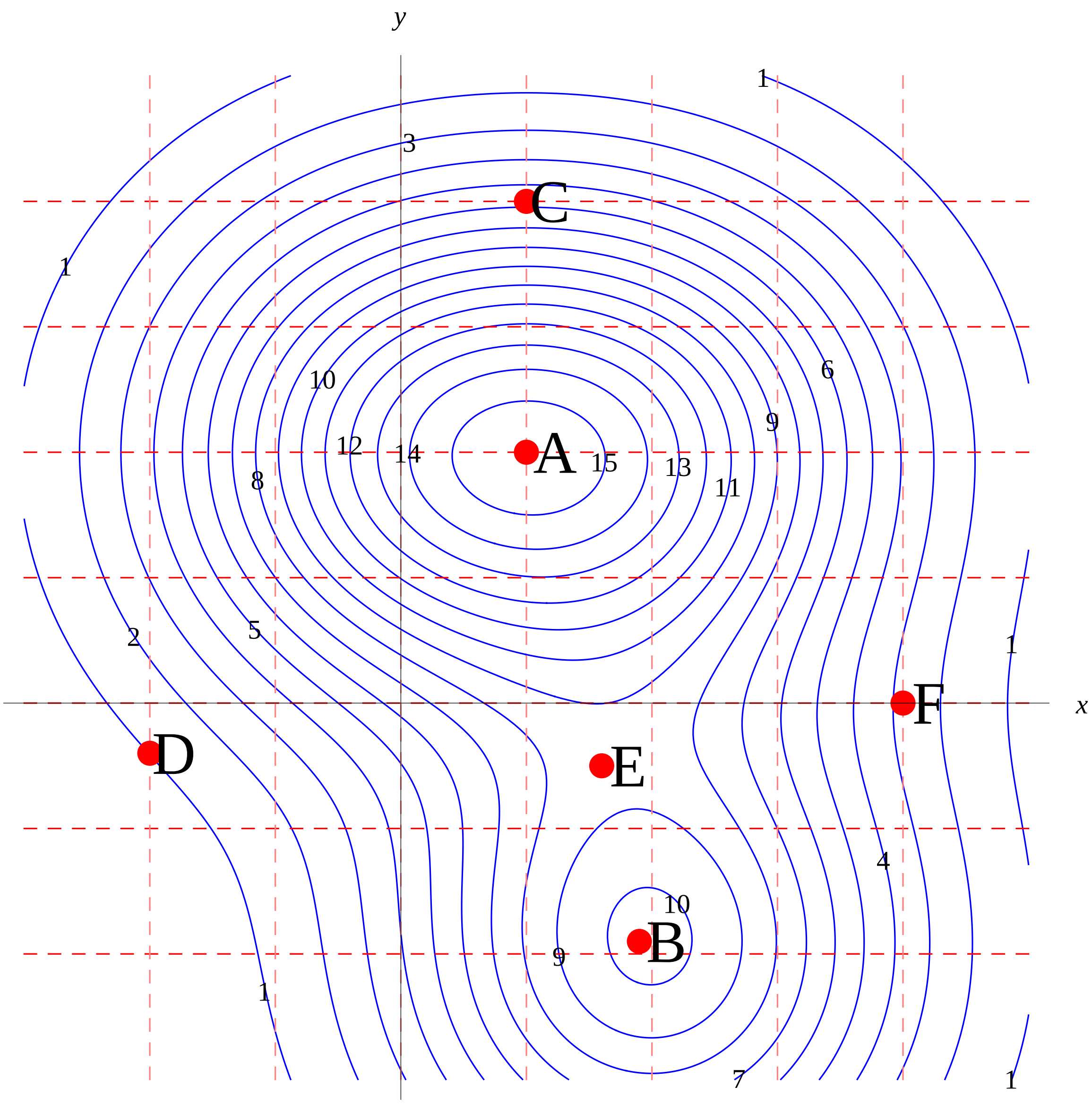
Concept related to gradient	Enter A-F
critical point	
estimation	
directional derivative	
linearization	
tangent space	
chain rule	

	Formula or notation
A	$L(x,y) = L(x_0,y_0)$
B	$f(x+a), y+b) \sim L(x+a, y+b)$
C	$\nabla f(x,y) = (0,0)$
D	$D_{\vec{v}} f$
E	$\frac{d}{dt} f(\vec{r}(t))$
F	$L = f(x_0,y_0) + \nabla f(x_0,y_0) \cdot \langle x - x_0, y - y_0 \rangle$

Problem 3) (10 points) (no justifications are needed)

A function $f(x, y)$ of two variables is shown as a contour map. You see equally spaced level curves. Check whatever applies. In each line there could be multiple points to be checked or zero checks. In the last question, there is only one check.

	A	B	C	D	E	F
points which are local maxima						
points which are local minima						
points which are saddle points						
points, where $f_x > 0$						
points, where $f_x < 0$						
points, where $f_y < 0$						
points, where $f_y > 0$						
global maxima on the region shown						
point among $\{A, B, C, D, E, F\}$ with maximal $\|\nabla f\|$						



Problem 4) (10 points)

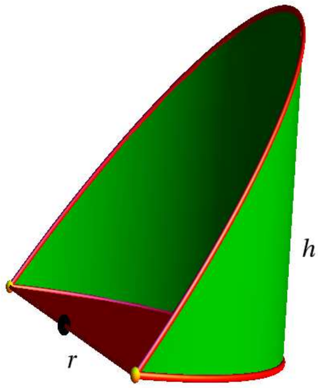
The **hoof of Archimedes** has the volume

$$V(r,h) = 2r^2h/3 .$$

The surface area without the top is

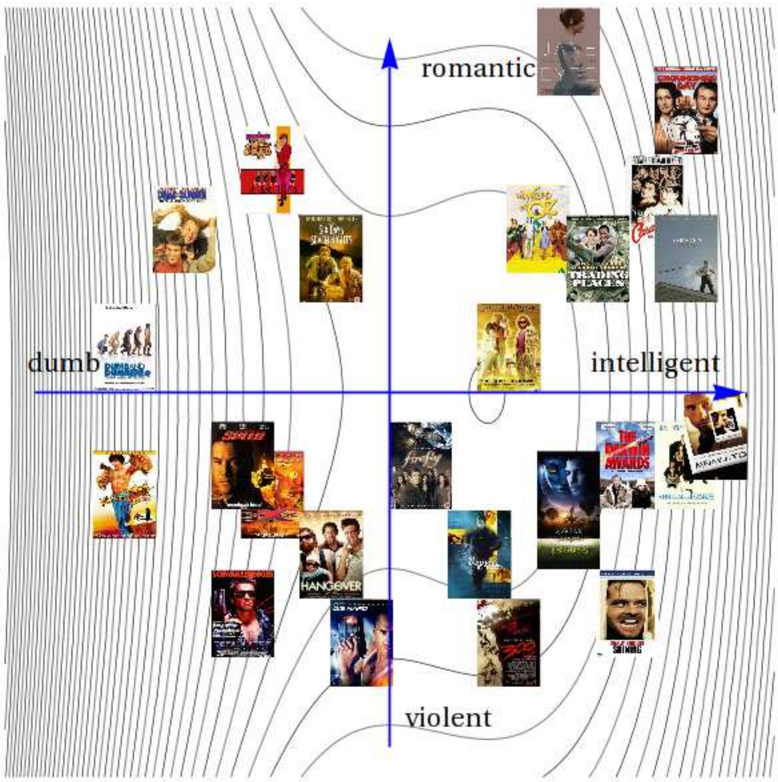
$$A(r,h) = \pi r^2/2 + 2rh .$$

Find the hoof which has minimal volume $V(r,h)$ under fixed surface area $A(r,h) = \pi/2$.



Problem 5) (10 points)

Companies like **Netflix** or **Hulu** track movie preferences. One can visualize preferences on parameter spaces. Let the x axes be the **intelligence - emotion** plane. Based on viewing habits, the service decides what you want to see. Your profile is a function $f(x,y)$. Maximizing this function allows the company to pick movies for you. Assume that your user profile is the function $f(x,y) = -2x^3 + 9x^2 - 12x - y^2$. Find and classify all the critical points and especially find the local maxima of f.

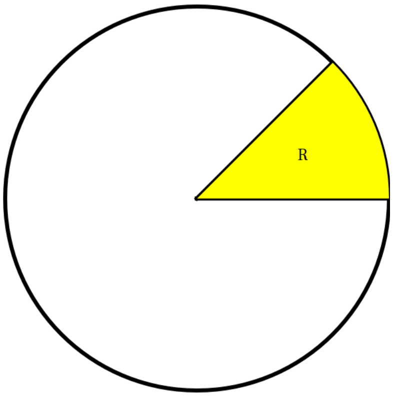


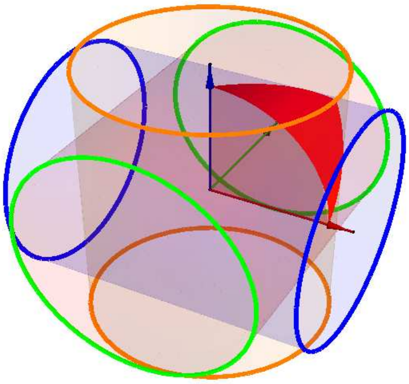
Problem 6) (10 points)

Find the surface area of the graph of $f(x,y) = \sqrt{1-x^2}$ over the sector region

$R = \{x^2 + y^2 \leq 1, y > 0, y \leq x\}$.

Remark: Multiply this result by 16 and you have computed the surface of the solid which is obtained by intersecting the three cylinders $x^2+y^2 \leq 1, x^2+z^2 \leq 1$ $y^2+z^2 \leq 1$.





Problem 7) (10 points)

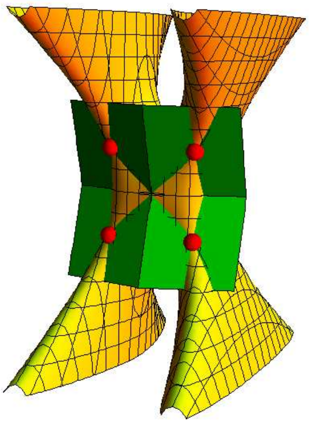
Estimate $f(0.999, 0.01) = 0.999^{10} \cdot (1 + \sin(0.01))$ by linear approximation.

Problem 8) (10 points)

Part of a "transformer" action figure given by the surface

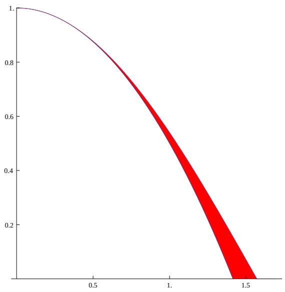
$$-2x^2y^2 + x^6 + y^2 + 3z^2 = 3 .$$

In order to put some armor around the body, we have to find the tangent planes at the "arm pits" $(1,-1,-1), (1,1,-1)$ and "kidney regions" $(-1,-1,-1), (-1,1,-1)$.
Find the tangent plane at the left armpit $(1,-1,-1)$.



Problem 9) (10 points)

a) (5 points) Since the graph of $1-x^2/2$ is always below the graph of $y = \cos(x)$, we can ask about the area of the "eye lash" region between the two graphs and the x axes. Write down a double integral which gives the area of the region above the positive x axes. You do not have to evaluate the integral in this part of the problem.



b) (5 points) Find the "mini" moment of inertia

$$\int\int_R \sqrt{x^2+y^2}\, dxdy$$

of the region R bound by the **Archimedean spiral** given in polar coordinates as $r \leq \theta$ and the x axes. You **should** of course evaluate the integral in this part of the problem.